Calculation and Analysis the Magnetic Parameters of the Minors Hysteresis Loop for Steels from the Basic Magnetic Parameters

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Abstract. Interpolating formulae to account for the variation of magnetization of steels on the normal magnetization curve and on the symmetrical saturation and minors of the hysteresis loop were elaborated from the basic magnetic parameters of the material - coercive force $H_{cs}$, saturation magnetization $M_s$, and remanent magnetization $M_{rs}$, measured on the saturation hysteresis loop. The legitimacy of their use was verified.

The formulas designed magnetic parameters of minors hysteresis loops of steels (coercive force $H_A$, maximum magnetization $M_m$, and remanent magnetization $M_r$) is developed. It is shown that values of $H_c$, $M_m$, and $M_r$ of steels in any minor hysteresis loops can be calculated at the accuracy sufficient for practice using $H_{cs}$, $M_s$, $M_{rs}$ of steels and maximum magnetizing field $H_m$ in a minor hysteresis loop.

The sensitivity of $H_c$, $M_m$, and $M_r$ of steels to the structure- and phase-sensitive parameters of steels is analyzed. A technique for an effective use of the results of measuring $H_c$ in magnetic structural analysis is developed, and the problems of controlling metal manufacture conditions to by solved by measuring $H_c$ are formulated.

Introduction

Steels have ferromagnetic properties: if the magnetic field $H$ is changing, the magnetization $M$ of steels varies nonlinearly and not single-valued - along the magnetization curve and a hysteresis loop (Fig. 1).

The coercive force $H_{cs}$, the saturation magnetization $M_s$, and the residual magnetization of $M_{rs}$ are the main magnetic parameters of the saturation hysteresis loop of steels (Fig. 1). The minor hysteresis loops of steels are characterized by the maximal intensity of the magnetizing field $H_m$, the maximal magnetization $M_m$, the coercive force $H_c$, and the residual magnetization $M_r$ (Fig. 1). There exists no exact analytical form for the function $M(H)$ on the main magnetization curve and the saturation and minor hysteresis loops of ferromagnetic steels, which would account for the processes of formation, growth and reorientation of the magnetic domains. Thus, in order to calculate the magnetic state of the steel, different approximating relations are used. An approximation of the main magnetization curve and the saturation and minor hysteresis loops of a steel with a precise and simple function will allow to simplify calculation of the field inside and outside of
ferromagnetic objects in electro-technical applications and magnetic structural analysis, eliminating cumbersome iterative calculations and graphical representations.

The purpose of this report is to develop and analyze formulas describing the change in the magnetization of steels on the main magnetization curve and the minor hysteresis loops based on the basic parameters of the magnetic material – $H_{cs}$, $M_s$ and $M_{rs}$, measured on the maximum hysteresis loop.

1. Formulas for calculating the magnetization of steels on the main magnetization curve and on minor hysteresis loops based on $H_{cs}$, $M_s$ and $M_{rs}$.

The experiments [1] have shown that equations [2] with the following compact notation most accurately describe the magnetization of steels:

- Formula for the main magnetization curve:

$$
M = \chi_a \frac{H_{cs}^2 H_m}{H^2 + H_{cs}^2} + M_s \frac{H_{m}^2}{\pi (H_{m}^2 + kH_{cs}^2)} \left[ \sum_{n=0}^{1} (-1)^n \arctg \frac{H_{cs} + (-1)^n H_m}{H_0} \right].
$$

(1)

- Formula for the branches of the hysteresis loop (the sign "+" refers to the descending branch, the sign "-" -- to the rising branch):

$$
M = \chi_a \frac{H_{cs}^2 H}{H^2 + H_{cs}^2} + M_s \frac{H_{m}^2}{\pi (H_{m}^2 + kH_{cs}^2)} \left[ 2\arctg \frac{H_{cs} \pm H}{H_0} - \sum_{n=0}^{1} \arctg \frac{H_{cs} + (-1)^n H_m}{H_0} \right].
$$

(2)

where $M$ is the magnetization of the material in a field $H$ on a branch of the hysteresis loop after magnetization in the field $H_m$ to the magnetization $M_m$ (Fig. 1),

$$
H_0 = \frac{H_{cs}}{\chi_a}, \quad k = \frac{M_s}{\pi M_1} - 0.5 \chi_a H_{cs}
$$

(3)

$\chi_a$ is the initial magnetic susceptibility, $M_C$ - magnetization on the main magnetization curve at $H_m = H_{cs}$.

A direct application of (1) - (3) in technical calculations is not possible because of their dependence on $M_C$, which is absent in reference books on magnetic characteristics of steels. Moreover, results of studies of the influence of production regimes of materials on their $\chi_a$ are listed less frequently in reference literature than their influence on $H_{cs}$ and $M_{rs}$. 

![Fig 1](image-url)
The measurement results for \( \chi_a \) are not always reliable. This is due to the fact that measurements of \( \chi_a \) should be carried out on thermally demagnetized material [3]. Repeated measurements of \( \chi_a \) on samples that were heat treated at a given regime are not possible after the samples were used for magnetic measurements. The lack of precision of the reference data on \( \chi_a \) (initial magnetic permeability \( \mu_a \)) can be demonstrated, for example, on the following result: for the steel 95X18, heat-treated at 1150ºC (tab. 51.5 in [4]), \( \mu_a \) appeared to be 10% greater than the maximum magnetic permeability \( \mu_m \), which is impossible by definition [5].

It was possible to exclude the values \( \chi_a \) and \( M_C \) from Eqs. (1) – (3) [6, 7]. To achieve this, the relationships between magnetic parameters of steels were used.

Thus, Gumlich and Schmidt [8] have empirically established a formula that relates \( \mu_m \) of steels and cast irons with their \( H_{cs} \) and \( M_{rs} \). In the SI, it is written as:

\[
\mu_m = (0.476 + 0.0712zH_{cs})M_{rs}/H_{cs},
\]

where \( \tau = 1 \text{ m/kA} \) is a dimensional factor.

The legitimacy of use of formula (4) for estimation of \( \mu_m \) of modern steels was proven in [9].

In [6, 7] on the basis of experimental studies [1] and the results presented in [10] (Fig. 7 – 9 in [1] and Fig. 4 – 9 in [10]), the ratio between \( \chi_a \) and \( \mu_m \) of steels was used:

\[
\chi_a \approx \frac{\mu_m}{3} - 1.
\]

Based on (4) and (5) for calculating \( \chi_a \) and \( M_C \), the following equations are used in [6, 7]:

\[
M_C = 0.67(0.476 + 0.0712zH_{cs})M_{rs},
\]

\[
\chi_a = 0.33(0.476 + 0.0712zH_{cs})\frac{M_{rs}}{H_{cs}} - 1.
\]

Based on formulas (1) – (3), (6), and (7), the main magnetization curve and the saturation and minor hysteresis loops of steels can be calculated using the main parameters of the magnetic material: \( H_{cs} \), \( M_s \) and \( M_{rs} \).

To validate the legitimacy of using Eqs. (1) – (3), (6), and (7), Figs. 2 and 3 relate the calculation results using these formulas to calculation results using formulas (1) -- (3) [2], which include five magnetic parameters: \( H_{cs} \), \( M_s \), \( M_{rs} \), \( \chi_a \) and \( M_C \). The magnetic parameters used in this computational experiment for several materials are listed in the table:

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>Status steel</th>
<th>( \chi_a )</th>
<th>( H_{cs} ), kA/m</th>
<th>( M_s ), kA/m</th>
<th>( M_{rs} ), kA/m</th>
<th>( M_C ), kA/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>ShX15</td>
<td>Delivery condition</td>
<td>110</td>
<td>0.848</td>
<td>1115</td>
<td>780</td>
<td>290</td>
</tr>
<tr>
<td>ShX15</td>
<td>quenching of the 850°C</td>
<td>44</td>
<td>4.750</td>
<td>1165</td>
<td>713</td>
<td>390</td>
</tr>
<tr>
<td>40X</td>
<td>quenching of the 850°C, tempering at 200°C</td>
<td>75</td>
<td>2.200</td>
<td>1381</td>
<td>723</td>
<td>380</td>
</tr>
</tbody>
</table>

In Figs. 2 and 3, the results of calculating the \( M(H) \) using Eqs. (1) – (3), (6) and (7) are compared with the results calculated using Eqs. (1) – (3) [2], using five magnetic parameters: \( H_{cs} \), \( M_s \), \( M_{rs} \), \( \chi_a \) and \( M_C \).

For the analysis, the same steels were used (steel 40X, ShX15 in the yield state and steel ShX15 tempered), as in [1] for the experimental verification of the results of calculations by formulas (1) – (3). In their magnetic parameters, they adequately cover the range of variation of the magnetic properties of steels.
Data shown in Figures 2 and 3 illustrate that the results of calculation according to known [2] formulas (1) – (3) substantially coincide with the results of calculation according to these formula using the expressions (6) and (7). Thus, the use of the formulas (6) and (7) together with the formulas (1) – (3) eliminates the need to know the parameters \( \chi_a \) and \( M_C \) to calculate the main magnetization curve and the minor hysteresis loops of ferromagnetic steels. These relationships can be calculated from \( H_{cs} \), \( M_s \) and \( M_{rs} \) of steel on the saturation hysteresis loop. Data on \( H_{cs} \), \( M_s \) and \( M_{rs} \) for almost all known steels are referenced in the literature.

![Figure 2](image2.png)

**Fig. 2.** The results of the calculation of basic magnetization curves of materials from the table (1, 1' - of grade ShX15 steel in delivery condition; 2, 2' - of grade 40X steel; 3, 3' - of grade ShX15 steel of quenching) using Eqs. [2] (1, 2, 3) using develop Eqs. (1', 2', 3').

![Figure 3](image3.png)

**Fig. 3.** Calculation results of the descending branches of the saturation (1, 1' - \( H_m = 49.1 \) kA/m) and minor (2, 2' - \( H_m = 8.49 \) kA/m; 3, 3' - \( H_m = 5.2 \) kA/m; 4, 4' - \( H_m = 2.0 \) kA/m) hysteresis loops of materials from the table (0 - steel ShX15 tempered; b - steel ShX15 in yield state) using Eqs. [2] (1 – 4) and using develop in [6, 7] Eqs. (1' – 4').

2. Refinement of the developed formulas.

Studies [11 – 15] have shown that the formulas (6) and (7) must be made more precise. An analysis [11] of the results of measurements performed in [4] of \( H_{cs} \), \( M_{rs} \) and \( \mu_m \) of 855 different steels has shown that an improved accuracy of calculation of \( \mu_m \) compared to (4) is provided by the equation:
The average relative error $\delta$ of calculation of $\mu_m$ for steels according to Eq. (8) is less than 10%. It is 1.58 times smaller than $\delta$ of calculation of $\mu_m$ according to Eq. (1) and is less than the sum of measurement errors obtained using standard methods [3] of measuring parameters in this equation.

Even better results were obtained in [12] for calculating $\mu_m$ of magnetically hard steels (with $4 \text{kA/m} \leq H_{cs} < 8 \text{kA/m}$) (SI):

$$\mu_m = [0.5 + 0.053H_{cs} - (0.068H_{cs})^2]M_{rs}/H_{cs} \quad . \tag{9}$$

Calculation of $\mu_m$ by (9) provides an order of magnitude smaller $\delta$ than the calculation of $\mu_m$ of these steels by (1), and a two-times smaller error than the $\delta$ of measuring $\mu_m$ according to a standard procedure [3].

In [13, 14], it was established that the ratio (3) is valid for steels with $2 \text{kA/m} \leq H_{cs} \leq 7.4 \text{kA/m}$. Data from [10], based on which the equation (3) in [6, 7] has been extended to lower values of $H_{cs}$, were shown to be incorrect. This is due to the fact that the experiments on the influence of the annealing temperature of steels on their magnetic properties, the results of which are summarized in [10] and used in [6, 7], were performed in an open magnetic circuit on samples with a demagnetizing factor $N = 0.00172 \div 0.002$. The inappropriateness of use of the measured $\mu_m$ and $\mu_a$ of samples in an open magnetic circuit to determine the $\mu_m$ and $\mu_a$ of the material of samples has been shown in [16].

Using the formula (8) and findings of [13, 14], a reliable relation between $\mu_a$, $\mu_m$ and $H_{cs}$ of steels was established in [15]:

$$\mu_a = \frac{[0.5 + 0.06H_{cs} - (0.068H_{cs})^2]M_{rs}}{(2.9 + 35e^{-1.75H_{cs}})H_{cs}} \quad . \tag{10}$$

The results of statistical analysis of the relation between reliable measurement results for $\mu_a$ in [4] and the results of calculation of $\mu_a$ using Eq. (10) have shown [15] that Eq. (10) can be used to calculate $\mu_a$ of steels based on $H_{cs}$ and $M_{rs}$. The value $\delta = 10.7\%$ for the calculation of $\mu_a$ using Eq. (10) is smaller than the sum of the measurement errors obtained using the standard methods [3] for measuring parameters in Eq. (10).

Taking (8) and (10) into account, the following equations must be used in order to calculate changes in the magnetization of steels on the main magnetization curve and on the minor hysteresis loops along with Eqs. (1) – (3) instead of Eqs. (6) and (7):

$$M_c = 0.67[0.5 + 0.06H_{cs} - (0.068H_{cs})^2]M_{rs} \quad , \tag{11}$$

$$\chi_a = \frac{[0.5 + 0.06H_{cs} - (0.068H_{cs})^2]M_{rs}}{(2.9 + 35e^{-1.75H_{cs}})H_{cs}} - 1 \quad . \tag{12}$$

Using Eq. (9) for magnetically hard steels, even more accurate results may be obtained with the following equations:

$$M_c = 0.67[0.5 + 0.053H_{cs} - (0.06H_{cs})^2]M_{rs} \quad , \tag{13}$$

$$\chi_a = \frac{[0.5 + 0.053H_{cs} - (0.06H_{cs})^2]M_{rs}}{3H_{cs}} - 1 \quad . \tag{14}$$

3. Calculation of the residual magnetization and the coercive force of the minor hysteresis loops.

For calculation of the residual magnetization $M_r$ and the coercive force $H_c$ of the minor hysteresis loops from the $H_{cs}$, $M_s$ and $M_{rs}$, taking Eqs. (11) – (14) into account, a formula developed in [2] may be used, which has the following compact notation [17]:
\[
M_r = \pm \frac{M_s}{\pi} \frac{H_m^2}{H_m^2 + kH_s^2} \left[ \pi M_{rs} - \sum_{n=0}^{1} \arctg \frac{H_{cs} + (-1)^n H_m}{H_0} \right], \tag{15}
\]

\[
h_c = 1 - T^{-1} \arctg \{0.5 \sum_{n=3}^{2} \arctg [T (1 + (-1)^n h_m)] \}, \tag{16}
\]

where: \(h_c = H_c / H_{cs}\); \(T = \arctg (\pi K_S / 2); K_S = M_{rs} / M_s\); \(h_m = H_m / H_{cs}\).

Thus, the changes \(M_r\) and \(H_c\) in a minor hysteresis loop of steels are fully determined changes in the magnetic parameters of its saturation hysteresis loop. The values of \(M_r\) and \(H_c\) of steels in any minor hysteresis loops can be calculated by Eqs. (15) and (16) at the accuracy sufficient for practice using \(H_{cs}, M_s, M_{rs}\) these steels and maximum magnetizing field \(H_m\) in a minor hysteresis loop.

As an example, Fig. 4 shows the results of calculation the dependence of \(H_c\) os grade 50 steel on temperature \(T_t\) of tempering after quenching using Eq. (16).

![Fig. 4. \(H_c\) of grade 50 steel vs. temperature \(T_t\) of tempering after quenching at \(H_m = 60\) kA/m (1); 5 kA/m (2); 3 kA/m (3); 2 kA/m (4) and 1 kA/m (5). Calculation using Eq. (16).](image)

In order to find the dependencies \(H_c(T_t)\) at different \(H_m\), the dependencies \(H_{cs}(T_t), M_{rs}(T_t)\) and \(M_s(T_t)\) for the steel 50 were used. The latter dependencies were presented in [18, Fig. 4a, 4b and 5a]. The form of the calculated dependencies \(H_c(T_t)\) changes with decreasing \(H_m\) in the same manner as the form of the experimentally measured [18, 4a] dependencies \(H_c(T_t)\) at decreasing maximum induction \(B_m\) of minor hysteresis loops of the steel 50. The \(B_m\) of a minor hysteresis loop of steel can be calculated from its \(H_{cs}, M_s, M_{rs}\) and maximum strength of the magnetizing field \(H_m\) using Eq. (15).

The authors of [18 – 21] and other articles tried to explain the observed changes in the form of the dependencies of \(H_c\) of steels on the mode of the technological impact with a changing \(B_m\) by some "peculiarities" of the impact of structural changes of steel on its magnetic properties measured in weak fields.

These explanations are not satisfying, however: in fact, the changing form of the dependencies of \(H_c\) of steels on the mode of the technological impact at changing \(H_m\) (or \(B_m\)) on a minor hysteresis loop is completely determined by changes in its \(H_{cs}, M_s, M_{rs}\) and reflects the physics of the magnetization reversal of ferromagnetic materials in weak magnetic fields. Which structural changes of the metal caused changes in its \(H_{cs}, M_s, M_{rs}\) is not significant. The phenomenon that \(H_c\) of a magnetically soft material is larger than \(H_c\) of a harder magnetic material in the magnetic reversal in weak fields follows (Fig. 5) from the dependence Eq. (16).
Fig. 5. Coercive force $H_c$ of a minor hysteresis loop at $H_m = 1$ kA/m (1, 1'); 3 kA/m (2, 2') and 5 kA/m (3, 3') vs. coercive force $H_{cs}$ of a saturation hysteresis loop for steels with $K_S = (1, 2, 3) 0.8$ and $(1', 2', 3') 0.4$.

Calculation by Eq. (16).

In [17], the structural and phase sensitivities of the coercive force $H_c$ of minor hysteresis loops of steels are analyzed. This analysis made it possible to develop the following practical recommendations on the use of this parameter for the magnetic structural analysis of steels:

1. If the dependence of the coercive force $H_{cs}$ of steel from a certain technological factor has a monotonic and unambiguous, to measure coercive force $H_c$ of this steel in minor hysteresis loop for its magnetic structure analysis is meaningless.

2. The prerequisite of a possible application of parameter $H_c$ for magnetic structural and phase analysis is the ambiguous character of the change in $H_{cs}$ of a steel during a change in technological factors in the course of a monotonic change $M_s$ at constant or oppositely changing residual magnetization $M_{rs}$. Steels with rather low values of $K_S$ ($K_S \leq 0.6$) from the region of an effective use of the proposed method.

3. The condition of a structural sensitivity of $H_c$ of steel in minor hysteresis loop is a monotonic change of residual magnetization $M_{rs}$ of the steel under the action of technological factors at constant $H_{cs}$ and $M_s$. In this case, the sensitivity of coercive force $H_c$ of steel to the structural transformations in it that change its $M_{rs}$ can be higher than the sensitivity of parameter $M_{rs}$ by factor of 1.5.

4. The condition of a phase sensitivity of $H_c$ of steel in minor hysteresis loop is a monotonic change of magnetization $M_s$ of the steel under the action of technological factors at constant $H_{cs}$ and $M_{rs}$. In this case, the sensitivity of coercive force $H_c$ of steel to the phase transformations in it that change its $M_s$ can be higher than the sensitivity of parameter $M_s$ by factor of 1.5.

5. When implementing the control to items (2) – (4), the optimum condition of using $H_c$ of steel for controlling the change in its structural or phase state that are caused by breaks in the technological process of steelmaking are met at the maximum magnetizing field $H_m$ in a minor hysteresis loop, which is close to the coercive force $H_{cs}$ of the steel treated according to a given for a given technological schedule.


1. The validity of use in technical calculations of the Eq. (1) – (3), (11) and (12) or (13) and (14) is established for describing the change in the magnetization of ferromagnetic steels on the main magnetization curve and the symmetrical saturation and minor hysteresis loops using the main magnetic parameters of the material – the coercive force $H_{cs}$, the saturation magnetization $M_s$ and the residual magnetization $M_{rs}$, measured on the maximum hysteresis loop.
2. It is shown that specific changes in the form of dependency of the coercive force $H_c$ of the minor hysteresis loops of steels on the mode of technological impact as the field strength $H_m$ of the magnetizing field (or the maximum induction $B_m$) changes on a minor hysteresis loop are completely determined by the changes in its $H_{cs}$, $M_s$ and $M_{rs}$ and match the physics of the magnetization reversal of the ferromagnetic material in weak magnetic fields.

3. Important practical recommendations are provided on the use of the coercive force $H_c$ of minor hysteresis loops for the magnetic structural analysis of steels.

References