Image Reconstruction from Compressed Measurements for Ultrasound NDT

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Abstract
In ultrasound NDT, Delay-and-Sum schemes are commonly used to reconstruct images from the measurement data. For single channel pulse-echo measurements, this is called Synthetic Aperture Focusing Technique (SAFT). In a multi-channel setup, SAFT is extended to the Total Focusing Method (TFM), where the focused image is reconstructed from measurements of all transmit-receive combinations, called Full Matrix Capture (FMC). In previous work, we showed that both SAFT and TFM can be cast as a sparse recovery problem that enables enhancing the Delay-and-Sum scheme to a physically motivated forward model that leads to improved image quality. The reconstruction is performed using l1 minimization. Further, this also allows to perform the reconstruction from compressed/subsampled measurements following compressed sensing theory. This subsampling was achieved by only measuring a subset of the frequency coefficients of the signal. In both the single channel and the multi-channel setup the amount of measurement data is thereby significantly reduced. Additionally, for the TFM, we added a spatial subsampling by only considering a subset of transmit and receive pairs, further reducing the measurement data and at the same time also reducing the measurement time.

In our previous work, the frequency and spatial dimensions were considered separately for simplicity, using the same compression strategy for all spatial measurements. In this work we consider joint frequency/spatial compression schemes. We show that a joint multidimensional compression strategy where different Fourier subsets are considered for each channel pair leads to a superior reconstruction performance when the compression rate is fixed. Analysis based on an analytical model for a single defect and a real-world example with multiple defects shows that the approach works in practice.

Keywords: Full Matrix Capture (FMC), Compressed Sensing, Sparse Array

1. Introduction
Defect localization is a central task in Ultrasound Nondestructive Testing (UNDT). Imaging approaches and techniques aimed at improving localization accuracy and mitigating measurement and processing costs are of widespread interest. In particular, inverse problem approaches for the post-processing of measurement data are currently being studied from different perspectives. A sparse recovery formulation for the Total Focusing Method (TFM) is presented in [1] with the goal of improving resolution. An alternative with reduced computation was later formulated in [2] where sparse recovery is performed on the TFM data instead of the Full Matrix Capture (FMC) data. We have shown in [3] that such sparse recovery models lend themselves well to compressed sensing and can be employed to carry out reconstructions from subsampled FMC data where only a small number of channels is present, and further, when only few Fourier coefficients from these channels are available. Similarly, the authors of [4] have studied the impact of the aperture size when applying compressed sensing to the reconstruction of Radio Frequency (RF) ultrasound data when the channels have been subsampled.

In this work, we build upon these inverse problem formulations by studying channel and Fourier coefficient selection techniques based on Cramér-Rao bounds with the goal of reducing measurement times without sacrificing resolution. This approach, which we have theoretically elaborated upon in [5], offers geometric explanations to observations related to the importance
of array aperture size that have been made in [4] and [6]. Additionally, it comes as an alternative to sparse array design techniques that are based on the minimization of side lobes such as the ones studied in [7] via simulated annealing and in [8] with genetic algorithms. These conventional techniques look for an array structure that remains fixed. In contrast, our approach aims to subsample a pre-existing array and offers more flexibility by allowing software-based subsampling techniques.

2. Sparse Array Design

Sparse arrays can be designed optimally based on a target performance metric. For unbiased estimators, the Cramér-Rao bound is a lower bound on the best (lowest) possible performance in terms of estimation variance [9]. It depends only on the data model and can be used to optimize system parameters independently of reconstruction techniques.

2.1 FMC Model

We focus on a scenario with a single scatterer for simplicity. Results derived from this scenario apply to multiple scatterers when they are well separated, and the theory can be extended for closely spaced scatterers. When only a single scatterer is present inside a test specimen, a single-channel measurement in the frequency domain can be written as

$$b_k(f) = a \cdot e^{i\phi} \cdot p(f) \cdot e^{-j2\pi f \tau_k(x,z)} \cdot g_k(x,z) + n(f).$$  \hspace{1cm} (1)

In this model, the scatterer has an associated amplitude $a$ and a phase $\phi$. This complex weight multiplies the a-priori known frequency spectrum $p(f)$ of a pulse shape emitted by the transmitting ultrasound transducer. For a channel index $k$, there is an associated transmitting (Tx) and receiving (Rx) element whose geometrical relationship with respect to the scatterer results in a time delay $\tau_k(x,z)$ that depends on the scatterer’s horizontal coordinate $x$ and depth $z$. Furthermore, there is an attenuation term $g_k(x,z)$ that considers the transducers’ directional responses. Finally, $n(f)$ represents the noise.

A total of $K = M_R \cdot M_T$ channels are collected, where $M_T$ Tx elements transmit sequentially, and a set of $M_R$ Rx elements receives during each transmission. These measurements are gathered in a vector $\mathbf{b}(f) \in \mathbb{C}^K$ that contains the frequency domain representation of an FMC measurement.

2.2 Cramér-Rao Bounds

Considering measurement data following model (1) with additive white Gaussian noise (AWGN), the Cramér-Rao Bounds (CRB) can be calculated straightforwardly. The Fisher information matrix for this scenario is given by [9]

$$J = \frac{2}{\sigma^2} \left( \frac{\partial \mathbf{b}}{\partial \xi^T} \right)^H \left( \frac{\partial \mathbf{b}}{\partial \xi^T} \right) \in \mathbb{R}^{4 \times 4},$$  \hspace{1cm} (2)

where $\sigma^2$ is the noise variance and $\xi^T = [x \ z \ a \ \phi]$ is a vector containing the unknown parameters in the model. The CRB is given by $\text{tr}(J^{-1})$, with $\text{tr}(\cdot)$ representing the trace of a matrix. This can be calculated numerically or in closed form in the case of a far field scatterer [5].

2.3 Array Design
The active elements in a sparse array can be selected by considering a cost function based on the CRB. To this end, we have proposed the expression

$$\min_{\mathcal{K}} \max_{x,z} \text{tr} \{ J^{-1} \}$$

(3)

in which $\mathcal{K}$ denotes the set of active Tx-Rx pairs. In this minmax optimization problem, the goal is to, given a fixed $M_R$ and $M_T$, find the set of channels for which the worst-case scenario CRB for a single reflector is the lowest. Such a channel selection has the lowest estimation variance even for the worst case scatterer location. Solving this problem is costly, since a large number of scatterer locations and channel combinations must be tested in order to find the best one. However, scenarios that yield faster measurement times are of special interest: this is achieved when the number of transmitters is small. Two scenarios of this kind are compared.

2.1.1 Constant Rx Approach

In the constant Rx approach, a single set of $M_R$ receivers is selected. For each of the $M_T$ Tx elements, the same set of receivers is employed. This optimization constraint requires $\binom{M_T}{M_R} \cdot \binom{M}{M_R}$ channel setups to be tested, where $\binom{a}{b}$ denotes “a choose b” and $M$ is the number of elements of a Uniform Linear Array (ULA). This constraint admits an exhaustive search in a reasonable amount of time for small arrays.

2.1.2 Varying Rx Approach

The varying Rx approach lets each of the Tx elements have a different set of $M_R$ receivers. Here, $\binom{M}{M_T} \cdot \binom{M}{M_R}^{M_T}$ channel setups must be tested, and an exhaustive search is intractable. Instead, a suboptimal solution can be sought after through a greedy search. This can be done by first selecting one transmitter. For this Tx element, choose the Rx element that results in the lowest CRB. Rx elements are added in such a manner until $M_R$ have been selected. This is done for ever Tx element, and the one with the lowest overall CRB is chosen. Finally, this process is repeated $M_T$ times. This greedy varying Rx approach compares a number of channel setups on order of $K \cdot M^2$ and is much faster than both the full varying Rx and the constant Rx models. In the remainder of this study, we refer to “greedy varying Rx” simply as “varying Rx”, since the calculation of the full varying Rx approach is too costly and was not considered.

2.1.3 Fourier Coefficients

Fourier coefficients may be subsampled in several fashions. In [10], the authors propose different techniques in which this can be done. Two of them are studied in this work. The first strategy is to keep $N_F$ Fourier coefficients chosen symmetrically around the center frequency of the pulse shape. We refer to this strategy as “centered”. The second strategy utilizes the Amplitude Spectral Density (ASD) $p(f)$ as a pseudo probability density function to randomly draw the $N_F$ coefficients, and we refer to this approach as “ASD”. The former captures the most signal energy, while the latter has a larger effective bandwidth.

3. Results and Discussion

The aforementioned approaches are combined in order to yield the four combinations of “Constant Rx centered”, “Constant Rx ASD”, “Varying Rx centered”, and “Varying Rx ASD”. 
In order to compare them, a synthetic data set \( b \) with a single scatterer located in the far field under a ULA’s centroid is generated. The underlying reflectivity information is reconstructed by using the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [11]. The result of doing this is a Point Spread Function (PSF) for each of the design approaches. These PSFs stem from the presence of noise and non-convergence of FISTA in a set amount of iterations. After amplitude normalization, the Array Performance Indicator (API) metric [12] is used as a point of comparison. The API measures the cross section area of a PSF for a given amplitude threshold level. Additionally, the Frobenius norm of the PSF above each amplitude threshold level quantifies the pseudo energy of the PSF.

![Example of point spread functions obtained from different array design techniques. The horizontal (a) and vertical (b) cross sections are shown.](image)

**Fig. 1** Example of point spread functions obtained from different array design techniques. The horizontal (a) and vertical (b) cross sections are shown.
The PSFs obtained from compressed sensing reconstructions are shown in Fig. 1. Qualitatively, narrower PSFs indicate better imaging performance, since the ideal case would be a single peak. Observe that varying Rx approaches outperform constant Rx approaches in horizontal resolution regardless of the choice of Fourier coefficients. Furthermore, the centered strategy for Fourier coefficient selection results in smaller side lobes. The difference among PSFs along the vertical axis is not as pronounced. In this case, the overall combination of channel and Fourier coefficient selection is more important than the channel selection approach, and the constant Rx ASD method shows the best performance, but its large side lobes in the horizontal direction must also be considered. To account for this, the API is employed next.

Fig. 2 Aperture performance indicator (a) and pseudo energy (b) of each point spread function. The curves correspond directly to the point spread functions in Fig. 1.
A quantitative evaluation of the array design approaches is shown in Fig. 2. After normalizing their amplitudes, narrower PSFs have smaller cross section areas and lower pseudo energy content. The plots portray that varying Rx approaches have smaller API values at low threshold amplitudes and are therefore less likely to obscure small scatterers with artefacts. The pseudo energy also reveals that the varying Rx ASD approach performs the best, with overall smaller side lobes.

As a final way of comparison, we take the constant Rx centered and the varying Rx ASD array design approaches and apply them to real FMC measurement data of a specimen with holes drilled along its diagonals.
Fig. 3 Reconstructions of a specimen with a single set of holes drilled along its diagonal. They correspond constant Rx centered (a) and varying Rx ASD (b) array designs.

Qualitative assessment of these reconstructions shows that the constant Rx centered approach results in artefacts that make the estimation of location and number of defects difficult. In contrast, the varying Rx ASD array design has allowed a satisfactory reconstruction from a small number of channels and Fourier coefficients.

4. Conclusions

In this work we have studied sparse array design approaches for compressed sensing applied to Full Matrix Capture data. The design approaches revolve around the minimization of a cost function based on Cramér-Rao bounds with different structure constraints. This is a different approach from conventional side lobe minimization techniques and inherently considers the geometric properties that lead to performance improvements stemming from larger aperture sizes that have been observed in prior studies.

Qualitatively and quantitatively, the proposed CRB-based array design technique with a varying Rx channel structure and Fourier coefficients chosen using the waveform’s amplitude spectral density as a pseudo probability density function offers the best performance. This multidimensional compression strategy results in a small number of transmit events for which different receiving elements are active, and each channel uses a different set of Fourier coefficients. Reduced measurement times without loss of reconstruction accuracy entail, and experiments with real data.

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References


