Vibration Signal Modeling for a Planetary Gear Set

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Abstract. For a planetary gear set, the measured vibration at the sensor location contains vibration components from multiple sources including planet gears, the sun gear, and the ring gear subject to corresponding transmission path effects. In this study, considering multiple vibration sources and the corresponding transmission path effects, we propose a comprehensive vibration model for a healthy planetary gear set. Individual vibration components are generated with a lumped-parameter dynamic model of a planetary gear set. The overall transmission path for each individual vibration component consists of two parts: the path inside the gearbox to the casing and the path along the casing to the sensor point. The effect of overall transmission path on the resultant vibration is investigated. Through the analysis of the modeled resultant vibration signals, we reveal some vibration characteristics of a planetary gear set. At the end, the proposed vibration model is validated with lab experimental data.

1. Introduction

Planetary gear sets are widely used in heavy industry applications such as wind turbines and helicopters as they can provide a high speed ratio and torque-to-weight ratio within a compact structure. A planetary gear set, normally, consists of a centrally rotating sun gear, a ring gear, several planet gears that mesh with the sun gear and the ring gear simultaneously, and one carrier that holds the planets. Different transmission kinematic and transmission ratios can be achieved by one planetary gear set with different input and output configurations. In this study, the planetary gear set has the sun gear as the input, the carrier as the output and the ring gear as the fixed component.

In a planetary gear set, multiple sun-planet gear pairs are in mesh simultaneously with similar vibration forms but different phases. Similar principles also apply to the multiple ring-planet meshes [1]. Moreover, ring-planet meshes mesh simultaneously with the sun-planet meshes. In addition, transmission paths of the vibration signals to the transducer are time varying due to the carrier rotation [1]. Multiple vibration sources and time-varying transmission path effects lead to the complexity of the vibration for a planetary gear set.

For the understanding on the vibration characteristics of planetary gear sets, researchers have conducted extensive investigations with both mathematical models and dynamic models [1]. McFadden and Smith [1] presented one model to explain the asymmetry of the modulation sidebands and the suppression of the meshing frequency in the vibration spectrum for a simple planetary gear set. Inalpolat and Kahraman [2] proposed a mathematical model to describe the mechanisms leading to modulation sidebands of planetary gear sets. Their model simulation shows that any planetary gear set can be classified into one of five distinct groups according to their sideband behavior in terms of...
frequencies and amplitudes. Feng and Zuo [3] proposed one mathematical model to summarize the spectral characteristics of planetary gear set vibration signals for the purpose of fault diagnosis. Inalpolat and Kahraman [4] adopted the gear mesh force spectra to illustrate the modulation sidebands of planetary gear sets with gear manufacturing errors. Liang, Zuo and Hoseini [5] obtained the resultant vibration signal of a planetary gearbox by incorporating the multiple vibration sources and the transmission path effect. The resultant vibration in [5] was obtained by the weighted summation of each planet gear vibration but without sun gear vibration and ring gear vibration. In practical applications, the sensors are commonly mounted on the housing of a gearbox to acquire the vibration from every gear component, which is illustrated by Feng et al. [3] and Forrester [6]. Accordingly, in our study, a more comprehensive signal model for a planetary gear set is proposed. The proposed vibration model contains the sun gear vibration, ring gear vibration and each planet gear vibration. All these vibration components are subject to corresponding transmission path effects. A paper with more detailed work has been submitted to Measurement for possible publication [7].

2. Modeling of Vibration Signals

2.1 Dynamic Modeling of a Planetary Gear Set

The lumped-parameter vibration model developed by Liang et al. [5] is adopted in this study. Fig. 1 shows the planetary gear set dynamic model. The local coordinate systems for gears are fixed on the carrier. The global coordinate system is fixed on the ground.

The gear mesh interface is modeled as a spring-damper system. Equations of motion for the dynamic model can be found in [5]. The mesh stiffness evaluation for a perfect gear set can be found in [8]. The calculation for the gear mesh damping can be found in [10]. The physical parameters of the planetary gear set are listed in Table 1. In this dynamic model, the ring gear is fixed and the four planet gears are spaced equally. The rotational speed of the sun gear is 46.667r/min. By solving the equations of motion numerically, vibration signals induced by the sun gear, each planet gear and the ring gear can be obtained.
Table 1. Physical parameters of the planetary gear set [8]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sun gear</th>
<th>Planet gear</th>
<th>Ring gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>19</td>
<td>31</td>
<td>81</td>
</tr>
<tr>
<td>Module (mm)</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>20°</td>
<td>20°</td>
<td>20°</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>0.7</td>
<td>1.822</td>
<td>5.982</td>
</tr>
<tr>
<td>Young's modulus (Pa)</td>
<td>2.068×10^{11}</td>
<td>2.068×10^{11}</td>
<td>2.068×10^{11}</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Base circle radius (mm)</td>
<td>28.3</td>
<td>46.2</td>
<td>120.8</td>
</tr>
<tr>
<td>Bearing stiffness</td>
<td>$k_{sx} = k_{sy} = k_{rx} = k_{ry} = k_{cx} = k_{cy} = k_{pnx} = k_{pny} = 1.0 \times 10^8 \text{ N/m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bearing damping</td>
<td>$c_{sx} = c_{sy} = c_{rx} = c_{ry} = c_{cx} = c_{cy} = c_{pnx} = c_{pny} = 1.5 \times 10^3 \text{ Ns/m}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2 Modeling of Transmission Path Effect

As discussed in [3], the vibration signal is mainly transferred through the following transmission path to the transducer position: meshing point → planet → ring → casing → sensor. Accordingly, in this study, only this transmission path is considered and modeled. The transmission path contains two parts: part inside the gearbox from the meshing point to the casing and the other part along the casing to the sensor point.

2.2.1 Modeling for the transmission path inside the planetary gearbox

As the transmission path inside the gearbox has a constant length even with the carrier rotation, in this study, constants $S_p$, $S_r$ and $S_s$ are used to represent the transmission path effects inside the gearbox for planet gears, the ring gear and the sun gear, respectively. For their possible values we have following considerations and assumptions:

As sensors are mounted on the casing, it is reasonable to assume there is no attenuation for the ring gear vibration transferring to the casing. Thus, we assume $S_r = 1$;

As the sun gear vibration transfers to the casing through the planet gears, one value smaller than 1 should be chosen for $S_p$. The value is varying with different levels of attenuation caused by the transmission path;

As modeled by the equations of motion in the dynamic model for planet gears, the planet gear vibrations have two excitation sources [5]: sun-planet gear meshing dynamic force and ring-planet gear meshing dynamic force. Based on the lengths of transmission paths from these two excitation sources to the casing, the transmission path effect inside the gearbox for planet gear vibrations can be expressed as

$$S_p = \alpha S_r + b S_s,$$

(1)

where $\alpha$ and $b$ are constants to represent the weights of ring-planet meshing induced planet vibration and sun-planet meshing induced planet vibration in the overall planet vibration.

2.2.2 Modeling for the transmission path along the casing

Depend on the properties of a planetary gearbox, like its size and the ring gear flexibility, the sensor-perceived vibration signal has two different situations when one planet gear is at the farthest position from the transducer point: (1) the vibration signal is perceived by the transducer but with a certain attenuation and (2) no vibration signal is perceived by the transducer. The transmission path effects of these two situations are modeled with different
window functions as expressed by Eq. 2 and Eq. 3, generated based on the Hamming function, in this study.

\( w_i(t) = \alpha - (1 - \alpha) \cos(w_c t + \psi_n) \),

where \( \alpha \) controls the bandwidth of the window function with \( 0 < \alpha < 1 \).

\( w_i(t) = e^{\beta(\text{mod}(w_c t + \psi_n, 2\pi) - \pi)^2} (0.54 - 0.46 \cos(w_c t + \psi_n)) \),

where \( \beta \) controls the bandwidth of the Hamming function with \( \beta < 0 \).

In Eq. 2 and Eq. 3, \( w_c \) is the carrier angular frequency and \( \psi_n \) is the initial phase angle for \( n \)th planet gear. Fig. 2 demonstrates the window shapes defined by Eq. 2 and Eq. 3 with different values of \( \alpha \) and \( \beta \).

Fig. 2. Transmission path effects

2.3 Modeling of Resultant Vibration

The overall resultant vibration for a planetary gear set is modeled by Eq. 4 as the weighted summation of the sun gear vibration, ring gear vibration and planet gear vibrations.

\[
\alpha(t) = \sum_{i=1}^{N} S_s w_i(t) \frac{1}{N} a_s + \sum_{i=1}^{N} S_r w_i(t) \frac{1}{N} a_r + \sum_{i=1}^{N} S_p w_i(t) a_{pi},
\]

where \( \alpha(t) \) is the resultant vibration; \( N \) is the number of planet gears; \( a_s \), \( a_r \) and \( a_{pi} \) are the sun gear vibration, the ring gear vibration and the \( i \)th planet gear vibration, respectively.

In this proposed model, the overall transmission path effect is modeled as the product of the transmission path effect inside the gearbox and the transmission path effect along the casing. The sun gear vibration is assumed to transfer through the \( N \) planet gears to the casing equally, meaning that \( 1/N \) of sun gear vibration will go through a planet gear to the casing, as expressed by the first term. Similarly, the ring gear vibration is assumed to transfer from the \( N \) different planet-ring meshing points to the transducer position equally so that there is the second term in Eq. 4.

3. Resultant Vibration Signal Analysis

3.1 Properties of Resultant Vibration Signal

In this section, resultant vibration of a planetary gear set without fault is modeled and the vibration properties are investigated. Acceleration signals are all in vertical (y-) direction.

Fig. 3 illustrates the wave form of the resultant vibration with transmission path effect parameters as \( S_s = 0.8 \), \( S_p = 0.9 \), \( S_r = 1 \) and \( \alpha = 0.6 \) (case 1). The resultant vibration is obtained by the weighted summation of the vibration components. Observed from Fig. 3, the resultant vibration signal is subject to the amplitude modulation (AM) but it is not necessary to be symmetric about horizontal axis any more. This phenomenon is in good agreement with
the experimental vibration signal as shown in Fig. 8 later. The possible rationale is the phase differences between vibration components which lead to some vibration is strengthened while some is attenuated. The envelope of the resultant vibration signal fluctuates 4 times within one revolution of the carrier, consistent with the number of planet gears.

Fig. 4 illustrates the frequency spectrum of the resultant vibration signals for case 1. With tooth numbers in Table 1 and the input rotational speed as 46.667r/min, characteristic frequencies of the planetary gear set can be calculated [3] and obtained as follows: \( f_s = 0.7778 \text{Hz} \) (sun gear rotation frequency), \( f_p = 0.23836 \text{Hz} \) (planet gear rotation frequency), \( f_c = 0.1478 \text{Hz} \) (carrier rotation frequency), \( f_{p-p} = 0.5913 \text{Hz} \) (planet gear passing frequency), \( f_m = 11.971 \text{Hz} \) (meshing frequency). The frequency spectrum is zoomed in from 0Hz to 90Hz for analysis, i.e. the low frequency band of the frequency spectrum is focused.

The planetary gear set is with equally spaced planets and sequentially phased gear meshes as defined in [2] according to parameters in Table 1. As illustrated in Fig. 4, there are nearly zero amplitude at the meshing frequency and its harmonics, which agrees well with the result in [2]. Sizable amplitudes are found at following locations: \( n f_m \pm f_c \) (11.83Hz, 36.05Hz, 59.7Hz, 83.94Hz) if \( n \) is an odd integer (\( n=1,3,5,7 \)), and \( n f_m \pm 2f_c \) (23.65Hz, 24.24Hz, 71.53Hz, 72.12Hz) if \( n \) is an even integer but not 4 (\( n=2,6 \)). These locations are compatible well with the conclusions in [5].

Overall, the proposed resultant vibration not only reflects the amplitude modulation induced by the time-varying transmission path but also indicates that the transducer-perceived vibration signal is not necessary to be symmetric about horizontal axis in time domain which is one result of the phase differences between multiple vibration sources. In frequency domain, no further frequency components are found by the proposed resultant vibration, compared with the results in [5]. It is reasonable because the frequency component caused by the sun gear vibration and ring gear vibration can be indicated by the planet gear vibrations [5].

![Fig. 3. Resultant vibration of case 1](image)

![Fig. 4. Frequency spectrum of case 1](image)

3.2 Influence of Different Transmission Path Effects

To illustrate the influence of different transmission paths on the resultant vibration, three more cases are considered: \( S_r = 1, S_s = 0.6, S_p = 0.8, \beta = 0.55 \) (case 2); \( S_r = 1, S_s = 0.5, S_p = 0.75, \alpha = -0.2 \) (case 3); \( S_r = 1, S_s = 0.4, S_p = 0.7, \alpha = -1 \) (case 4). The waveforms of resultant vibration signals for these three cases are illustrated in Fig. 5. The bandwidths of the transmission path effects along the casing is decreasing from case 1 to case 4 and the values of \( S_p \) and \( S_s \) are decreasing correspondingly. The smaller bandwidth and smaller \( S_p \) and \( S_s \) mean the greater attenuation induced by the gear system. As shown in Fig. 3 and Fig. 5, with the attenuation increasing, the amplitude of the resultant vibration
decreases while the degree of amplitude modulation increases. Similar as case 1, none of case 2, case 3 and case 4 has the symmetric wave form with the horizontal axis.

Fig. 5. Resultant vibration wave forms of case 2, case 3 and case 4

Fig. 6 illustrates the frequency spectra of the 4 cases. As demonstrated in Fig. 6, for case 1 to case 3, with the attenuation increasing, amplitudes at $f = n f_m + f_c$ where $n$ is odd decrease while the amplitudes at $f = n f_m + 2f_c$ where $n$ is even almost keep constant; moreover, the amplitude at the planet gear passing frequency is almost 0. On the other hand, if the attenuation is such great as in case 4, greater non-zero amplitude appears at the planet gear passing frequency while amplitudes of other frequency components have smaller values when compared with those in case 1 to case 3.

Fig. 6. Frequency spectra of resultant vibration signals

4. Experimental Validation

In this section, the resultant vibration signal modeling method is validated by the experimental vibration signal. Fig. 7 shows the configuration of the planetary gearbox test rig. Table 2 lists the number of teeth for each gear. All gears are standard spur gears without tooth profile modification and manufacturing errors. An acceleration sensor was installed vertically on the casing of the second stage planetary gearbox. The second stage planetary
gear set has the same configuration and gear parameters as the simulated one does. The 2\textsuperscript{nd} stage of the experimental planetary gear set had the same characteristic frequencies with the simulated planetary gear set with the driving motor speed as 1200r/min.

![Fig. 7. Planetary gearbox test rig](image)

<table>
<thead>
<tr>
<th>Table 2. Parameters of experimental planetary test rig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gearbox Gear</td>
</tr>
<tr>
<td>Teeth No.</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>18</td>
</tr>
</tbody>
</table>

Fig. 8 shows the wave form of the experimental vibration signal in the vertical direction. As demonstrated, the amplitude modulation presents and signal envelope fluctuates four times within one carrier revolution period. Moreover, the experimental vibration signal is not symmetric about the horizontal axis. As modeled resultant vibration reflects these two properties as discussed in Section 2, the proposed model is validated in time domain by the experimental vibration to some extent.

Fig. 9 illustrates the frequency spectrum of the experimental vibration. The spectrum was zoomed in from 0Hz to 90Hz. As case 1 matches best with the experimental vibration in time domain, the spectrum of case 1 was chosen to compare with the spectrum of the experimental vibration. It can be found that the experimental vibration frequency spectrum is much more complicated with more frequency components and sidebands. This makes sense because of the complexity of the practical planetary gearbox test rig and the fact that the transducer mounted on the second stage planetary gearbox can acquire the vibration from other components of the test rig. On the other hand, when looked into details in Fig. 9, there are sizable amplitudes at following positions: 11.83Hz, 23.65Hz, 24.24Hz, 36.05Hz, 59.7Hz, and 83.94Hz. All these frequency components are with sizable amplitudes in the spectrum of case 1. Moreover, the amplitude at the planet gear passing frequency, 0.5913Hz, is almost 0. These characteristics are compatible well with those of case 1. Thus, the experimental data validates the modeled resultant vibration in frequency domain to some extent.

![Fig. 8. Experimental vibration in time domain](image)

![Fig. 9. Frequency spectrum of experimental vibration](image)
5. Conclusions

In this study, one resultant vibration signal modeling method for a planetary gear set is proposed. Vibration sources from the sun gear, the planet gear and the ring gear are simulated by a lumped parameter dynamic model. The overall transmission path has two parts: the part inside the gearbox from each gear to the casing and the part along the casing to the transducer position. The transmission path effects are modeled to reflect the attenuation induced by the gear system. Incorporating all the vibration sources and corresponding transmission path effects, the resultant vibration signals at the transducer position are modeled. In the time domain, the resultant vibration signal gets increasing level of amplitude modulation with the increase of the attenuation and vice versa. Moreover, the resultant vibration signal is not necessary to be symmetric about the horizontal axis in time domain because of the phase differences between vibration sources. In frequency domain, the spectrum structure is analyzed and the frequency components with sizable amplitude are located, which agrees well with reported studies. Finally, the resultant vibration signal modeled by the proposed modeling method is validated by experimental data in both time and frequency domains.

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References