On Determination of Focal Laws for Linear Phased Array Probes as to the Active and Passive Element Size

Andreas GOMMLICH 1, Frank SCHUBERT 2
1 Institute of Radiooncology, Helmholtz-Zentrum Dresden-Rossendorf, Dresden, Germany
2 Fraunhofer-Institut für Keramische Technologien und Systeme, IKTS, Dresden, Germany
Contact e-mail: Frank.Schubert@ikts.fraunhofer.de

Abstract. Ultrasonic linear phased array probes consist of several single transducer elements. By exciting each single element at a particular time wave fronts can be tilted, focused or both combined. The required set of time delays is called "focal law". Hence, the shape of the resulting wave front depends significantly on focal law calculation. The typical single transducer element in linear phased array probes has a rectangle shape with a width-to-length-ratio of approximately 0.1 to 0.5. The width of the short side is called "active aperture size", the larger one is called "passive aperture size". In state of the art calculations of the focal laws only the pitch between the single elements is considered and the elements are approximated by a point source in the centre of their aperture. Therefore, the real dimensions of the elements - both the active and the passive dimension - have no further influence. Moreover the wave propagation process itself is modelled by geometrical acoustics.

The numerical CEFIT (Cylindrical Elastodynamic Finite Integration Technique) in combination with transient PSS (Point Source Synthesis) enables flexible and fast simulation of 4-D spatio-temporal sound fields in homogeneous and layered half spaces, respectively. Thereby all wave physical effects like diffraction, scattering and mode conversion will be considered. The calculations with different geometrical parameters for the probes show that both the active as well as the passive aperture size influence the time and frequency characteristic of the signal in the focal point. Based on the focal laws calculated with and without respect to the aperture size, sound fields were simulated for selected focal points. The results were qualitatively and quantitatively compared whereby the differences between both methods are distinguishable. It becomes evident that geometrical focus and acoustical focus are different. The feasibility of corrections for the focal laws with respect to time and frequency characteristics in the focal point as well as the improvement of resolution will be discussed. The results further allow an estimation of the optimal width-to-length-ratio for single transducer elements of linear phased arrays.

Introduction

In the field of medical diagnostics, nondestructive testing and materials characterization ultrasonic phased arrays are well established. They consist of several single transducer elements (typically 16, 32, 64 up to 128 and more) in regular arrangements (Fig. 1).
By means of individual excitation the transmitted wave front can be tilted and focused within the near field length of the array. For this, it is necessary to excite each single element with a specific delay time. The state of the art for calculating these set of delays, which is called “focal law”, is traditionally based on geometrical acoustics. The corresponding focal point is therefore called “geometrical focus”. In fact this geometrical focus is the only focus that can be defined with a commercial phased array system nowadays.

**Focal law calculation based on geometrical acoustics**

In case of the geometrical approach the whole aperture is approximated by one single point source in the center of the transducer element and therefore the path between the element and the focal point will be reduced to a straight line. Hence the transit time depends only on the ratio of distance between aperture center and focal point as well as on the sound velocity (Fig. 2).

![Fig. 2. Geometrical sound paths from the center of the transducer elements to the focal point.](image)

This approach leads directly to the formulas for shaping the wave front as shown in Fig. 3.

![Fig. 3. Superposition of elementary wave fronts with constructive interference in the focal point (from left to right: wave front parallel, tilted, focused, tilted and focused)](image)

If all elements are exited simultaneously (Fig. 3, left), the wave front will propagate parallel to the aperture of the phased array.

The wave front can be tilted if the time delay between adjoined elements is unequal to zero but constant (Fig. 3 – second picture). For the time delay we have

\[ t_n = n \frac{p}{c} \sin \theta + t_0 \quad \text{with} \quad n = 1, \ldots, N \]
(\(N\): number of array elements; \(n\): element under consideration; \(p\): pitch; \(c\): sound velocity; \(\theta\): tilting angle).

A wave front focused to one point on the acoustic axis in front of the array (\(\theta=0^\circ\)) can be realized with the following formula (Fig. 3 – third picture):

\[
t_n = \frac{F}{c} \left[ 1 - \sqrt{1 + \left( \frac{np}{F} \right)^2} \right] + t_0
\]

Both tilting and focusing can be combined. This allows for specific focusing to arbitrary points with distance \(F\) and angle \(\theta\). A universal calculation specification without the offset \(t_0\) was introduced in [1] (compare Fig.3 – picture on the right).

\[
t_n = \frac{F}{c} \left[ \left( \frac{8np}{F} \right)^2 \right] + \frac{2np}{F} \sin \theta - \sqrt{1 + \left( \frac{(n-N)p}{F} \right)^2} \sin \theta
\]

with \(N = \frac{N-1}{2}\)

All presented formulas are not affected by the width or length of the single transducer elements! The only geometrical value which influences the focal laws is the distance between the centers of adjoined elements, the so-called pitch \(p\).

**Sound field simulation with 4D-CEFIT-PSS**

The 4D-CEFIT-PSS enables the simulation of wave propagation in homogeneous or layered half-spaces. Thereby all wave physical effects like diffraction, scattering and mode conversion will be considered [2].

This cascaded method consists of three successive steps. First the sound field based on a broad-band point source is calculated by the CEFIT method (Cylindrical Elastodynamic Finite Integration Technique). Due to the axisymmetric properties of a point source on the boundary layer of a half-space it is sufficient to calculate the radial and axial components of wave motion. (Fig. 4). In the present case the center frequency is approximately 1 MHz.

![Fig. 4. CEFIT simulation: Axisymmetric sound field in the half-space (left: \(v_r\) component, right: \(v_z\) component) initiated by a normal force point source in the origin (snap-shot).](image)

Secondly an aperture is generated as a grid of point sources. The superposition of all single transient sound fields by transient PSS (Point Source Synthesis) yields the complete sound field of the whole aperture. For analysis of linear phased arrays mostly the \(xz\)-plane at \(y=0\) is...
notable. After sound field reconstruction the complete 3D-result can be reduced to a 2D-map by disregarding the y-component of particle velocity (Fig. 5).

![Figure 5](image.png)

Fig. 5. Vector scheme of sound field components related to the focal point F (left: 3D, right: reduced to the xz-plane)

In the final step all sound fields from the single transducer elements are added up considering the applied focal laws for the resulting sound field of the whole array.

**Single element modelling**

As mentioned above the pitch between two elements is the only geometrical parameter which affects the (geometrical) focal law calculation. However, the single element sound field also depends on the aperture size as is shown in this section (see also [3]). For comparison four different aperture models will be introduced: a centered point source, a centered line source in x direction (y=0), a centered line source in y direction (x=0) and an area source (Fig. 6). In the present case the dimensions of the single element amount to $4 \times 10 \text{ mm}^2$.

![Figure 6](image.png)

Fig. 6. Aperture modelling with different point source configurations (from left to right: point source, line source (active aperture), line source (passive aperture) and area source).

**4.1 Model influence in the time domain**

Figure 7 shows the particle velocity $v(t, \theta)$ in the defined receiving point in the near field of the full phased array aperture ($F = 20 \text{ mm}$) projected on the normal vector (array center to focal point) as a function of time and tilting angle. The four diagrams correspond to the four aperture models introduced above.
Fig. 7. Particle velocity $v_R(t, \vartheta)$ as a function of time (vertical axis) and tilting angle (horizontal axis) for different aperture models (from left to right: point source, line source (active aperture), line source (passive aperture) and area source).

The point source as well as the passive aperture generate a longitudinal and a transversal wave almost over the full range of angles. In case of an active aperture source or an area source destructive interferences yield to an extinction of these different wave types with increasing angle. Especially the Rayleigh wave is extinguished almost completely.

In Figure 8 the particle velocity $v_R(t)$ for $\vartheta = 40^\circ$ is displayed for the different aperture models. It is recognisable that the wider the elements ($e$) the earlier the first arrival at the focal point is detected. For the element length $l_{pas}$ holds: the larger the length the later the first arrival.

Fig. 8. Particle velocity $v_R(t)$ for $\vartheta = 40^\circ$ for different aperture models

4.2 Model influence in the frequency domain

In Fig. 9 the spectra of the time domain signals, $\mathcal{F}\{v_R(t, \vartheta)\}$ are shown. Both the spectra of the active line source and of the area source are changing at small angles. In contrast to that the spectra of the point source and of the passive aperture keep their shape within a large range of angles ($\vartheta = 0 ... 40^\circ$).

Fig. 9. Spectra $\mathcal{F}\{v_R(t, \vartheta)\}$ for different aperture models (from left to right: point source, line source (active aperture), line source (passive aperture) and area source)
Analogous to the time domain analysis the corresponding spectra for a tilting angle of $\Theta = 40^\circ$ are shown in Figure 10. There are two effects: The shape of spectra will change dramatically, if the active element size increases. If the passive size increases the maximum of the spectra shifts to lower frequencies.

![Fig. 10. Spectra $\mathcal{F}[v_n(t)]$ at $\Theta = 40^\circ$, for different aperture models.](image)

5 Phased array focal law calculation considering full wave physics

The previous analysis of the aperture modelling of single elements and its influence to the resulting wave field illustrate that focal law calculation with the pure geometrical approach on the one hand and the consideration of real aperture size and full wave physics on the other hand will deviate. The real aperture size $e \times l_{pas}$, the distance $F$ and the tilting angle $\Theta$ have an impact on the directional characteristic of the single element. Figure 11 shows the focal laws for an 8-element array based on the geometrical approach compared to the focal laws based on the wave physical simulation with 4D-CEFIT-PSS:

![Fig. 11. Focal laws for a linear array with 8 elements for different tilting angles (bottom: 0°… top: 70°, $c = 5900 \text{m/s}$, $f = 1.1 \text{MHz}$, $e = 0.7\lambda$, $l_{pas} \approx 4\lambda$, $F \approx 3\lambda$).](image)

From Fig. 11 the following two results can be extracted: The real sound field arrives earlier in the focal point then the geometrically calculated transmit time because of the active aperture part which is considered. This leads to smaller delay times (focal laws) compared to the focal laws based on pure geometrical calculation. The bigger the tilting angle $\Theta$ the larger the difference in delay times.
The differences between both methods are shown in Figure 12. All focal laws are triggered on the farthest element (element 1). Even tilting angles $\vartheta > 20^\circ$ cause time differences of $\Delta t > 100\text{ns}$.

The evaluation for tilting angles $\vartheta = 0^\circ \ldots 90^\circ$ is shown in Figure 13. In the range $\vartheta = 0^\circ \ldots 40^\circ$ the signal enhancement grows to approximately 5%. Beyond $40^\circ$ the signal enhancement increases to 10%. For all parameters one receive better conditions compared to the conventional geometrical approach.

**Conclusions**

The 4D-CEFIT-PSS method represents a powerful and flexible numerical tool for wave field simulation and optimization of ultrasonic phased array probes. In the reviewed case the transient broadband signal in the isotropic and homogeneous half-space was calculated. The influence of possible aperture models for focal law calculation on the wave field was investigated in both time domain and frequency domain. It was shown that the real aperture size, in particular the active size, has an important impact on the resulting signals in the focal point.

The simulated signals in dedicated focal points can be used for calculating improved focal laws by considering the real aperture size and the full wave phenomena in the phased array near-field. The comparison between geometrical and modified focal laws shows a significant signal enhancement for the latter. Therefore this new approach offers an individual and optimal excitation of real ultrasonic phased array probes. In the future it should be possible to develop phased array systems with a focal law generator based on wave physics instead of geometrical acoustics as in all commercial systems nowadays.
References