Ultrasound Imaging for Quantitative Measurement of Immersed Plastic Waste Particles

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Abstract

Ultrasound imaging techniques are proposed for measuring the shape and thickness of immersed waste particles (10-20 mm size) using a linear sensor array from a fixed position. For these purposes both the front and back surface of a particle needs to be reconstructed. Raw ultrasound pulse-echo and plane wave datasets and different imaging techniques were investigated in a case study using a single, generic test object. Phase shift migration proved the most efficient technique with a good performance, provided the front surface of the particle is plane. In other cases the non-stationary phase shift extrapolator may be applied to generalize this migration technique to particles with a non-plane front surface. Alternatively, SAFT and wave equation redatuming were coupled to obtain an acceptable reconstruction independent of the shape of the front surface. The case study shows that ultrasound imaging has potential as a quantitative assessment tool in immersion solid waste processing.

Keywords: Ultrasound imaging, wave equation redatuming, phase shift migration, non-stationary phase shift migration, SAFT, solid waste, quality control

1. Introduction

The breakthrough technology magnetic density separation (MDS) is currently being developed for separation of different types of polyolefin particles from either shredded automobile waste, WEEE or household waste [1]. An online monitoring system is required to visualize the plastic particles flowing inside the MDS, for example for the purpose of detecting possible turbulence, measuring throughput and optimizing the splitter positions where the different plastic products are extracted from the flow. However, the magnetic liquid in which the particles are separated is essentially black and optical camera systems proved useless. This motivated research into ultrasound imaging systems for the purposes of monitoring and quantitative analysis of the streams of polyolefin particles [2, 3]. It was shown that the medical ultrasound technology could be adapted owing to its real-time performance. Moreover, the ultrasound video-streams obtained from a commercial imager contained information relevant for controlling the separation process. Ultrasound research then focused on improving the reliability of the image forming technique and on extracting more quantitative particles information from the raw data. One important example is volumetric throughput measurement for which advanced imaging techniques are required to deliver shape consistent images of the particles. In particular, the position and shape of the back of the particle, i.e. the side not facing the sensor array, gives information about thickness and volume. In addition, occurrences of turbulence in the magnetic fluid flow in the MDS can be made quantitative by tracking the particle centre of gravity and its orientation in the fluid.

Imaging methods are effective if objects to be imaged are sparse and in a uniform embedding; i.e. if the object to be viewed is relatively removed from other objects and if it is embedded in a material with a constant acoustic wave speed. In that case the multiple wave paths between the objects that arise by refraction and scattering at material boundaries are negligible. Thus, the object front surface directly in view of the sensor array can easily be imaged using only the direct reflected waves. But in contrary to visible light, ultrasound
waves can penetrate any elastic object. This, in principle, allows one to also image the back of an object using one-sided access with a single fixed sensor array. The sensor array should preferably be fixed since it may otherwise interfere with laminar fluid flow in the MDS.

The ultrasound data available for back surface imaging also contains mode converted waves, i.e. waves interchanging between compression and shear modes of propagation. Besides velocity also the amplitudes and angles of propagation change during mode conversion, which complicates the imaging process. SAFT can be adapted to any type of wave path, but at the cost of much increased computational effort. It would have been simpler if the sensor elements were in direct contact with the particle in question, eliminating the wave paths inside the fluid. An approximation to this thought experiment has been studied in seismology using redatuming of the wave equation [4]. Once the transform domain wave field is calculated, the acoustic path between the transmitter and the reflector becomes a straight line. This scenario is similar to the conventional contact testing and allows SAFT to reconstruct the back surface of the object using direct reflected waves. In geophysics, phase shift migration was developed for the same purpose and performs (theoretically exact) migration in a horizontally layered configuration [5]. This method employs operators that extrapolate the data to more distant interfaces, in which it proves computationally faster than SAFT. Phase shift plus interpolation (PSPI) was developed to deal with media involving lateral velocity variations [6]. Its frequency-space domain migration principle is to apply linear interpolation between introduced reference velocities to account for the lateral velocity variations in each back-propagation step. The success of PSPI relies heavily on the choice of the reference velocities. The performance improves by introducing more reference velocities, but the computational effort increases proportionally. The PSPI method may be extended to what is referred to as non-stationary phase shift (NSPS) [7]. In NSPS the problem of selecting reference velocities or interpolating is resolved by taking an exhaustive set of reference velocities for every distinct velocity component from the known lateral velocity variation. A disadvantage of migration methods with respect to SAFT is that they are not flexible in the way the dataset is built, as receivers have to be distributed in accordance with the Nyquist criterion and if FFT is used also equidistant. In the present work, image forming methods are investigated for reliable quantitative measurements of the geometry of plastic waste particles, which are typically 10-20 mm in size. Different imaging techniques are implemented in Matlab and raw ultrasound datasets are acquired in the laboratory. To that end a single test object is used with a generic shape, i.e. an object which displays all shape dependent wave effects typically encountered in this type of waste particles. A ray-tracing approach is employed to identify wave types and evaluate the useful information present in the datasets for successful imaging. Imaging performance is discussed in terms of shape consistency and signal-to-noise-ratio. Finally, a statistical analysis is performed for a real-time scenario, exploiting the common factors in sampling and measurement errors in order to minimize the effort needed to achieve the desired accuracy in an online application.

2. Methods and Materials

2.1. Imaging methods

2.1.1. Wave field redatuming in SAFT processing

Figure 1(a) shows a 2D cross section with a plastic particle immersed in water. In the following analysis \( g(x, y, t) \) is the solution to the acoustic wave equation with wave speed \( c \).

The ultrasound data is denoted as \( g(x, y_0 = 0, t) \) which is recorded at line \( y_0 = 0 \). The thought experiment depicted in Figure 1(b) is considered as a step towards improving image
resolution in the plastic interior. There, the sources and receivers are located on the front surface of the particle instead of their original location $y_0 = 0$.

![Figure 1. Concept of redatuming. (a) Real experiment. (b) A thought experiment.](image)

To approximate the situation in Figure 1(b) the data measured at $y_0 = 0$ has to be transformed into data that could have been measured at the particle surface. This technique is often called ‘redatuming’ which is a way to eliminate the propagation effects of the fluid waves. The homogeneous acoustic wave equation has the following solution for the downward travelling waves in the transform domain

$$G(k_x, y, \omega) = G(k_x, y_0 = 0, \omega) \exp(i k_y y), \quad k_y = \sqrt{(\omega/c_{\text{Water}})^2 - k_x^2}, \quad (1)$$

where wave number $k_y$ is derived from the dispersion relation. The wave field in a given point $(x_p, y_p)$ of the particle surface can be determined by first applying an inverse Fourier transform with respect to $k_x$ and $\omega$ to obtain:

$$g(x, y_p, t) = \int \int G(k_x, y_p, \omega) \exp(i k_x x) \exp(i \omega t) dk_x \, d\omega. \quad (2)$$

Next, the wave field at $(x_p, y_p)$ is extracted from $g(x, y_p, t)$ and written as the transformed wave field $g(x_p, y_p, t)$ and used as input data for the SAFT kernel to reconstruct the back surface reflections within the plastic particle. In time-domain SAFT, the stored signal from each receiver position is delayed by $\Delta t_n$ as determined by the distance travelled by the wave from the source to the synthetic focal point and back again to the receiver. SAFT is implemented as the sum of coherent waves

$$S_{\text{SAFT}}(t) = \sum_{n=1}^{N} g(x_{p,n}, y_{p,n}, t - \Delta t_n), \quad (3)$$

where $N$ indicates the number of measurements included in the summation. The coherent summation increases the SNR as provided in the data by a maximum of $10\log_{10}(N)$. 


2. 1. 2. Phase shift migration

In phase shift migration the wave field at a given distance from the plane where data were collected is calculated by extrapolation and making recursive use of phase shift operators. In that procedure the data effectively serve as a boundary condition. Requirements are that the boundaries are plane and parallel and that the wave speed is constant within each layer. The following analysis assumes a configuration with N horizontal layers (cf. Figure 2). First a single sensor is scanned along a line at \( y = y_0 \) to detect the pressure, which gives \( G(x, y_0, t) \) where \( x \) denotes scan position. The objective of phase migration is to determine the wave field \( G(x, y, 0) \). First, a 2D Fourier transform with respect to \( x \) and \( t \) gives \( \hat{G}(k_x, y_0, \omega) \). An image line at the new distance \( y \) follows from extrapolation of the phase

\[
\hat{G}(k_x, y_1, \omega) = \hat{G}(k_x, y_0, \omega) \exp(ik_{y,1}(y_1 - y_0)), \quad k_{y,1} = \sqrt{(\omega/c_1)^2 - k_x^2}, \quad (4)
\]

where \( k_{y,1} \) is the \( y \)-component of the wave vector in the first layer. This extrapolated serves as a new boundary condition to determine the wave field at the greater depth \( y_2 \). In a recursive approach the pressure at any depth \( y_N \) can then be solved by

\[
\hat{G}(k_x, y_N, \omega) = \hat{G}(k_x, y_{N-1}, \omega) \exp(ik_{y,N}(y_N - y_{N-1})), \quad k_{y,N} = \sqrt{(\omega/c_N)^2 - k_x^2}. \quad (5)
\]

Having found the wave field at all the layer boundaries \( (y_1 \cdots y_N) \), the field within the specific layer number \( N \) can be calculated as

\[
\hat{G}(k_x, y, \omega) = \hat{G}(k_x, y_{N-1}, \omega) \exp(ik_{y,N}(y - y_{N-1})). \quad (6)
\]

Next, this transform domain wave field is integrated with respect to \( \omega \) to obtain the image at \( t = 0 \) after which a spatial inverse Fourier transform gives the final result

\[
G(x, y, t = 0) = \int \int \hat{G}(k_x, y, \omega) d\omega \exp(ik_x x) dk_x. \quad (7)
\]

![Figure 2. A horizontally layered medium allows for phase shift migration.](image-url)
2. 1. 3. Non-stationary phase shift migration
Consider a 2D configuration where the velocity \( c(x) \) varies in a horizontal direction but not in depth. The objective is again to extrapolate the field from \( y_0 \) to another depth \( y \). In PSPI a set of reference velocities are introduced to account for the horizontal velocity variation in each extrapolation step. First, phase shift extrapolation is performed for each reference velocity \( c_N \) to produce the corresponding reference wave field \( \hat{G}_{c,N}(x,y,\omega) \)

\[
\hat{G}_{c,N}(x,y,\omega) = \int \hat{G}(k_x,y_0,\omega) \exp(ik_y(y-y_0)) \exp(ik_x x) dk_x,
\quad k_y = \sqrt{(\omega/c_N)^2 - k_x^2}.
\] (8)

The main assumption in PSPI is that the extrapolated field \( \hat{G}(x,y,\omega) \) is identical to the reference wave field \( \hat{G}_{c,N}(x,y,\omega) \) in points where the local velocity \( c(x) \) matches the reference velocity \( c_N \). The final step is to apply linear interpolation (LI) using the reference wave fields in the frequency-space domain. Based on the available information of the velocity \( c(x) \) and given the reference velocities, we obtain

\[
\hat{G}(x,y,\omega) = LI(\hat{G}_{c,N}(x,y,\omega),\hat{G}_{c,N+1}(x,y,\omega)), \quad c_N < c(x) < c_{N+1}.
\] (9)

In NSPS a complete set of reference velocities is taken for all actual velocities. Consequently, a reference wave field is computed for each velocity and the final result is expressed by the Fourier integral

\[
\hat{G}(x,y,\omega) = \int \hat{G}(k_x,y_0,\omega) \exp(ik_y(y-y_0)) \exp(ik_x x) dk_x,
\quad k_y = \sqrt{(\omega/c(x))^2 - k_x^2}.
\] (10)

Note that here, unlike the conventional phase shift migration, \( k_y \) is a function of \( c(x) \) and therefore equation (10) is not an inverse Fourier transform. In other words, the phase shift extrapolator is applied simultaneously with the transformation from \( k_x \) to \( x \) domain. Solving equation (10) is equivalent to creating a complete set of reference wave fields for all distinct velocities. A windowing operation, defined as \( w_N(x) \), is applied to assign the reference wave fields to points where \( c_N = c(x) \) and to form a single and final wave field

\[
\hat{G}(x,y,\omega) = \sum_{n=1}^{N} w_n(x) \text{ IFFT}[\hat{G}_{c,n}(k_x,y,\omega)].
\] (11)

An alternative approach would be the direct numerical integration of equation (10). Up to this point the velocity profile changed only in a horizontal direction. Depth variations in wave speed can be accounted for by using recursive wave field extrapolation. In recursive migration, data recorded at the top surface is continued in depth through many small steps. In each step it is assumed to involve only horizontal velocity variations, but depth-dependent velocities can also be included.

2. 2. Experimental data
The discussed imaging methods were derived on the basis of an exploding reflector model, i.e. scatterers in the medium are replaced by sources that simultaneously activate at \( t = 0 \). Thus, the extrapolated wave field at \( t = 0 \) shows the spatial distribution of the exploding reflectors. A complication in experimental data is that the incident waves first must propagate from the
transmitter to the scatterers, which time delay is quite specifically linked to the method of building the data set. In pulse-echo (PE) dataset, the direct reflected sound wave travels up and down the same path, which in the imaging may be accounted for using half the wave speed. In a plane wave (PW) scan using a linear array all sensor elements are fired simultaneously, creating a plane wave front. The path between the sensor element and a reflector at depth \( y_0 \) is a straight line with delay time \( \Delta t = y_0 / c \). This time delay may be corrected for in the imaging procedure using the phase operator

\[
H(k_x, y_0, \omega) = \exp(i\omega\Delta t + iy_0\sqrt{\left(\omega / c\right)^2 - k_x^2}) .
\]  

(12)

The data are scanned at regular intervals along a straight line, which is facilitated by using a linear sensor array (probe) of 5 MHz with 128 elements. A semi-automated data acquisition setup was built to assist in scanning a complete dataset \( (128^2 \) time traces) from a static scene involving a plastic test object. Note that this dataset contains all information that could linearly be retrieved from a given, fixed position. Data were recorded by a 40 MHz/16 bit data logger. Various subsets can be extracted to reduce the computational effort. As a consequence, the information contained in those subsets will be different, leading naturally to differences in imaging methods and performance. In this study a PE dataset and a PW dataset were subtracted, both containing 128 time traces.

2. 3. Test object
A PVC test object was chosen as a generic case for plastic waste particles flowing in the MDS channel. The object shown in Figure 3(a) has a plane side and a circular side. These two sides are quite representative of shredded plastic particles from consumer waste products that tend to be either flat (like flakes) or round (in case of thicker products). The object is also generic in that it covers the maximum range of reflection and refraction angles. The object was large enough (in out-of-plane direction) in that it approximated a 2D layered cross section for the 35 mm long sensor array. Thus, the 2D layers for imaging front and back of the object consisted of water \( (V_{\text{water}} = 1480 \text{ m/s}) \) and PVC \( (C_p = 2341 \text{ m/s}, C_s = 1060 \text{ m/s}) \). Below the object is a layer of water that is large enough (considered as a half space) not to cause interfering reflections from the bottom of the water tank. A complete dataset was scanned after which a PE and a PW data subset were extracted. Next, the sample was flipped upside down so the other type of surface faced the array and the scanning procedure was repeated. A ray-tracing procedure was employed to identify the different types of wave in the dataset and especially those related to reflections at the back surface of the object. The rays taken into account are the first order reflected rays involving at most one mode conversion, which proved sufficient to account for all energetic wave events in the data. Each identified ray was counted as a 'hit-rate; for that specific point of the back surface. The relative energy in a ray, and thus the relative strength of the events in the data, was estimated using the two plane wave transmission coefficients at the front surface and the reflection coefficient at the back surface of the object.

3. Results and Discussion

3. 1. Information in the data-sets
Figure 3(b) shows the experimental dataset for a PE scan of the test object with its circular surface facing the array. Five dominant types of wave could be identified using the ray-tracing method, denoted by the successive modes of water pressure (P), solid compression (C) and
shear (S). The five different rays labelled 1-5 in Figure 3(b) are PP (1), PCCP (2), PSSP (3), PCSP (4) and PSCP (5). The SNR was 60 dB in the PE data and 40 dB in the PW data, where it is noted that the PE scan was averaged over 100 scans during data acquisition.

Based on the ray-tracing, for a PE dataset, the average hit-rates for PCCP and PSSP waves along the plane back surface were determined as 1 and 2, respectively. Both these hit-rates were zero near the corners of the test object due to critical reflection of the incident waves at the strongly curved front surface. In fact, the waves covered 89% (PCCP) and 67% (PSSP) of the back surface. These hit-rates were 2 and 4 in the PW dataset, thanks to the larger synthetic receiving aperture in a plane wave scan. However, in the PW dataset the waves covered only 67% (PCCP) and 58% (PSSP) of the back surface. The ray amplitude for PCCP waves in PE data varied only 2 dB, while it varied up to 90 dB for PSSP waves. Similar behaviour was observed in a PW dataset. Therefore, even though PSSP waves have a relatively higher hit-rate for the back surface, they mostly carry little energy. Moreover, it was found that the strongest PSSP waves reflected quite localized from the back surface and therefore do not contain information on the whole back surface. The mode converted waves 4 and 5 were also rather weak compared to the PSSP waves and also carried only partial information about the back surface. Concluding, only PCCP waves are useful for a full back surface reconstruction. For the case in which the flat particle surface is facing upwards essentially the same conclusions apply.

![Figure 3](image)

Figure 3. (a) Cross-section of the PVC test object. (b) B-scan of a PE dataset.

3. 2. Performance of imaging methods

As a first case the curved surface of the PVC object faced upwards towards the sensor array and a complete dataset was collected. All results in this section are based on the PCCP waves. Figure 4(a) shows the image using the PE dataset by the wave equation redatuming method after scaling to a SNR of 30 dB. The front surface is reconstructed using the PP waves in SAFT. Only the flattest middle part of the circular front surface shows up correctly as the waves reflected critically from the steeper angled parts of the circle are missing in the data.

The back surface is strongly curved towards the edges since there is also no data coming from these parts of the back surface. In other words, critical reflection shields the back surface of the particle from a proper ultrasound reconstruction. Figure 4(b) shows the image for a PW dataset. The curving at the sides is suppressed better thanks to the effectively larger receiving aperture of the PW dataset. The background noise within the contours of the object is also less. Note that, the curving which is an imaging artefact is linked to possible non-uniqueness that is inherent to all kind of inverse problems.
Figure 4. Ultrasound images of the PVC object with its plane surface downwards. (a) PE dataset and redatuming. (b) PW dataset and redatuming. (c) PE dataset and NSPS. (d) PW dataset and NSPS.

Figure 5. Ultrasound images of the PVC object with its plane surface upwards. (a) PE dataset and redatuming. (b) PW dataset and redatuming. (c) PE dataset and phase shift migration. (d) PW dataset and phase shift migration.

This can be improved by employing more data and that is simply enlarging the viewing aperture in this case. The same PE and PW datasets were also processed using the non-stationary phase shift migration in Figures 4(c) and 4(d). The NSPS method produced a better
image with the PE dataset in Figure 4(c), even though the particle front surface is slightly blurred. This blurring is less obvious for the PW dataset in Figure 4(d).

As a second case, the PVC object was positioned with the plane surface facing the sensor array after which a full dataset was collected. The resulting images obtained by redatuming are shown in Figures 5(a) and 5(b). The plane surface is reconstructed correctly, since the waves reflected from this surface were captured quite successfully by the sensor array. However, the curved back surface is distorted or partly missing at the sides for the same reasons as the first case in Figure 4. The same datasets were also processed using phase shift migration. It is noted that the top of the PVC block provided the reference depth for the artificial divisional plane between the two layers (i.e. water and PVC) that is required for this method. The results are shown in Figures 5(c) and 5(d). Similar to the first case, the PE dataset shows the best performance with phase shift migration.

3.3. Statistics in online quantitative ultrasound

So far the research was limited to imaging of an individual particle in a static scene. However, the ultimate goal of the ultrasound system is to serve as a real-time tool for quantitative measurements of particles processed in an industrial version of the MDS. Due to the limited length of the sensor array its use will be necessarily restricted to the detection and measurement of a representative sample of the particles stream, i.e. a limited number of particles per unit of time. To assess the accuracy of the ultrasound sample measurements it is required to quantify both the sampling error and the obtainable measurement error. The statistical theory is demonstrated using the following assumptions:

- Each interval $\Delta t$ one sample consisting of $N$ particles is measured;
- Using imaging the thickness $d$ of each particle is measured;
- The width and length of the particles hardly varies and are given as $l$ and $w$.

Note that the sampling error is now related to how representative the thickness distribution in the sample is for the particles stream. Here, the objective is to measure the cumulative volume of $N$ particles in the sample, which is expressed as

$$V_{\text{measurement}} = \sum_{n=1}^{N} lw(d_n + \Delta d) = \sum_{n=1}^{N} lw(d_n + \Delta d) = V_{\text{sample}} + lw\sqrt{N}\sigma_{\text{imaging}},$$

where $V_{\text{sample}}$ is the total volume of the sample and $\Delta d$ is the error made in the imaging for thickness measurements, complying with a normal distribution with standard deviation $\sigma$. Equation (13) shows that for a cumulative measurement the error increases by the square root of the sample size. This error may be reduced by averaging over $M$ subsequent samples, according to

$$V_{\text{measurement}} = \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1}^{N} lw(d_n + \Delta d) = \sum_{n=1}^{N} lw(d_n + \Delta d) = \bar{V}_{\text{sample}} + lw\sqrt{N/M}\sigma_{\text{imaging}},$$

where $\bar{V}_{\text{sample}}$ is the volume of the average sample. This way the quantitative imaging system delivers an accurate average sample volume every time interval $M\Delta t$. The sample size $N$ and number of samples $M$ may be minimized by solving the following equations when the tolerated sampling error and measurement error are given.

$$E_{\text{sampling}} = 1/\sqrt{MN} \quad E_{\text{measurement}} = \sqrt{N/M}\sigma_{\text{imaging}}.$$
This result shows that a higher imaging accuracy per particle allows for faster measurements, since there is less need to average over subsequent samples.

**Conclusions**

Ultrasound imaging is proposed for quantitative measurements of immersed plastic waste particles in the size range 10-20 mm using a linear sensor array. Migration techniques were adapted to image the back surface of a generic plastic test object. The algorithms were investigated in Matlab for pulse-echo and plane wave datasets. Using ray-tracing the information available in a dataset for front and back surface imaging was analysed. Among all primary rays, i.e. up to and including one mode conversion, only the pressure-compression type of waves carried sufficient information and signal strength to allow for back surface reconstruction. In general, all tested imaging methods provided a good shape consistency for the front surface. The back of the particle could be imaged only if the incident waves were not shielded from the particles interior due to critical reflection. A larger synthetic aperture will improve coverage of the back surface and thus help improve the shape consistency of the imaging when scanning data from a fixed position. A sufficient and technically feasible option for aperture enhancement appears to be a combination of two linear arrays, allowing the complete back surface to be interrogated by compression waves. For a pulse-echo dataset, phase shift migration techniques demonstrated the best performance, especially if the front surface is plain. With a plane-wave dataset both the migration and SAFT-redatuming techniques gave a similar performance. The statistics of real-time quantitative ultrasound in an online application was analysed, showing that a more accurate imaging method allows for faster quantitative measurements.

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