An insight on the impact of the local and global nonlinearities on the features of propagating guided ultrasonic waves

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Abstract

The propagation of guided ultrasonic waves is a phenomenon of a burgeoning interest in the fields of structural health monitoring and non-destructive testing. In particular, Lamb waves are a peculiar representative of the guided ultrasonic waves propagating in plate-like structures. In spite of their multi-modal and dispersive features, they are often utilised in damage detection methods. However, those methods are based on examination of the so-called linear features of Lamb waves (i.e. change of time-of-flight or wave amplitude), which in some damage cases may not be sufficient to observe the degradation of the structure. Thus, the nonlinear features of Lamb waves exhibited in the spectral domain are exploited. Various sources of nonlinearity can be differentiated, i.e. with local characteristics (breathing motion of crack) and global (nonlinear characteristic of the structures’ material), where each of them has their own impact on the spectral features of the propagating wave. As the result, a complex behaviour is obtained which can only be analysed in the proper way through the numerical simulation. Hence, this work discusses the results of numerical simulations, where the impact of the excitation magnitude and choice of the frequency on the wave spectral features is investigated. Moreover, influence of the respective local and global sources of nonlinearity is also examined. This leads to the conclusion of the appropriate approach to observe the desired phenomenon, which can be further applied in the experimental investigation.

Keywords: Structural Health Monitoring, Modelling and Simulation, Wave Propagation, Lamb Waves, Fatigue Crack, Nonlinear Material

1 Introduction

The analysis of the nonlinear wave propagation in the plate-like structures became a burgeoning area of expertise in the fields of Non-Destructive Testing (NDT) and Structural Health Monitoring (SHM) in recent years. The advantage of the aforementioned analysis methods is exhibited through the possibility to detect fatigue damage in its embryo stage. The methods based on the linear change of the propagating wave (i.e. change of the wave amplitude or time-of-flight) are not capable to achieve such a goal. In contrast to the linear methods, nonlinear ones investigate the spectrum of acquired time responses of examined structures. A number of techniques is available in order to investigate the excited nonlinearity in the structure response. A comprehensive review on those methods can be found in [1]. The method based on the analysis of the generated higher-order harmonics [2] is of the particular interest. Mainly due to its sensitivity to the nonlinear events exhibited in the examined structure while the guided ultrasonic waves propagate through it. The higher-order harmonics can be generated due to numerous sources, such as: the breathing phenomenon between the interfaces of the fatigue crack [3], de-bonding of the interfaces [4], the adhesive which bonds a transducer to the structure [5] and also the material of the structure itself. The most popular approach to observe the influence of the above-mentioned sources, is to examine the characteristics of the second harmonic. Such an approach is dictated by the fact that the magnitudes of the higher-order harmonics drop dramatically with the
increasing order [6]. This makes the second harmonic (the next after the fundamental frequency) the most approachable one in terms of observability of changes to investigate the impact of different sources of nonlinearity on the propagating wave. However, this feature brings also the complexity to the analysis. In the real life system it is nearly impossible to differentiate level of contribution of each nonlinearity source to the overall magnitude of the second harmonic. Therefore, to overcome this problem, one should direct his attention to the numerical solution, which allows analysing the influence of each source of nonlinearity separately.

A number of numerical techniques are available in the field of wave propagation modelling. Among the most popular, one can find the Finite Element Method (FEM), Finite Difference Method (FDM), Local Interaction Simulation Approach (LISA) or Cellular Automata for Elastodynamics (CAFE). However, a new local method – recently implemented for nonlinear wave propagation [7] and based on the last to methods – is used in this paper. Moreover, the hybrid method was adjusted, based on theory of the spring model [8], to allow the interaction between the element interfaces. Such an action allows modelling a “breathing” crack phenomenon.

The structure of the paper goes as follows. The hybrid modelling approach is briefly discussed in Section 2, along with the theory of the Lamb wave internal resonance. The modelling scheme of the “breathing” crack phenomenon is explained in Section 3. In Section 4 the results of the numerical investigation are presented and discussed. Finally, the paper is concluded in Section 5.

2 Modelling of Nonlinear Lamb Waves

2.1 Nonlinear Lamb Waves – Internal Resonance

The internal resonance is a phenomenon exhibited between two wave modes which propagate in the nonlinear medium. The constitutive relation of such medium can be described through the strain energy density function as [6]

$$W = \frac{\lambda}{2} I_1^2 + \mu I_2 + \frac{\mathcal{A}}{3} I_3 + B I_1 I_2 + C I_1^3, \tag{1}$$

where $\lambda$ and $\mu$ are the Lamé constants and $\mathcal{A}$, $B$ and $C$ are the third-order elastic constants. In (1) the strain energy density function has been limited to quadratic and cubic term in strains only. The $I_1$, $I_2$ and $I_3$ are the first, second and third isotropic invariants of the Green-Lagrange strain tensor $\varepsilon_{GL}$, which takes form

$$\varepsilon_{GL} = \frac{1}{2} \left( H + H^T + H^TH \right) \tag{2}$$

The displacement gradient is signified by $H$ and $T$ in the superscript implies the transposition of the matrix. In order to obtain the nonlinear wave equation, the second Piola-Kirchhoff stress tensor $\sigma$
should be found through the differentiation of the energy density in the domain of the Green-Lagrange strain tensor as

$$\sigma = \frac{\partial W(\varepsilon_{GL})}{\partial \varepsilon_{GL}}.$$  (3)

Thus, one can retrieve the nonlinear wave equation in the form

$$\nabla \cdot (F \sigma) = \rho \frac{\partial^2 \mathbf{U}}{\partial t^2},$$  (4)

where $\mathbf{U} = [u \ v]^T$ is the particle displacement vector in 2D space, $\rho$ is material's density, $t$ is time and $F$ denotes the deformation gradient tensor.

The solution of (4) is difficult to obtain, particularly due to the presence of the higher-order strain components. However, assuming that the nonlinear wave equation describes only two wavefields: (i) of the primary wave and (ii) of the secondary wavefield, the perturbation method can be applied in the form as

$$\mathbf{U} = \mathbf{U}^{(1)} + \mathbf{U}^{(2)}, \quad \mathbf{U}^{(1)} \gg \mathbf{U}^{(2)},$$  (5)

which divides the nonlinear wave equation in to the set of two linear ones, one homogeneous and one inhomogeneous, each for one wavefield. This condition holds only when the magnitude of the primary wave field ($\mathbf{U}^{(1)}$) is much larger than of the secondary one ($\mathbf{U}^{(2)}$). Therefore, assuming that the considered strain terms from (1) are taken up to and including those multiplied by the third-order (i.e. neglecting the fundamental and the DC components in the secondary wavefield) elastic constants, the following equation is obtained for the description of the secondary wavefield [17],

$$\mathbf{U}^{(2)}(x, y, t) = \frac{1}{2} \sum_{m=1}^{\infty} A_m(x) U_m(y) e^{-2\omega t} + c. c.,$$  (6)

where $c. c.$ is the complex conjugate. $A_m(x)$ is the modal amplitude of the $m$th mode in the secondary wavefield at the frequency of $2\omega$ and it takes form as

$$A_m(x) = f_{n, m}^{\text{surf}} + f_{n, m}^{\text{vol}} \begin{cases} \frac{i}{2\xi - 2\xi} \left( e^{i\xi x} - e^{i\xi_n x} \right) & \text{for } \xi_n \neq 2\xi \\ x e^{i\xi x} & \text{for } \xi_n = 2\xi \end{cases}$$  (7)

In the above-presented equation the power flux transferred through structures surface and volume due to the propagating primary mode (designated with the wavenumber $\xi$) to the secondary mode (designated with the wavenumber $\xi_n$) are $f_{n, m}^{\text{surf}}$ and $f_{n, m}^{\text{vol}}$, respectively.

For more details regarding the derivation of the presented equations, the following reference is recommended [10]. Through the analysis of the received solution for the secondary wavefield, the conditions for the exact and approximate internal resonance between the primary and the secondary
mode can be stated. The exact internal resonance is obtained, when the wavenumber of the secondary wave at the frequency of $2\omega$ is of the double of the wavenumber corresponding to the primary wave at the $\omega$ frequency and the non-zero power flux is present ($f^{\text{surf}}_n + f^{\text{pol}}_n \neq 0$). In the result, the cumulative growth of the secondary wavefields magnitude can be observed. In the case, however, if the aforementioned relation between the corresponding wavenumbers is not satisfied, the approximate solution will be obtained as described in the first case in (7). The non-zero power flux still has to be presented for the approximated internal resonance.

2.2 The Hybrid Local Modelling Approach for Nonlinear Wave Propagation

Local computational modelling techniques, e.g. LISA or CAFE, offer great flexibility and efficiency for high frequency wave propagation simulations. Previous works [11, 12] demonstrated great application potential of those methods, but indicated certain deficiencies. For instance, LISA offers the Sharp Interface Model for accurate treatment of discontinuities, however – due to specific displacement-based derivation – its application to nonlinear media leads to non-unique formulations. CAFE, on the other hand, handles arbitrary constitutive relations but requires special treatment of boundary conditions. A hybrid approach, combining advantages of LISA and CAFE, was therefore of interest, particularly for wave propagation in nonlinear media and for wave-damage interaction studies and it was presented in [7]. A brief explanation is shown in the next part of this section.

The hybrid method, follows from a combination of the Local Interaction Simulation Approach and the Cellular Automata for Elastodynamics. The method – in contrast to classical LISA – relies on the stress-based form of the equilibrium equation as shown in (4). The stress formulas, are evaluated from displacements in respective ith cell nodes only, following the relation $\sigma = f(U)$. Please note that – analogously to the LISA and CAFE methods – the cells are treated as discontinuous, and a set of eight independent equations describes the four cells time-varying displacement independently for the two-dimensional model. Subsequently, it is assumed that accelerations in all four cells converge towards a common value. Finally, equilibrium equations from the cells are summed and stress continuity between adjacent cells faces are imposed, leading to the final – stress based – iteration equations. It should be emphasized that particular numerical properties of LISA and CAFE – related to accurate treatment of material interfaces – are due to the Sharp Interface Model obtained through stress continuity conditions. Moreover, the iteration equations depend on the character of function $f(U)$ relating stresses and displacements. For the linear case, it resembles the standard LISA iteration equations [11]. However, the form of iteration equations of the hybrid approach enables arbitrary constitutive and geometric relations, e.g. $f(U)$ may include nonlinear stress-strain dependence and the Green-Lagrange strain formulation.
3 Modelling of Higher-order Harmonics Generation Due to Nonlinear “Breathing” Crack

The “breathing” crack phenomenon is observed when a propagating ultrasonic wave reaches the interface of the crack, causing the opening and closing of the gap between the crack interfaces. As a result, the higher-order harmonics are induced into the incident wave [1]. The crack closes due to the compressional part of the propagating wave, which reaches the crack interface. While the crack is closed, it is assumed that the wave penetrates through it undisturbed and propagates further into the structure. The tensile part causes the opening of the crack resulting in the partial wave reflection.

In order to simulate such a phenomenon within the framework of the hybrid local approach, a concept of the spring model has to be implemented between the interfaces of the considered elements. This approach follows the derivation framework of the Spring Model (SM) method presented in [8]. The SM method was introduced as a spin-off of the LISA to analyse the interaction between bulk waves and a crack, with a demonstrated capability to depict the non-contact and flawed interfaces. The discretization scheme and material definition in SM follow those of LISA. The major difference between the two methods is found in the nodal displacement analysis. In a 2-D case in LISA, each nodal point belongs to four cells; while in SM, each nodal point is divided into four sub-nodal points, each of which belongs to a cell. Then, the relations between the sub-nodes are defined as “tensorial” springs and used for force representation. The forces between the sub-nodes within one cell are named external forces \( F \) and the forces between the cells, introduced in order to keep the continuity of the structure, are named internal forces \( f \). When summing the discretized forces applied on each of the four cells, one can receive four sets of interaction equations. The consideration of internal force relations between the cells allows to model different kinds of imperfect contacts between the two interfaces. In order to consider an imperfect contact between the interfaces in a fatigue crack, the contact quality factor \( Q_{kl} \) was introduced in SM. Its value, modeling the contact imperfection, varies from 0 to 1. In this study, the factor \( Q_{kl} \) was used to model the “breathing” crack phenomenon. \( Q_{kl} \) was set to 1 when the compressional part of the propagating wave was reaching the crack surfaces; while it was set to 0 for the tensile part.

4 Numerical Results

A two-dimensional numerical model was prepared within the framework of the hybrid local approach for wave propagation simulation. Aluminium was chosen as the material with the following properties: Young’s modulus \( E = 68.9 \) GPa, Poisson’s ratio \( \nu = 0.33 \), density \( \rho = 2700 \) kg/m\(^3\) and Landu’s third order elastic constants as \( A = -320 \) GPa, \( B = -200 \) GPa and \( C = -190 \) GPa. The thickness of the beam was set as 4 mm and the length was equal to 1000 mm. For the damaged case, the “breathing” crack was set at the distance of 300 mm from the left hand side of the beam and its depth was equal to 50% of the beam thickness. The gap between the crack interfaces was assumed as infinitesimal. The excitation was assigned to the top and bottom left corners of the beam as the in-plane displacement with a set of amplitudes in order to check their influence on the excited second harmonic.
due to the material nonlinearity and the presence of the “breathing” crack. Such a type of excitation will impose only the symmetric Lamb wave modes to be imposed into the structure. Twenty-cycle sine signal multiplied by the Hanning window with the centre frequency 892.5 kHz was applied. This will lead to the excitation of the S1-s2 resonant mode pair will be excited if the material of the beam is defined as nonlinear, which is also exhibited in Figure 1.

![Lamb wave dispersion curves](image)

Figure 1: Lamb wave dispersion curves for 4 mm thick aluminum beam used in numerical simulations.

Finally, the in-plane displacement responses were collected from the upper surface of the beam every 50 mm. In order to extract the second harmonic waves, each simulation setup was performed twice, however the signals were in the opposite phases. In the result, through a simple summation of the signals, the second harmonic field can be extracted.

First, the propagation characteristics of the second harmonic s2 mode are presented in the propagation distance domain (see Figure 2). The results are obtained from the models when the “breathing” crack and material nonlinearity are executed separately. Third type of models takes both nonlinearities into consideration.

It can be noticed that the approximate synchronisation is achieved between the first harmonic S1 mode and second harmonic s2 mode, which is exhibited by the cumulative growth of the second harmonic amplitude with the propagation distance (see Figure 2a). Moreover, it can be seen that as the amplitude of the excitation decreases the trend of the second harmonic goes toward the exact solution – linear growth – as shown in [10]. Next, the influence of the “breathing” crack and the change of the excitation amplitude on the generated second harmonic should be taken into consideration as shown in Figure 2b. It can be observed that the second harmonic amplitude obtained on the distance after the crack has rather constant value. Such a behaviour is expected to appear due to the local characteristics of the crack itself. The first harmonic interacts with the crack and due to such an event the second harmonic is generated into the structure. Then it continues to propagate into the structure undisturbed with constant amplitude as predicted within 2-D plane strain framework. The excitation amplitude does not have a major influence on the second harmonic mode features, which is in contrast to the results obtained from the models with both sources of nonlinearity being present.
One can notice that in the area before the crack, the cumulative characteristic of the second harmonic is obtained. Such behaviour is the result of the defined nonlinear material. Next, after interaction with the “breathing” crack, for two highest amplitudes of the excitation (i.e. 1.1 μm and 1 μm) a drop in the amplitude can be observed which continues to grow with the propagation distance. However, different characteristic of the secondary mode can be seen for the lower excitation amplitude. The obtained behaviour is very similar to the one obtained when the model had only the “breathing” crack as the source of nonlinearity. Based on an observation it can be said that for the lower values of the excitation amplitude, the materials nonlinearity does not have a significant influence on the generated second harmonic mode. In order to have a deeper understanding of the presented phenomena the comparison of the time signals of the second harmonic magnitude for two excitation amplitudes – 1.1μm and 0.95μm – is presented in Figure 3.

One can notice that the second harmonic generated from the nonlinear material and the one generated from the interaction with the “breathing” crack are in the opposite phase. Such behaviour may explain the drop of the amplitude of the second harmonic in the area after the crack, when both sources of nonlinearity are taken into consideration. Moreover, it can be clearly noticed that for the lower (0.95
μm) excitation amplitude the second harmonic from the “breathing” crack has the dominant value and for the higher value (1.1 μm) the dominant is the secondary wave generated due to the nonlinear material.

5 Conclusions

It was observed in the presented paper that the excitation amplitude has a significant influence on the generated second harmonic, when a high frequency S1-s2 Lamb wave mode pair is taken into consideration. It was noticed that for the amplitude equal or higher than 1μm both considered sources of nonlinearity have an observable impact on the second harmonic magnitude. However, if the excited amplitude is lower than the mentioned value, the “breathing” crack have a greater impact on the second harmonic value than the nonlinear material. Further investigations are required to explain such a phenomenon.

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References