Numerical Analysis of Probability of Detecting Defects in Engineering Materials

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Abstract
The probability theory has found its major application in many engineering and industrial areas for the detection of defects inside structures. It can be used to assess the risks involved in the decision-making process. Probabilistic methods provide essential tools used in the better assessment of the impact of uncertainties on component life and risk of failure. The nondestructive inspector is to be aware of the probabilistic nature of component failure and nondestructive inspection. Knowledge of risks and assurances accompanying nondestructive evaluation leads to the better choice of the appropriate nondestructive techniques that can enhance efficiency and optimization of the engineering design processes. The paper presents the application and capabilities of a probabilistic method in nondestructive testing of engineering materials.

Keywords: Probability of detection, nondestructive testing, defects, size distributions, component failure

1. Introduction
Nondestructive Testing (NDT) is one of the methods used for quality control and maintenance examination of engineering materials. It is crucial that engineering materials should be inspected for their integrity and safe operation so that information on the existence and nature of defects is provided. The principal objective of NDT includes finding the flaws and cracks as well measuring their dimensions. NDT is used in critical physical assets such as aircraft, pressure vessels and nuclear reactor components. Knowledge of risks that accompany NDT leads to the better choice of the appropriate NDT techniques that can enhance efficiency and optimization of the engineering design processes. It is only now that it is not sufficient to just assume that an inspection is a perfect process of unbounded capability, but what is more important is to know the probability of locating defects in engineering materials. This activity is known as the probability of detection (POD) which is a means to describe how well an inspection procedure can detect the required defects. Probability is a way of viewing the world and forms a core mathematical discipline. POD can also be considered as a combination of the intrinsic capabilities of an inspection which can be spoiled by the application parameters and the human factor. By assigning the POD values to a procedure, it becomes possible to compare the effectiveness of different NDT inspection techniques.

Recently, it became evident that the power of probabilistic reasoning as NDT method of scientific inquiry has led to a rapid growth in the significance of probability theory in NDT research. The used value of a probability is a number between 0 and 1 where 0 is a number used for an event that cannot occur while 1 is for an event that is certain to occur. The POD is usually plotted against a defect size and its curve can be used to optimize an inspection procedure. The POD curve can also be used to identify the parameters that influence an inspection (namely, human factors and environmental conditions) as well as to evaluate the performance of the
operators [1]. The human factor is regarded as the interface between a machine and its operators and is a very crucial factor in NDT [2].

The human factor contributes greatly towards the safety and reliability of engineering materials. The human factor includes the mental and physical make of the individual, where the individual’s training in NDT, experience and the conditions under which he or she must operate greatly affect the ability of the NDT procedure to achieve its intended purpose [3].

In this paper, a theoretical background and the numerical experiment of using empirical data to augment a POD estimate are discussed. Thus, POD is described in terms of flaws, but not limited to flaws only it can also be used to corrosion loss, impact damage and delamination.

2. Probability of detection concept

The probability of detection of an NDT system to certain types of flaws with dimensions \( a \) is given in the form [4]

\[
POD(a) = \int_0^a P f_a(p) dp
\]

(1)

where \( P \) is the detection probability of a flaw and \( f_a(p) \) is the density function of detection of flaws with dimension \( a \). It must be noted that POD is different for different types of dimensions of flaws.

If the largest defects should be detected with some smaller defects left in the items of interest, then the inspection system depicted in the hypothetical curve for \( P(D|a) \) should be applied to the defect distribution. The POD at a specific crack size \( a \) can be estimated from a series of inspection of cracks of size \( a \) as [5]

\[
POD(a) = \frac{M_d}{N}
\]

(2)

with POD \((a)\) being the probability of detection at the crack size \( a \) and \( M_d \) represents the number of cracks of size \( a \) detected, and \( N \) stands for the total number of cracks.

There is a need for those involved in POD provision to be fully aware of the various models and which are best at providing the specific output being requested. These models should include factors such as noise, geometry and defect visibility. The models cover a range of inspection methods which include ultrasonic, radiography and magnetic methods.

Some methods of modeling the nondestructive inspection data to determine POD curves were examined [5]. It was then concluded that the log odds distribution was the most consistent distribution for determining a POD curve as a function of crack length \( a \). The log odds distribution is mathematically defined as [5, 6].
where $POD_i(a_i)$ is the mean probability of detection of crack $a_i$, $\alpha$ and $\beta$ are constants of the log odds curve.

3. Reliability

The safety, reliability, cost effective operation and maintenance of steam boilers, pressure vessels, aircraft, highway and railway bridges as well as nuclear power plants greatly rely on the effectiveness of NDT methods. The term reliability can be understood as the probability that a material is able to perform its intended function for a prescribed life when subjected to certain environmental conditions stated by the manufacturer [7]. Hence it is safe to say that the inspection reliability is to study the probability of detection of the flaws with the same dimensions and the smallest dimension of the flaw that can be detected under a certain confidence level, $CL=0.95$, with probability of detection, $POD=0.90$. This inspection reliability of NDT can be understood to mean the reliability of detection of an NDT system to certain types of flaws. A value of 0.90 represents the probability that 90 out of 100 components would function for a period stated by the manufacturer and 10 components would not function for the same prescribed period.

The probability distributions such as exponential, normal and Weibull are often used in reliability studies. The formula $R_t = e^{-t/\theta}$ signifies the exponential distribution while the formula $R_t = e^{-sp}$ represents the Weibull distributions [7]. In reference [7] the normal distribution is mathematically given by

$$R_t = 1 - \int f(t)d(t) = 1 - P(t)$$  \hspace{1cm} (4)

where $P(t)$ is the probability of failure or area of the normal curve to the left of time $t$ and $R_t$ is the reliability at time $t$.

4. Numerical experiment

Results of the numerical experiment can be used to establish the probability of detection as a function of flaw size for a given system denoted by $P(D|x)$ [8, 9] which means that the probability that the defect ($D$) will occur given that the size of the defect ($x$) is known to have occurred. For the purpose of this numerical experiment, a sample of 200 items delivered in a company from which 10 of them contain defects is considered. This company has a manager who understands the importance of NDT methods. The wise manager then demanded that this sample must be inspected using the NDT method which is known to have the capability of detecting defects as shown in figure 1 with POD being formulated as

$$P(D \mid a_i < x < b_i) = \begin{cases} \frac{x}{11} & 0 < x \leq 11 \\ \frac{1}{1} & x > 11 \end{cases}$$  \hspace{1cm} (5)
where \( P(D \mid a_i < x < b_i) \) means the probability of detecting a defect in the given size range and \( a_i \) and \( b_i \) are the lower and upper bounds of each interval. The wise manager further demanded that the sample of 200 items must be NDT inspected without replacement (which is statistically correct). The curve for the probability of detection as a function of the defect size is given as shown in figure 1. Eq. (5) and figure 1 show the capability of flaw detection of the NDT method used.

![Probability of detection curve obtained by using Eq. (5)](image)

**Figure 1.** Probability of detection curve obtained by using Eq. (5)

In order to understand the calculations of involving the probability of detection, the data in table 1 is considered. This data can now be used to determine the flaw distribution and it should be taken as fictitious so that it can only be used for the purpose of understanding the use of probability of existence of a defect in the given size range.
## Table 1. The distribution of defect depth

<table>
<thead>
<tr>
<th>Depth range</th>
<th>Number of defects ($M_d$)</th>
<th>$P(x)$ ($P = \frac{M_d}{N}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0.0426</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0.1277</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.2128</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>0.2553</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.0851</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.0426</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>0.0638</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.0213</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0.0426</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.0426</td>
</tr>
<tr>
<td>x</td>
<td>3</td>
<td>0.0638</td>
</tr>
<tr>
<td></td>
<td>47 ($\sum M_d$)</td>
<td>1.002 ($\sum P$)</td>
</tr>
</tbody>
</table>

The data in this table are those of NDT inspection of the 200 items to check for the existence of fatigue cracks.
Figure 2. The distribution of crack depths obtained by using table 1

If $\sum P(D|x)$ means the probability of locating a defect within the distribution then the probability of finding a defect in the items is expressed in the form [10]

$$P(D) = \sum_x P(D | x)P(x)$$  \hspace{1cm} (6)

where

$$\sum_x P(D | x) = \sum_x P(D | a_i < x \leq b_i)P(a_i < x \leq b_i)$$  \hspace{1cm} (7)

where $P(a_i < x \leq b_i)$ draws its data from table 1 and $P(D | a_i < x \leq b_i)$ gets its data from Eq. (5). Eq. (7) represents the probability of locating the defect within the distribution as given in figure 2. When Eq. (6) is multiplied by 100%, the number of defects is obtained. If the number of known defects is greater than the number of defects in the items of interest then the number of missed defects is calculated as the number of known defects minus the number of defects estimated by using the probability of detection of a defect equation.

5. Results and discussion

Results from the numerical experiment are presented to show the power and capability of the POD theory under discussion. In this numerical experiment, the probability of an item selected randomly is calculated using Eq. (2) and the result is found to be $P(a) = 0.05$. Calculating Eq. (7) we find the value 0.4333. The result of Eq. (2) is multiplied by that of Eq. (7), to determine the result of Eq. (6) which is $P(D) = 0.022$. When this value is multiplied by 100%, the number obtained gives the number of defects detected in the sample, that is 2.2. Since there were 10 defects in the sample of 200 items, it can be concluded that there are 8 defects which are not detected by the NDT method used.

6. Conclusions

This study was an attempt to numerically analyze the probability of detecting defects in engineering materials. According to the calculations from the numerical experiment two defects have been detected and six of them were missed. The method was simplified so that those researchers that are engaging in the probability of detection of defects can follow.

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