A Local Defect Resonance to Enhance Wave-Defect Interaction in Nonlinear Spectroscopy and Ultrasonic Thermography

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Abstract
It is experimentally shown that in order to provide maximum acoustic wave-defect interaction the concept of a local defect resonance should be applied. The frequency match between the defect frequency response and the probing ultrasonic wave results in enhancement of local vibration amplitude and substantial rise in efficiency for ultrasonic methods applicable in linear, nonlinear and thermographic non-destructive materials testing. The resonance frequencies are estimated for basic types of defects and found to be in ultrasonic frequency range. By using laser vibrometry, the resonance frequency responses are measured and visualized for delaminations, cracks and impacts in various materials. An increase in the higher harmonic nonlinear response of the defect at its resonance frequency exceeds considerably the one obtained at natural frequencies of the specimen. The strong wave-defect interaction is confirmed by a resonance rise of a local temperature of the defect in the frequency band of its mechanical resonance.

Keywords: Defect resonance, nonlinear ultrasonic spectroscopy, ultrasonic thermography.

1. Introduction

Elastic wave-defect interaction is a background of ultrasonic nondestructive evaluation (NDE) of materials and industrial components. In linear ultrasonic NDE, it is responsible for sound attenuation and scattering that results in the wave amplitude and phase variations as indicators of the presence of defects. The efficiency of the interaction (acoustic response of the defect) becomes particularly crucial for recently developed and power-dependent techniques such as nonlinear ultrasonic spectroscopy and ultrasonic thermography.

In both cases, an increase in efficiency was found by choosing the wave frequency equal to one of the resonance frequencies of the specimen providing that the defect is outside the nodal areas of the standing wave pattern [1-3]. This condition was found to enhance the nonlinear defect response and was applied in multiple experiments on nonlinear acoustic characterization of materials [e.g. 1-2]. The amplitude dependent shift of a resonance frequency of the whole specimen is also widely used as an indicator of damage [e.g. 4-5]. In order to eliminate the effect of nodal lines and to exclude “missing” the defect, the overall acoustic nonlinear response of the specimen was averaged over a certain frequency range [6].

A drawback of single-frequency ultrasonic excitation concerned with the nodal lines was also realized in ultrasonic thermography. To increase the probability of detecting a crack it was suggested to excite the specimens at multiple resonance frequencies instead of using a single-frequency excitation [7]. The frequency selection was attributed to avoiding the areas of low-sensitivity due to nodal line patterns [7] or to providing particular displacement pattern (clapping or rubbing) of a vibrating crack [8]. An enhancement in crack detectability by producing a wide-band excitation (“acoustic chaos”) was reported in [9]. The authors acknowledged that the origin of the effect might be related to elimination of vibrational nodes of the sample or correlated with the presence of higher frequencies [9].
The alternative approaches, used in nonlinear acoustics of gas bubbles in liquids [10] introduce a resonance frequency of the bubble as a key factor to increase an ultrasonic response of the insonified inclusion. The discovery of anomalous nonlinear ultrasonic response of medical contrast agents whose local nonlinearity enhanced remarkably due to resonance vibrations of incorporated gel bubbles made a breakthrough in ultrasonic medical diagnostics [11, 12].

Likewise, the opportunity of a resonance interaction of ultrasound with defects in solids was theoretically analysed in [13] for plate waves. The model of a resonance inclusion in soil was also applied to development of the nonlinear seismo-acoustic land mine detection methodology [14]. The phenomenology of “resonant” defects was used by us to describe the nonlinear dynamics of flaws in solid materials [15]. Recent numerical simulations and shearographic imaging demonstrated a modal vibration structure for delaminations in composites [16].

An efficient ultrasonic activation of the defect by the driving wave is the way to optimize both ultrasonic thermography and nonlinear ultrasonic spectroscopy. In this paper, it has been experimentally shown that to provide maximum acoustic energy transfer to the defect, the concept of a local defect resonance (LDR) should be applied. The LDR provides an efficient energy pumping from the wave to the defect, which results in a higher sensitivity of ultrasound to the presence of defects. By a frequency match between the probing ultrasonic wave and the defect resonance, a substantial enhancement in efficiency of ultrasound-based NDE techniques can be obtained.

2. LDR phenomenology and estimation of defect resonance frequency

The LDR concept is based on the fact that inclusion of a defect leads to a local drop of stiffness for a certain mass of the material in this area, which should manifest in a particular characteristic frequency \( f_0 \) of the defect. To illustrate the reduction in local stiffness we first use a model of the defect as a spherical cavity of radius \( R \). For a stress normal to the cavity surface, the local stiffness \( E_L \) of such a defect is given by [17]:

\[
E_L = \frac{2(1-2\sigma)}{3(1-\sigma)} E_i \approx 0.3 E_i .
\] (1)

According to (1), the stiffness of intact material \( E_i \) reduces about three times for materials with Poisson’s ratio \( \sigma \approx 0.3 \). Similar estimations for a disk-like crack of elliptical cross-section with semiaxes \( R \) (radius of the disk) and \( d \) (half of maximal opening) show that the local Young’s modulus is [17]: \( E_L \approx (d/R) E_i \). Assuming that for realistic cracked defects \( R \sim (10^{-3} - 10^{-2}) \) m and \( d \sim 10^{-6} \) m, the local drop in stiffness can be very substantial: \( E_L/E_i \sim (10^{-3} - 10^{-4}) \).

By deriving the local rigidity of the void and vibrating mass of the material, from (1) the estimate of the LDR frequency for a spherical cavity in a solid takes the form similar to the one in a liquid:

\[
f_0 \approx \frac{E_i}{16\pi^2 \rho R^2} . \] (2)

By using conventional parameters for a composite material \( E_i \approx 30 \) GPa; \( \rho \approx 2 \cdot 10^3 \) kg/m\(^3\) for a void of 1 mm-radius we obtain: \( f_0 \approx 174 \) kHz.
For such a typical defect in composites as a sub-surface delamination, the local resonance is apparently attributed to the resonance vibrations of the material layer (thickness $D$) above the defect and identified with a flexural resonance of a plate of thickness $D$ clamped around the boundary. By using a circular plate approximation (radius $a$) the estimate of the fundamental resonance frequency of the delamination is given by [18]:

$$f_0 \approx \frac{3.2D}{2a^2} \sqrt{\frac{E}{12\rho(1-\sigma^2)}}. \quad (3)$$

For a 1cm-delamination at 1mm depth in a typical composite material with ($E \approx 30$ GPa; $\sigma \approx 0.3$ and $\rho \approx 2000$ kg/m$^3$), from (3) the fundamental LDR is expected at $f_0 \approx 75$ kHz. Both evaluations given above show that the LDR frequencies belong to ultrasonic frequency range and are located quite distantly from the specimen natural frequencies (low-kHz range) that facilitates their experimental observation.

### 3. Experimental observation of LDR

A direct way to experimentally reveal a local defect resonance is to measure an individual contribution of each point of the specimen in its overall frequency response in a wide frequency range. For this purpose, an ultrasonic excitation by a wide-band piezoelectric transducer is combined with a laser vibrometer C-scanning of the specimen surface. It enables to probe and indicate all possible resonances in every point of the specimen.

An example of application of such an approach is shown in Figure 1 for an in-plane oval delamination (25x18 mm$^2$) in a glass fibre-reinforced composite (GFRP) plate (200x25x2.5 mm$^3$). A piezoelectric transducer embedded in the plate was used for a wide-band (400 Hz – 40 kHz) excitation of flexural waves. For this purpose, a pseudo-random input electrical voltage (flat frequency response within 0-50 kHz (see inset in Fig. 1); amplitude 1 – 70 V) was applied to the transducer. The out-of-plane particle velocity components and the wave vibration pattern in the specimen were measured and visualized by a scanning laser vibrometer (Polytec 3001S).

![Figure 1. Frequency response of GFRP specimen with inter-ply delamination measured with a scanning laser vibrometer.](image)

The origin of each maximum in Fig. 1 was verified by imaging the wave pattern in the specimen at the corresponding frequency. The pattern in Fig. 2 (a) illustrates one of the specimen length resonances, which are located in the frequency range below 10 kHz in Fig. 1. Similarly, a series of peaks in the frequency band 10–18 kHz was found to be associated with the specimen width resonances. The specimen vibration pattern measured at ~20 kHz – resonance (Fig. 2 (b)) reveals a strong enhancement (more than 20 dB) of the vibration pattern.
amplitude locally in the defect area and is identified as a fundamental defect resonance. The zoom-in 2D-image (Fig. 3 (a)) shows that the resonance response is inhomogeneous over the defect area with a particularly high-amplitude in the upper (core) part of the delamination. Multiple higher-order resonances with characteristic nodal lines in the defect area were also found in the frequency range up to 40 kHz (Fig. 3 (b)).

Figure 2 (a), (b). Laser vibrometry of vibration pattern for GFRP-strip with a local delamination: (a)-driving frequency 6.8 kHz; (b)-frequency 19.75 kHz.

Figure 3 (a), (b). Zoom-in 2D-images of the vibration modes for the fundamental (a) and higher-order (b) local defect resonances.

For elastic constants of the material derived from the measurements of longitudinal and shear wave velocities: $E \approx 30$ GPa; $\sigma \approx 0.3$ and $\rho \approx 2500$ kg/m$^3$; and the delamination parameters $D \approx 1$ mm; $a \approx 9$ mm, from (3) we obtain $f_0 \approx 20.7$ kHz, that is in a good agreement with the experimental value of $\approx 20 \pm 1$ kHz.

In fact, the LDR suggests a new NDE and imaging approach to ultrasonic spectroscopy, which enables to identify the defect position (as well as its size and type) by visualizing the LDR-induced increase in the vibration amplitude. In further experiments, wide-band (vacuum sucked) piezo-transducers (ISI SYS GmbH) were applied for ultrasonic excitation of the specimens. The electronic unit comprised an HP 33120A function generator (bandwidth up to 15 MHz) and a high voltage amplifier HVA 3/450 (SI GmbH). Another example of LDR-methodology is shown in Fig. 4 (left) for imaging of a 50μm-wide crack in a GFR-concrete plate ($45 \times 10 \times 1$ cm$^3$). The LDR-frequency is substantially lower in this case (4.188 kHz for a ~10 cm long crack) while more than 20dB dynamic range of the image proves a feasibility of the LDR-approach. The higher LDR frequency is expected in high-stiffness materials, such as carbon fibre-reinforced composite (CFRP) ($E > 100$ GPa). This is illustrated in Fig. 4 (right): to observe the LDR image of an impact area in a CFRP specimen the driving frequency had to be raised up to 66.22 kHz.
4. Nonlinear LDR-spectroscopy

At low amplitude of monochromatic excitation, LDR manifests as a conventional (linear) resonance and enhances the defect vibrations at the resonance frequency. As was shown above, a local stiffness in the area of a small crack can reduce dramatically: \( \frac{E_L}{(d/R)E_R} \approx \frac{1}{2} \), so that for the stress applied, the crack acts as a local strain “amplifier” by \((R/d)\) factor. As a result, even at moderate ultrasonic excitation level, the faces of the cracked defect start “clapping” and its vibrations become strongly nonlinear [19]. The nonlinear defect response is widely used as a sensitive “tag” for incipient defect recognition and imaging in nonlinear ultrasonic spectroscopy [20]. Subject to both nonlinearity and LDR, the defect exhibits the properties of a nonlinear LDR.

A strong increase in defect nonlinearity due to LDR is illustrated in Fig. 5 for a delamination in a GFRP specimen. The zoom-in frequency response of a delamination obtained for a wide-band excitation (Fig. 5 (a)) reveals the LDR-frequency \( f_0 \approx 20900 \) Hz. For driving frequency outside the LDR-bandwidth \( f \approx 19000 \) Hz, the defect vibration spectrum is close to monochromatic even at input voltage raised up to 50V (Fig. 5 (b)). As the driving frequency matches the LDR-frequency, a drastic enhancement of the higher harmonic (HH) spectrum is observed (Fig. 5 (c)). The HH-measurements within LDR-bandwidth (Fig. 6) show that the resonance induced increase in the HH-amplitudes on average is about an order of magnitude.

When the frequency of the driving wave \( f \) is different from LDR-frequency \( f_0 \), the resonance conditions can be satisfied for the combination frequencies \((nf \pm mf_0) \approx f_0\) produced by the
interaction between the natural and driven defect vibrations. It, therefore, occurs for \( f \approx [(m+1)/n]f_0 \), i.e. for the driving frequency to be fractional (higher (ultra-) harmonic resonance) or integer multiple of \( f_0 \) (ultra-subharmonic (USB) resonance). Both types of nonlinear resonances enhance substantially the vibration amplitude at \( f_0 \) and due to subsequent interaction with the driving wave cause “cascading” of the higher harmonics or subharmonics. The phenomenology of such “densely packed” nonlinear spectra was developed in our paper [21] in the assumption that the LDR exists even before it was experimentally observed.

The spectra of nonlinear resonance are modified for driven vibrations of a more complex cracked defect represented as a set of coupled nonlinear oscillators. The lower-order nonlinear response of the two oscillators (normal frequencies \( f_1, f_2 \)) for \( f \approx f_1 + f_2 \) contains the combination frequencies \( f - f_1 \approx f_2 \) and \( f - f_2 \approx f_1 \), respectively, that provide resonance excitation of the frequency pair \( f_2 \) and \( f_1 \). The higher-order nonlinear terms result in generation of a line spectrum, which comprises multiple ultra-frequency-pair (UFP) side-lobes around ultra-harmonics [21]. For high-amplitude excitation, the UFP bring the system to a quasi-continuous spectrum which indicates a build-up of chaotic vibrations.

An example of USB-UFP-resonance is shown in Fig. 7 for a delamination with LDR frequency \( f_0 = 20900 \) Hz (Fig. 5 (a)). For excitation frequency \( 2f_0 \), a resonance increase in the amplitudes of the HH, USB, and UFP-components is observed.

All nonlinear spectral components are generated locally and highly localized in the defect area that provides a background for high-contrast defect-selective imaging. The benefit of the higher harmonic LDR imaging is illustrated in Fig. 8. A strong improvement of the image quality (10x20 mm² delamination in 1 mm-GFRP plate) is clearly seen by comparing the fundamental (left) and the second harmonic LDR (right) images.
5. Ultrasonic LDR-thermography

In ultrasonic thermography, the defect thermal response is caused by a local dissipation of mechanical energy, which is converted into heat. The variation of mechanical energy (\( \Delta W \)) can be estimated by using phenomenology of internal viscous (friction) forces. In the frame of Newton’s law of viscosity, the viscous stress in the material is determined by the gradient of velocity: \( \sigma_v = \eta \varepsilon \), where \( \eta \) is the dynamic viscosity factor and \( \varepsilon = \varepsilon_0 \exp(j\omega t) \) is the strain. Hence, the time-averaged power dissipated in a unit volume of the material is:

\[
(\Delta W / \Delta t) = (\eta/\rho) \int_0^t (\varepsilon)^2 dt = \eta \omega^2 \varepsilon_0^2 / 2.
\]

The ultrasonic heating is then proportional to the square of both vibration amplitude and frequency.

As was mentioned above, in the defect area the local strain amplitude increases strongly that provides a high temperature contrast of flaws in ultrasonic thermography. The effect of viscosity causes the phase shift between stress and strain: \( \sigma = \varepsilon_0 + \eta \varepsilon = (E + j\omega\eta)\varepsilon \) and hysteresis loss even for small strains (in a linear strain/stress range). The area of the hysteresis loop (energy dissipation per a vibration cycle) increases substantially for the higher strain (nonlinear and plastic deformation), which is expected to develop particularly due to deformation of asperities on the rough interfaces of cracked defects. Therefore, the use of LDR, which strongly intensifies local defect vibrations, is beneficial for enhancing the efficiency and sensitivity of ultrasonic thermography. These benefits are illustrated in Figs. 9, 10, where the thermal responses of the delamination are visualized (Fig. 9) and measured (Fig. 10) for the same input amplitudes (input voltage 50 V) but different frequencies.

![Figure 9](image9.png)

Figure 9. Thermographic images of the delamination at the fundamental defect resonance frequency (~ 20 kHz, (a)) and at a frequency of one of the specimen length resonance (6.8 kHz, (b)).
The thermographic image of the defect taken in the vicinity of the resonance frequency ~ 20 kHz (Fig. 9 (a)) demonstrates that the heating is mainly produced in the core part of the delamination where maximum vibration amplitude is observed (Fig. 3 (a)). The temperature variation in this area (for insonification time 3 s) (Fig. 10) confirms the resonance character of the effect: At the LDR central frequency (20900 Hz, Fig. 5 (a)), the temperature rise (0.85 °C) is almost an order of magnitude higher than that outside the LDR bandwidth. For larger deviation of the driving frequency from LDR (including the specimen natural frequencies), the thermal response of the defect was barely measurable with the IR-camera (IRCAM Equus 327K, sensitivity ≈ 20 mK).

As shown above, for a linear elastic defect response, ultrasound-induced heating is proportional to the square of excitation frequency. However, the frequency increase of injected ultrasound is accompanied by the higher ultrasonic attenuation in the material and enhancement of the background heating. The nonlinear LDR suggests a unique opportunity to generate the higher frequency components directly in the defect and thus to enhance a local heating of the defect selectively. In the nonlinear LDR mode, the defect generates multiple spectral components (HH, USB, UFP), which extend the local excitation spectrum to a much higher frequency range. Each spectral line of the high frequency vibrations contributes to the defect thermal response, which is enhanced due to both the high number of the nonlinear spectral components and their higher frequencies.

A feasibility of such a nonlinear thermography mode is demonstrated in Figs. 11, 12.

Figure 10. Thermal responses of delamination at LDR-frequency (20900 Hz) and outside the LDR bandwidth (19000 Hz).

Figure 11. Laser vibrometry measurements of vibrations at excitation frequency (fundamental LDR 20900 Hz) and higher harmonics as functions of excitation level.
The laser vibrometry measurements of vibration velocity averaged over the thermally active area of the delamination in the GFRP plate (Fig. 9, a) show that as the defect excitation increases (input voltage > 60 V), the vibrations at the excitation frequency (squared vibration velocity proportional to a “linear” elastic energy (Fig. 11)) saturate and then reduce considerably. This is possibly caused by the amplitude-induced frequency shift of LDR and the generation of nonlinear spectral components (nonlinear distortion). The relevance of the latter factor is seen in Fig. 11: the sum of squared HH-vibration velocities (“nonlinear” elastic energy) measured in the frequency band up to 1 MHz (over 47 HH) increases strongly at higher excitation level. According to Fig. 11, about half of the “loss” of elastic energy at the excitation frequency exists in the form of the higher harmonic vibrations.

The “depletion” of the vibrations at fundamental frequency, however, does not affect the thermal response of the defect (Fig. 12), which increases steadily over the whole range of excitation level (up to 90 V, Fig. 12). An additional source of heating comes from nonlinear vibrations of the defect: the ultrasound-induced temperature (Fig. 12) rises synchronously with the growth of nonlinear elastic energy (Fig. 11). The nonlinearity of the defect, therefore, contributes substantially to its thermal response and enhances the sensitivity of ultrasonic thermography.

![Figure 12. Thermal response of the active delamination area as a function of excitation level. Ultrasonic exposure time is 2 sec.](image)

6. Conclusions

It has been experimentally demonstrated that the concept of local mechanical resonance is applicable to defects in solid materials. A single-frequency ultrasonic excitation is generally not an optimal way to inspect the material for defects. To optimize acoustic wave-defect interaction a frequency match between the defect frequency response and the probing ultrasonic wave is required. In this condition, a substantial enhancement in efficiency is observed for ultrasonic spectroscopy methods applicable to linear, nonlinear, and thermographic non-destructive materials evaluation and testing.

References