A Novel Crack Location Method Based on the Reflection Coefficients of Guided Waves

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Abstract
As the guided waves can be partially reflected by the cracks in the waveguides, the reflection characteristic may be adopted to indicate the crack location. This paper proposes a novel crack location method by using measured wave amplitudes at two points between which a crack may exist. In this method, the reflection coefficients can be experimentally obtained by a generalized discontinuity approach without the information of the crack location. Meanwhile, the reflection coefficients can also be determined by a predicted model with respect to the possible crack location in the detection region. Then the crack location involved in the reflection coefficient model can be identified by using the measured reflection coefficients. The identification process is based on the principle that the error between the experimental reflection coefficients and the predicted ones reaches the minimum in the least-square sense. In order to evaluate effectiveness of the crack location method, a 25-meter 60-kg/m rail containing a transverse-type crack in the rail head was simulated by using the spectral super-element method. For practical considerations, different boundary conditions and measurement noise were investigated. Simulation results show that this location method is insensitive to the boundary conditions of the waveguide and robust against the measurement noise. An experiment was carried out on an infinite 60-kg/m rail. The location of the transverse crack with 20 mm depth in the rail head was detected accurately with an error of 0.5%.

Keywords: Crack, location, guided wave, reflection coefficients, rail

1. Introduction
Guided waves can propagate long distance along the length of waveguides with a little amplitude loss in contrast with conventional ultrasonic techniques. Therefore, the guided wave method becomes promising in the long-distance online crack detection of waveguides, such as rails and pipes [1-6]. Currently, the pulse-echo method is one of the most popular guided wave methods [7], which detects the crack by sending a beam of ultrasonic waves into the waveguide and measuring the time interval between the forward and returned waves [8]. Then the distance between the crack and the measuring point can be evaluated by multiplying the arrival time and the wave speed. However, the crack location could not be detected accurately since the speed of the guided waves is difficult to be obtained precisely due to uncertainty of the waveguide parameters. Additionally, only the waves with the low dispersion characteristic can be employed in the pulse-echo method, which constrains the application of this method in the long-distance crack detection, since the waves with low dispersion may not propagate with a low decay rate.

This paper proposes a novel crack location method based on the reflection coefficients of a set of guided waves which may be partially reflected by a crack. Compared with the pulse-echo method, this reflection coefficient method is more robust against waveguide uncertainty and measurement noise, since plenty of waves involving crack information are employed in the crack location. Furthermore, since the detecting guided waves do not require the low dispersive property, more low-decay-rate guided waves become candidates for the crack detection and therefore the long-distance online monitor of cracks is possible to be realized.

In this paper, a reflection coefficient model of the crack is developed in Sections 2.1. Meanwhile, the reflection coefficients are experimentally obtained by a generalized discontinuity method in Sections 2.2. Based on the reflection coefficient model and the
measured reflection coefficients, a crack location method is proposed in Section 3. In order to evaluate effectiveness of the crack location method, a 25-meter rail section containing a transverse-type crack was simulated by using the spectral super-element method in Section 4. For practical considerations, different boundary conditions and measurement noise of the wave amplitudes and wavenumber were investigated in Sections 4 and 5. Finally in Section 6, an experiment was carried out on an infinite 60-kg/m rail to validate the simulated results.

2. Reflection coefficients at a crack

2.1 Reflection coefficient model based on crack location

In Figure 1, the waveguides $\alpha$ and $\beta$ lying along the $x$-axis are connected by the discontinuity $D$. Under the excitation of the vibration source at $\Gamma$, the positive- and negative-going (only for the finite waveguides) waves are incident upon $D$ and then produce the propagating reflected and transmitted waves on both sides of $D$.

$A_\alpha$ and $A_\beta$ are the wave amplitudes at $x = x_1$ and $x = x_2$, respectively, and the superscripts + and − denote the positive- and negative-going waves, respectively. Then, the wave amplitudes at $D$, $x = x_1$ and $x = x_2$ can be related by

$$
\begin{bmatrix}
A^-_{D\alpha} \\
A^+_{D\alpha} \\
A^-_{D\beta} \\
A^+_{D\beta}
\end{bmatrix} =
\begin{bmatrix}
e^{-ik(x_0-x_1)} & 0 \\
0 & e^{ik(x_0-x_1)} \\
e^{ik(x_2-x_0)} & 0 \\
0 & e^{-ik(x_2-x_0)}
\end{bmatrix}
\begin{bmatrix}
A^-_\alpha \\
A^+_\alpha \\
A^-_\beta \\
A^+_\beta
\end{bmatrix}
$$

(1)

where $A_{D\alpha}$ and $A_{D\beta}$ are the wave amplitudes at $D$ on the sides of waveguides $\alpha$ and $\beta$, respectively, and $x_D$ is the location of the discontinuity. $k$ represents the wavenumber corresponding to the wave.

The wave amplitudes at $D$ can be related by

$$
\begin{bmatrix}
A^-_{D\alpha} \\
A^+_{D\alpha} \\
A^-_{D\beta} \\
A^+_{D\beta}
\end{bmatrix} =
\begin{bmatrix}
r_{\alpha\alpha} & t_{\alpha\alpha} \\
t_{\alpha\beta} & r_{\beta\beta}
\end{bmatrix}
\begin{bmatrix}
A^+_\alpha \\
A^-_{\alpha} \\
A^+_\beta \\
A^-_{\beta}
\end{bmatrix}
$$

(2)

where $r$ and $t$ represent the wave amplitude reflection and transmission coefficients of the propagating waves, respectively, and the subscript $gh$ ($g, h: \alpha, \beta$) represents the wave propagates from waveguide $g$ to waveguide $h$.

Considering $D$ as a physically symmetric discontinuity, the relationship between $r_{\alpha\alpha}$ and $r_{\beta\beta}$ can be written as

$$
r_{\alpha\alpha} = r_{\beta\beta} = r_p
$$

(3)

and the relationship between $t_{\alpha\alpha}$ and $t_{\beta\beta}$ can be written as
Substitution of Eqs. (1), (3) and (4) into Eq. (2) gives the reflection coefficient as

\[
    r_p = \frac{A^+_{a1} - A^+_{b1}}{e^{-2ik_{x1}}A^+_{a2} - e^{-2ik_{x2}(L_{D2}-L_{D1})}A^+_{b2}}
\]

(5)

where \( L = x_2 - x_1 \), and \( L_D = x_D - x_1 \).

### 2.2 Reflection coefficients based on the generalized discontinuity

The reflection coefficients can also be determined by the dynamic properties of the discontinuity but not by the discontinuity location \( L_D \). Therefore, a generalized discontinuity method is developed here to calculate the reflection coefficients.

Substitution of Eq. (1) into Eq. (2) gives

\[
    \begin{bmatrix}
        A^-_a \\
        A^-_b
    \end{bmatrix} =
    \begin{bmatrix}
        R_{aa} & T \\
        T & R_{bb}
    \end{bmatrix}
    \begin{bmatrix}
        A^+_a \\
        A^+_b
    \end{bmatrix}
\]

(6)

where the generalized reflection coefficients \( R_{aa} \), \( R_{bb} \) and the generalized transmission coefficients \( T \) corresponding the generalized discontinuity \( D_G \) (shown in Figure 2) are

\[
    R_{aa} = e^{-2ik_{x1}}r_p
\]

\[
    R_{bb} = e^{-2ik_{x2}(L_{D2}-L_{D1})}r_p
\]

\[
    T = e^{-ik_{x1}L}
\]

(7)

![Figure 2 Wave reflection and transmission at the generalized discontinuity \( D_G \).](image)

The generalized reflection and transmission coefficients can then be obtained by substituting two groups of wave amplitudes, measured at \( x = x_1 \) and \( x = x_2 \), into Eq. (6). Thus, the reflection coefficients \( r \) of the discontinuity \( D \) can be given as

\[
    r = \left( R_{aa} R_{bb} \right)^{1/2} e^{i\theta_{1,2}}
\]

\[
    = \frac{\left( A^-_{a1} A^-_{b2} - A^+_{a2} A^+_{b1} \right) \left( A^-_{a1} A^-_{b2} - A^+_{a2} A^+_{b1} \right)^{1/2}}{A^-_{a1} A^-_{b2} - A^+_{a2} A^+_{b1}} e^{i\theta_{1,2}}
\]

(8)

where the subscripts 1 and 2 of the wave amplitude \( A \) present the wave amplitudes measured in the first group and second group, respectively.
3. Crack location identification method

3.1 Location factor

In order to identify the location of the discontinuity, a dimensionless discontinuity location factor is proposed to describe probability of the identified crack location. Then the predicted reflection coefficients can be obtained by substituting a predicted discontinuity location \( L_p \) into Eq. (5) instead of \( L_D \).

The reflection coefficients can also be calculated by Eq. (8) with two groups of wave amplitudes as the measured reflection coefficients \( r_m \). Then the second moment norm of the difference between the amplitudes of \( r_p \) and \( r_m \) can be presented as

\[
F = \| \text{abs}(r_m) - \text{abs}(r_p) \|_2
\]  

(9)

The value of \( F \) will be changed when the predicted discontinuity location moves between \( x = x_1 \) and \( x = x_2 \). Note \( L_{pi} (i = 1, 2, \ldots, N) \) the distance between \( x = x_1 \) and the predicted discontinuity location, these \( N \) different locations distributed evenly along the waveguide between \( x = x_1 \) and \( x = x_2 \), where \( L_{p1} = 0 \) corresponds \( x = x_1 \) and \( L_{PN} = L \) corresponds \( x = x_2 \). Substituting \( L_{pi} (i = 1, 2, \ldots, N) \) into Eq. (9) yields the corresponding second moment norm \( F_i \) (\( i = 1, 2, \ldots, N \)).

The average of these second moment norms can be written as

\[
F_A = \frac{\sum_{i=1}^{N} F_i}{N}
\]  

(10)

Then the vector of the discontinuity location factor can be obtained based on \( F_A \) and \( F_i (i = 1, 2, \ldots, N) \) as

\[
C = \frac{F_A}{F}
\]  

(11)

where the vector of the second moment norm is represented as \( F = [F_1, F_2, \ldots, F_N] \).

In order to describe the ability to detect the discontinuity, the location index is defined as the location factor corresponding to the identified location.

3.2 Location identification method

The flow chart of the discontinuity location identification is show in Figure 3. Two sensor groups are respectively arranged on each end of the detection range. The wave amplitudes at the center of the sensor groups (\( x = x_1 \) and \( x = x_2 \)) are measured twice by exciting the rail at a point on the left side of the first sensor group and at a point on the right side of the second sensor group, respectively. Substitution of these wave amplitudes into Eq. (8) yields the measured reflection coefficients \( r_m \). In the case of \( |r_m|^2 = 1 \) for all the interested frequencies (except for some frequencies where the reflection coefficients are contaminated by measurement noise), there is no discontinuity in the area of the detection range. Otherwise, some discontinuities do exist in the detection range. Cases of only one discontinuity in the detection range are herein considered.

On the other hand, the predicted reflection coefficients can be obtained by using the wave amplitudes measured in one of the measurement and the predicted distance vector \( L_p = [L_{p1}, L_{p2}, \ldots, L_{PN}] \).

Substitution of the measured and predicted reflection coefficients into Eq. (9), with some arrangements, yields the vector of the discontinuity location factor \( C \). The predicted distance...
$L_{P1}$ corresponding the global maximum of $C$, is the distance between the identified location of the discontinuity and the left side of the detection range ($x = x_1$). This identification method can be used to detect a single discontinuity on the waveguides between the two sensor groups.

4. Numerical results and discussion

In order to introduce the discontinuity location identification method, a crack on the rail head was carried out by using spectral super element method (SSEM). Four different boundary conditions, free-free, fixed-supported, simply supported and infinite, were introduced to investigate the effects of the boundary condition on the discontinuity location identification. A 25 m and an infinite 60 kg/m rail were investigated. The material properties of the 60 kg/m rail are the density $\rho = 7800 \text{ kg/m}^3$, Young's modulus of elasticity $E = 2 \times 10^{11} \text{ Pa}$, and Poisson's ratio $\mu = 0.3$. The dimension of the crack along the rail is 10 mm, and the depth of the crack is 20 mm. The SSE models of the finite and infinite rails with crack discontinuity are shown in Figure 4.

The flexural wave, which is easy to be measured, is selected as the guided wave to detect the rail crack here. The frequencies of the flexural wave for the crack detection are considered to be lower than the cut-off frequency $f_c$.

$$f_c = \sqrt{\frac{T_1 GB}{\rho I}} \frac{1}{2\pi}$$

where $T_1$ is the shear correction factor, $G$ is the shear modulus, $B$ is the sectional area of the waveguide, and $I$ is the second moment of area. The near-field waves beyond the cut-off frequency would convert into propagating flexural waves, which induces the wave amplitudes could not be obtained by the wave decomposition method. Therefore, the frequencies considered here are up to 6000 Hz while the cut-off frequency is about 7000 Hz.

The measurement layout is shown in Figure 5, where $x_0 = 0 \text{ m}$ (the left edge of the rail section), $\Gamma_1 = 0.5 \text{ m}$, $x_1 = 4.5 \text{ m}$, $x_D = 14.5 \text{ m}$, $x_2 = 20.5 \text{ m}$, and $\Gamma_2 = 24.5 \text{ m}$. The detection
range is between $x_1$ and $x_2$, the predicted distance vector is $L_P = 0: 0.001: 16$ m, and the distance between $x_1$ and the location of the crack is 10 m. The spacing $\Delta$ between the two sensors in each sensor group is 0.15 mm which is about a quarter of the wavelength of the interested frequency (3000 Hz) [9].

![Figure 5 Measurement layout for the crack location.](image)

Figure 5 Measurement layout for the crack location.

The amplitudes of the measured reflection coefficients for the crack in the rails with different boundary conditions, i.e. free-free, cantilevered, clamped-clamped and infinite, are shown in Figure 6. Figure 6 shows that the magnitudes of the measured reflection coefficients in the rails with different boundary conditions are similar to each other. This is because that the reflection coefficients of the discontinuity (crack) are the local properties of the structure regardless the boundary conditions.

The location factors for the rail crack can then be obtained by using the amplitude of the measured reflection coefficients and the predicted distance vector $L_P$. The location factor
curve for the crack in the free-free rail is shown in Figure 7, and the location factor curves for the crack in the rails with other three boundary conditions are similar to the one shown in Figure 7. The identified distances, corresponding to the global maximum of the location factor curve, for the cracks in the free-free, cantilevered, clamped-clamped and infinite rails are all 10000 mm, which are the same as the real crack locations. The corresponding location indexes are 16.72, 21.94, 25.31, 18.58 and 14.33, respectively. According to the simulation results, no concrete evident shows relations between the location results and the rail boundary conditions.

5. Effect of measurement noise on crack location
The above numerical cases do not consider the influence of noise on the identification of discontinuity location. However, in practice, measurement noise is unavoidable. In this section a noise model is introduced into the simulated transducer outputs and the wavenumber. Then the effects of measurement noise on the identification of discontinuity location are investigated by considering different noise levels. The free-free rail with crack mentioned in Section 4 is used to investigate the effects of measurement noise on the identification of discontinuity location, and the statistical property of noise is assumed to be Gaussian distribution.

5.1 Effect of the transducer outputs
The noise is added to the response data, $V_{11}$, $V_{21}$, $V_{31}$, $V_{41}$, $V_{12}$, $V_{22}$, $V_{32}$ and $V_{42}$ (see Figure 5), to estimate the measured response data contaminated by measurement noise. The mean of the noise is zero, and the Signal-to-Noise ratio is set from 20 dB to 80 dB with increment of 5 dB.

![Figure 7 Location factor of the crack in the free-free rail.](image)

![Figure 8 Location index and location error identified by using the transducer outputs contaminated by measurement noise of different levels: ▲, location error; △, location index.](image)
The location error and location index with different Signal-to-Noise ratios are shown in Figure 8. The results show that the crack location can be detected accurately when the Signal-to-Noise ratio of the transducer outputs is larger than 40 dB. The crack location is just at the middle of the detection region if the Signal-to-Noise ratio of the transducer outputs is less than 40 dB. Figure 8 shows that the location index becomes larger with the Signal-to-Noise ratio changing from 20 dB to 80 dB. That is to say, the smaller the noise is, the more easily the discontinuity can be identified.

5.2 Effect of the wavenumber
The noise is added to the wavenumber to investigate the effect on the crack location introduced by the error of the wavenumber. The mean of the noise is zero, and the Signal-to-Noise ratio is set from 20 dB to 80 dB with increment of 5 dB.

As mentioned in Section 5.1, Figure 9 shows that the location of the discontinuity can be identified accurately with max error equal to the interval of two adjacent predicted discontinuity locations when the Signal-to-Noise ratio is larger than 40 dB, and the influence of the noise can be ignored when the Signal-to-Noise ratio is larger than 80 dB. Figure 9 also shows that the smaller the noise is, the more easily the discontinuity can be identified.

6. Experimental results and discussion
The experimental rig for the crack detection is shown in Figure 10. The length of the rail section is 3 m, and the material properties of the rail are the same as those mentioned in Section 4. Two sandboxes were placed on each end of the beam to approximate the infinite boundary condition. The dimension of the crack along the rail length is about 1 mm, and the depth of the crack is about 20 mm. The rail was suspended by flexible strings, and the excitation for the rail was provided by a PCB K2004E01 electro-dynamic shaker at the center of the rail head. The excitation and response signals were collected by a PCB force sensor (208C01) and four PCB accelerometers (352C22), respectively. The accelerometers were mounted at the four points with bee wax, as shown in Figure 10, with \( L = 1 \text{ m}, \ L_0 = 0.4 \text{ m} \) and \( \Delta = 0.15 \text{ m} \). The response signals were acquired and analyzed by the dynamic signal analyzer (B&K Pulse 3560) with the frequency range from 0 Hz to 6000 Hz, the frequency resolution of 1.56 Hz and the ensemble average of 200.

The location factor curve calculated based on the reflection coefficients is shown in Figure 11. The identified location for the crack in this rail is 0.402 m with a nominal location error of 0.5%. The corresponding location index is 13.71, less than those obtained in Section 4.
7. Conclusions

In this paper, a predicted model of the reflection coefficient is proposed, and the corresponding reflection coefficients were experimentally obtained based on the generalized discontinuity method. Based on the predicted model and the experimental results of the reflection coefficients, a crack location method is developed. Simulation results show that the location errors are nearly zero, and no concrete evident indicates the relations between the detection results and the rail boundary conditions. Simulation results also show that the location errors are nearly zero when the Signal-to-Noise ratios of the transducer outputs and wavenumber are larger than 40 dB. Finally, an experiment was carried out on an infinite 60-kg/m rail containing a crack in the rail head with 20 mm depth, and the corresponding location error is 0.5%.

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