Abstract
Stockbridge dampers are used to reduce the intensity of vibrations on power lines in order to extend their life. During its operation, the damper’s messenger cable vibrates and bending stress is developed. This can affect the performance of the Stockbridge damper. The aim of this paper is to analyse a mathematical model describing the bending stress of the Stockbridge damper’s messenger cable near the clamped end. The reliability of the mathematical model is assessed using experimental data obtained from the forced response test conducted at the VRTC laboratory. The damper is vibrated in the same range of frequency as wind motion at constant (peak-peak) amplitude. Results show that the model can be used to predict the stress at resonance frequency. The analysed model represents a component of a non-destructive procedure that can be used to predict the remaining life of Stockbridge damper as well as evaluating their condition.

Keywords: Wind induced vibration, Power line, Stockbridge damper, Messenger Cable, Bending Stress

1. Introduction
Aeolian vibration is one of the causes of fatigue failure of power line conductors. The commonest method used to protect conductors against this type of failure is to dissipate the energy transferred by the wind to the power line by means of suitable dynamic dampers. The most commonly used basic damper has been designed for the first time by G H Stockbridge in 1924. It is called Stockbridge damper and consists of two masses rigidly attached to the ends of a double strand wire cable called messenger cable. The messenger cable is rigidly assembled to the power line by means of clamps. Figure 1 shows the different parts of a Stockbridge damper.
Design improvements which have been made on the Stockbridge damper after its first invention are available in the open literature [1].

During its energy dissipation operation, the damper’s messenger cable oscillates and bending stress is developed. This is one of the mechanical propriety which can be investigated to determine the performance of Stockbridge damper. The bending stress is important in the determination of the stress distribution on the damper messenger cable and life prediction of damper. A fundamental analysis of damper has been presented by Claren and Diana who considered the damper as a cantilever beam with two degrees of freedom [2]. Wagner et al. established the mathematical model of bending stress of message cable. In their experiments, the damper has been vibrated at constant displacement 1mm peak to peak in the frequency range from 2.5 to 35 Hz [3]. Lara-Lopez and Colin-Venegas did the same as Wagner et al. [3] but at 2.7 mm peak to peak displacement. This constant displacement has been determined by taking into account the first resonance frequency [4]. The energy dissipation capacity of dampers has been presented by Richardson [5]. He found the gap between the wind energy input on conductor and the energy lost by means of a graphical method. The gap between both curves represents the energy which must be supplied by a Stockbridge damper. Research has been made by Diana and coworkers to determine the force and the torque exerted between the conductor and the damper’s clamp [6]. The Stockbridge damper has been modified for this purpose and the impedance of damper has been determined under the same conditions as its operation on real conductor lines. Canalase et al. [7] proposed a general optimization method to maximize vibration attenuation and minimize damper’s cost. However, in all these previous studies, no experimental measurements have been undertaken to validate the derived models for the bending stress and establish their applicability limits [3],[4]. The present paper deals with both the mathematical modeling and the experimental investigation of the bending stress of the stockbridge damper’s messenger cable near the clamp. In the present work, the damper is vibrated at its end at constant displacement (peak to peak) in the range of wind induced vibration.

To determine the remaining life of a damper, a destructive method can be used. It consists of subjecting the device to successive vibration cycles until mechanical failure is observed. An alternative procedure would be the use of a reliable model which relates vibration frequency of the Stockbridge damper’s messenger cable and the stress on it to. Hence the importance of this work as it analyzes the reliability of such a model, derived by Wagner et al [3].

2 Model description

Stockbridge damper is attached on the power line, therefore it moves with it through the wind. Bending stress is developed on the messenger cable which vibrates. Two mathematical models for bending stress calculation have been presented in the literature [4], [5]. In this paper, the model presented by Wagner et al. is used due to the availability of the required properties of the damper.

During the vibration of Stockbridge damper, the movement of damper’s weight is characterized by two degrees of freedom: the alternative motion of translation, y, and the rotation, θ. The damper’s equation of motion is [8]:

\[
\begin{align*}
\frac{d^2y}{dt^2} + k_y y &= F(t), \\
\frac{d^2\theta}{dt^2} + k_\theta \theta &= M(t),
\end{align*}
\]
\[ M \ddot{x} + K \dot{x} + C x = K \dot{y} + C y \]  

(1)

M, K, C are respectively the mass, the damping, the stiffness matrix and y and x are the excitation of the damper clamp and displacement.

**2.1. Calculation of the axial acceleration and the angular acceleration**

![Diagram](image)

Figure 2. Two degrees of freedom for the damper

The displacement of damper’s weight is determined [2, 3] by vector \( q \) which is given by equation (2)

\[
q = \begin{bmatrix} x \\ \theta \end{bmatrix}
\]  

(2)

The component expressions of \( q \) are given by equations (3) and (4). Figure 2 shows the half part of damper with \( x \) and \( \theta \) which are functions of driving frequency [3]

\[
x = X e^{(\omega t + \beta)} = Y \left(1 + i \mu \right) \frac{1 + i \mu - S_2 \eta^2}{\left(1 + i \mu - \eta^2\right)} \frac{\left(1 + i \mu - a \eta^2\right)}{\left(1 + i \mu - a \eta^2\right)}
\]  

(3)

\[
\theta = \Theta e^{(\omega t + \phi)} = \frac{Y}{L_2} \left(1 + i \mu \right) \frac{- a \eta^2 S_2}{\left(1 + i \mu - \eta^2\right)} \frac{\left(1 + i \mu - a \eta^2\right)}{\left(1 + i \mu - a \eta^2\right)}
\]  

(4)

With:

\( \omega = 2 \pi f \) is the angular frequency

\( t \) is the time.

\( \beta \) and \( \phi \) are phase angles.

\( r \) is the constant displacement of the damper’s clamp (peak-to-peak).

\[
\mu = \frac{\delta}{\pi}
\]  

(6)

\[
\eta = \frac{f}{f_1}
\]  

(7)
In the above three expressions, \( \delta \) is the logarithmic decrement of damper’s messenger cable whereas, \( f, f_1, f_2 \) represent the driving frequency, the first resonant frequency and the second resonant frequency.

In the expressions (3) and (4), \( S_1 \) and \( S_2 \) are respectively the first and the second dimensionless stiffnesses. They are given by:

\[
S_1 = \frac{\alpha (1 + \rho) c_{11}}{m \rho L_2 f_1^2}
\]

\[
S_2 = \frac{c_{12} - L_2 c_{11}}{m \rho L_2 f_2^2}
\]

Where:

\[
\rho = \left( \frac{r}{L_2} \right)^2
\]

is the square ratio between the radius of gyration, \( r \), and the distance between the center of gravity and the damper attachment point, \( L_2 \). The elements of the stiffness matrix \( C \) are:

\[
c_{11} = 4 \left( \frac{3 EI}{L^3} \right)
\]

\[
c_{12} = c_{21} = 2 L \left( \frac{3 EI}{L^3} \right)
\]

\[
c_{22} = \frac{4 L^2}{3} \left( \frac{3 EI}{L^3} \right)
\]

The axial acceleration \( \ddot{x} \) and the angular acceleration \( \ddot{\theta} \) are obtained respectively by second derivative of the axial and angular displacement

\[
\ddot{x} = -\omega^2 X e^{(\omega t, \rho})
\]

whose the argument is given by : \( |\ddot{x}| = \omega^2 x \)
\[ \dot{\theta} = -\sigma^2 \Theta e^{(\sigma + \varphi)} \]  
(17)

Whose argument is given by: \[ |\dot{\theta}| = \sigma^2 \theta \]  
(18)

### 2.2. Bending moment

![Figure 3. Moments and forces acting on the damper’s messenger](image)

The bending moment of the messenger cable \( M_B \) at the clamp attachment point of the cable is calculated by using the sum of all torques at the attachment damper. Applying this principle to **figure 3**, the expression of bending moment is:

\[ M_B = M_G - L_1 F_G \]  
(19)

\( M_G, L_1 \) and \( F_G \) are respectively the bending moment from the damper’s mass acting at its center of gravity, the length between the clamp attachment point, \( B \), and the center of gravity, \( G \), of the damper’s mass and the force at the clamp through the center of gravity.

With \( \dot{\theta} \) calculated in equation (18) and \( J_G \) as a known, the moment \( M_G \) may be calculated by using the principal of moment of inertia. Its expression is:

\[ M_G = J_G |\dot{\theta}| \]  
(20)

where:

\( J_G \) is the damper’s mass moment of inertia.

\( |\dot{\theta}| \) is the argument of the damper’s mass angular acceleration.
The force \( F_\varphi \) may be determined by using the Newton law applies on the damper’s mass. Referring to figure 2, the expression of \( F_\varphi \) is given by:

\[
F_\varphi = m \left( |\dot{x}| + L_0 |\dot{\varphi}| \right)
\]  \hspace{1cm} (21)

with:

- \( m \) is the damper’s mass.
- \( |\dot{x}| \) is the argument of the damper’s mass axial acceleration.
- \( |\dot{\varphi}| \) is the argument of the damper’s mass angular acceleration.

2.3. The dynamic bending stress

The dynamic bending stress of the messenger cable at the attachment point B as function of frequency \( f \) is given by:

\[
\sigma(f) = \frac{|M_s|}{W}
\]  \hspace{1cm} (22)

where:

- \( M_s \) is the bending moment,
- \( W \) is the modulus of the cross sectional area of the messenger cable.

As the messenger cable is the stranded cable, the modulus of the cross sectional area has for expression:

\[
W = \kappa \frac{\pi D^3}{32}
\]  \hspace{1cm} (23)

With :

- \( \kappa \) is the factor which takes into account the fact that the messenger cable is not a circle but a stranded cable (\( \kappa = \frac{2}{3} \) for the messenger cable with 7 wires).
- \( D \) is the overall diameter of the messenger cable.
3. Experimental procedure

Figure 3 shows the experimental setup used in this work to measure the bending stress of messenger cable as a function of frequency. The Stockbridge damper was mounted on the electrodynamic shaker (TIRA, Type 56263LS). The damper was vibrated in the range of wind induced vibration (sweep) at constant displacement (1 mm) peak to peak. The frequency ranged from 5 to 60 Hz as determined by Strouhal formula [1]. The characteristics of the tested damper are given in Table 1.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Symbol/ Unity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of messenger cable</td>
<td>L [mm]</td>
<td>115</td>
</tr>
<tr>
<td>Length between B and G</td>
<td>L_1 [mm]</td>
<td>65</td>
</tr>
<tr>
<td>Length between A and G</td>
<td>L_2 [mm]</td>
<td>50</td>
</tr>
<tr>
<td>Mass of weight</td>
<td>m [kg]</td>
<td>20533</td>
</tr>
<tr>
<td>Diameter of messenger cable</td>
<td>D [mm]</td>
<td>14</td>
</tr>
<tr>
<td>Moment of inertia about the center mass</td>
<td>J_G [kgm^2]</td>
<td>0.018</td>
</tr>
<tr>
<td>Logarithmic decrement</td>
<td>( \delta )</td>
<td>0.169</td>
</tr>
</tbody>
</table>

Five accelerometers (Bruel&Kjaer, Delta Tron, Type 4508002) were necessary during the experiments to measure the resonance frequency and the displacement. Four accelerometers, two on each Stockbridge damper’s weight were used and one on the shaker base. Strain gauges (half-bridge circuits) [9] were put both sides on the messenger cable near the clamped end.

The fifth accelerometer was put on the shaker base to control the displacement (1 mm) during the experiment. The two accelerometers permit the measurement two modes of vibrations of the Stockbridge damper (the first and the second mode) during the sweep. The strain data of the
messenger cable were conditioned by a MP55 strain gauge amplifier and data from accelerometers was captured by a PUMA control systems software.

All accelerometers were calibrated by means of a calibration exciter type 4294 (Bruel&Kjaer). A calibration unit, type K 3607 (HBM) was used to calibrate the strain gauges. Through instrument calibration, deviations were ± 10μm/m respectively for accelerometers and for strain gauges. The curve generated by the PUMA software was compared with the one from the mathematical model which was generated through MATLAB code (Figure 3). Basic characteristics of damper tested are summarized in the table 1. The length of messenger cable was measured by using a ruler. L1, L2, m, J, and G were determined by constructing a model using the Solid Edge Package. The messenger cable’s Young modulus determined by means of Solid Edge after determining the density of the messenger cable. The vibration has been made on the stockbridge damper to determine the logarithmic constant decrement, δ.

4. Results and Discussion

Resonance frequency and strain data were collected when the damper was vibrating in the frequency range of wind induced vibration. Table 2 shows the correlation of measurement from strain gauges and accelerometer, the average error is 0.6 % for the first resonance frequency and 1 % for the second resonance frequency. The comparison of curve from mathematical model and experiment is presented in figure 3. These values could be used as data to determine the life expectance of Stockbridge dampers on the basis of the relationship between resonance frequency changes and the fatigue history of the cable [10], [11].

<table>
<thead>
<tr>
<th>Measurement From</th>
<th>1st Resonance frequency [Hz]</th>
<th>2nd Resonance frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer</td>
<td>8.63</td>
<td>27.9</td>
</tr>
<tr>
<td>Strain Guage</td>
<td>8.69</td>
<td>28.23</td>
</tr>
</tbody>
</table>

In this work, experimental curve of stress as function of frequency at constant peak to peak displacements has been generated using the PUMA control systems software. Figure 3 presents a comparison between experimental data at 1 mm peak to peak and predictions using the model derived by Wagner et al.

The graph shape obtained from the mathematical model is similar to that from experiment as they present two peaks each. Peaks correspond to the two resonances frequencies of the symmetrical damper. The first peak corresponds to the first resonance which is the translation movement of the damper mass. The second frequency is corresponded to the second peaks. At the second resonance frequency, the mass of damper has an alternating rotational motion. The first and the second resonance frequency from experiment are closer to those from the mathematical model and the average error is below 1%. Stresses at the resonances frequency are also close in the frequency range from 5Hz to 14Hz. The mathematical model agrees with the experiment where the average error is equal to 5%. Therefore the messenger cable can be assimilated to the tube as all wirevibrate at the same moment. In addition, the less slipping
between wires make a gap between graphs in the same frequency range. From 14Hz to 27Hz, the gap between curves is created by the interactions of the two modes of damper vibration. Data from 14Hz up to 60Hz present a gap between the experiment and the model. However the curve from the experiment is the rotation of the curve from the model with the peak as the rotation centre. On the other hand, the strain gauges must glue exactly at the clamp end of messenger in order to reduce the gap between both curves.

![Bending stress of messenger cable versus frequency for constant displacement Y= 1mm](image)

**Figure 3.** Bending stress of messenger cable versus frequency for constant displacement Y= 1mm

### 5. Conclusion

In this work, we present the experiment set up of bending stress of messenger cable near the clamped end point. Data from experiments agreed with that from theory at resonance frequency also the related bending stress. Unfortunately, the friction between wires, the stiffness and the non-homogeneity of Stockbridge damper messenger cable are the main reason of the gap between the two graphs. Data from experiment confirm that the damper as two degrees of freedom as we have two peaks on the graphs. The same procedure may be used to determine the bending stress of the messenger cable near the weight point. The change of resonance frequency as well as the related bending stress of messenger cable can been used to make the life assessment of Stockbridge damper. This model will be used as part of non-destructive test to assess the remaining life of the damper.
References