Calculated bandwidth dependent DGS and DAC curves for Phased Array Sizing

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Abstract
Angle beam probes, single element or phased array, based on the trueDGS® technology have a fully modelled special transducer shape generating rotational symmetric sound fields. These probes reach an up to date unachieved DGS accuracy for angle beam probes using the general DGS diagram as published in the EN ISO 16811:2012. The limitation to evaluate only sound paths above 0.7 near field length can be overcome by calculating a bandwidth dependent DGS diagram for these probes resulting in high DGS accuracy for the entire range of sound paths. Furthermore, the mathematical model of these probes compensates for curved coupling surfaces, ensuring the DGS accuracy even in these cases. Recording DAC curves using phased array probes is very time consuming, because for each angle used a separate DAC curve has to be recorded. By using trueDGS® phased array probes it is sufficient to record a single DAC curve for one single angle, the DAC curves for all other angles are calculated.

Keywords: Modeling and Simulation, phased array, Ultrasonic Testing (UT), DGS, DAC, Accuracy, Simplification

1. Introduction and problem statement

The DGS method was developed in 1959 for flat circular straight beam transducers generating rotational symmetric sound fields [1]. Despite this fact the DGS method was extended to angle beam probes which do not generate rotational symmetric sound fields. Most of the angle beam probes in the market have rectangular transducers. But even if they contain circular transducers the sound field is not rotational symmetric due to the influence of refraction, phase shift and mode conversion taking place at the interface between probe wedge and test object.

Applying DGS evaluation for angle beam probes according to the state of the art based on the General DGS Diagram published in the EN ISO 16811:2012 results in oversizing of the reflectors (figure 1) [2], [3]. To avoid the oversizing a new probe technology was developed. The transducers have a special shape, being bent in all dimensions. These angle beam probes generate rotational symmetric sound fields.

![Figure 1: Comparison of DGS evaluation using a probe with rectangular transducer and using a trueDGS® transducer](image-url)
2. *trueDGS®* technology

2.1 Introduction

It was the task to develop an angle beam probe generating a rotational symmetric sound field to overcome the oversizing issue. Therefore the sound field of a flat circular straight beam probe was calculated for shear waves. The question was, how to rotate this sound field for a given angle of incidence. Figure 2 shows the development task [4].

Figure 2: Development task for the *trueDGS®* technology

2.2 Construction method

Given is a straight beam probe with frequency f, diameter D and near field length N calculated for shear waves as well as the angle of incidence β and the delay line length v₇ of the angle beam probe under construction. The left side of figure 3 shows the near field of the straight beam probe used as base for the construction. First the time of flight from the transducer along the acoustic axis to the end of the near field is determined. From this time the time of flight in the wedge is subtracted and the remaining part is applied to the test material under the selected angle of incidence defining the end of the near field of the angle beam probe (point T in the right part of figure 3).
The sound path for each angle $\gamma$ from the straight beam probe is transferred to the angle beam probe under construction considering Snell’s Law (in the plane of projection and in the plane perpendicular to the plane of projection). This method results in a point cloud defining the shape of the trueDGS® transducer. Three additional adaptations are needed:

- Angle dependent phase shift for each sound beam used for calculation
- Based on the different phase shifts of the rim beams a correction of the angle of incidence needs to be applied.
- The construction of the trueDGS® probes is not area invariant: the area of the circular transducer used as base is not the same as the resulting area of the trueDGS® transducer. For DGS evaluation this area ratio has to be taken into account.

Figure 4 shows on the left side the resulting transducer after applying the necessary adaptations. The right side displays the transducer after a coordinate transformation with an extremely spread z-axis. The trueDGS transducer is bent in all dimensions.

2.3 trueDGS® sound fields

For validating the rotational symmetry of the resulting sound field a CIVA simulation for the trueDGS® transducer has been carried out. This simulation has to be seen as an approximation. The starting point was a 2D array. The elements not on the area of the trueDGS® transducer were deactivated. The curvature of the transducer was realized using delay laws. Using CIVA, cross sections perpendicular to the acoustic axis in the distances of 0.7 N, 1 N and 2 N were calculated. Figure 5 shows the comparison of the sound fields of an angle beam probe with a rectangular transducer and a trueDGS® transducer.
In spite of the approximation applied the rotational symmetry of the trueDGS® sound fields is clearly recognizable.

2.4 trueDGS® phased array probes

The construction method described for trueDGS® transducers can be applied to phased array probes as well.

![trueDGS® calculation for phased array probes](image)

**Figure 6: trueDGS® calculation for phased array probes**

First for both, the original and the virtual transducer, longitudinal sections are calculated using the trueDGS® technology. Using positive steering angles the virtual transducer is shifted mathematically in such a way that the lower edges of both transducers coincide and that the beam from the upper rim of the virtual transducer crosses the upper edge of the original transducer. (For negative steering angles the calculations are performed accordingly.). Mathematically the required shift is described as a non-linear system of equations with the following variables:

- Diameter of the circular transducer used as base for the calculation
- Delay line length of the virtual transducer
- Sound exit point of the virtual transducer

This system of equations can be solved numerically. The right side of figure 6 shows the result. After shifting, the delay laws can be determined easily as can be seen from this figure.

2.5 DGS evaluation according to EN ISO 16811:2012

![DGS evaluation according to the EN ISO 16811:2012 using a trueDGS® phased array probe](image)

**Figure 7: DGS evaluation according to the EN ISO 16811:2012 using a trueDGS® phased array probe**
Using the General DGS Diagram published in the EN ISO 16811:2012 maintaining the restriction using only sound paths > 0.7 near field length results in a high accuracy, refer to the left part of figure 7. If the restriction for sound paths > 0.7 N is neglected oversizing occurs again. But the small subfigure on the right side of figure 7 proves there is a way to overcome this issue as well using bandwidth dependent DGS diagrams.

3. Bandwidth dependent DGS diagrams

3.1 Introduction

The DGS method was developed at a time when gain measurement with a resolution of 0.1 dB was not possible. At this time the probes used had lower bandwidths as the probes used today. Today having the possibility of PC based numerical solvers a bandwidth dependent DGS diagram can be calculated for truedGS® transducers. The trueDGS® technology is a precondition because the mathematics are significantly easier with the rotational symmetry. Thus these calculations can be performed without taking the refraction at the interface into account given the fact that a trueDGS angle beam probe behaves like a circular straight beam probe for shear waves.

3.2 Bandwidth dependent DGS evaluation

Bandwidth dependent DGS curves for the back wall echo (BW, equation (1)) as well as for a given equivalent reflector size (ERS, equation (2)) can be calculated (figure 8) based on the trueDGS® technology.

\[
\rho_{BW}(z, t) \propto 4\pi \omega Z \int_0^1 D e^{A(2z - \sqrt{x^2 + y^2})^2} \frac{x y}{\sqrt{x^2 + z^2} + 4z^2} \cos \left( \omega t - k \sqrt{x^2 + 4z^2} \right) \, dx \, dy
\]  

\[
\rho_{ERS}(z, t) \propto \rho_A(z) \int_0^1 D e^{A(2z - \sqrt{x^2 + y^2})^2} \frac{x y}{\sqrt{x^2 + 4z^2}} \cos \left( \omega t - k \sqrt{x^2 + 4z^2} \right) \, dx \, dy
\]

Figure 8: Calculation of bandwidth dependent DGS diagrams for flat bottom holes based on the trueDGS® technology

In this example the DGS curves are calculated for shear waves with a relative bandwidth of 30 %. A change of the bandwidth in the range of ±5 % does not have a significant influence on the evaluation. Figure 9 shows the bandwidth dependent DGS evaluation of measurements carried out with a 4 MHz trueDGS® phased array probe for incidence angles of 53° and 65°. The nominal angle of the probe used is 56°.
When using bandwidth dependent DGS diagrams for trueDGS® probes the restriction using only sound paths > 0.7 N in the EN ISO 16811:2012 can be neglected.

### 3.3 Curved coupling surfaces

Under defined circumstances the EN ISO 16811:2012 does not allow the use of DGS when having curved coupling surfaces.

The European standard EN ISO 16811:2012 requests the matching of the probe wedge for concave coupling surfaces if the diameter of the test object is not large enough to ensure good coupling. For convex coupling surfaces the matching is required when the conditions shown in figure 10 are valid.

The trueDGS® technology can be extended easily to curved surfaces if the probes are calculated accordingly. Thus the restrictions using DGS with curved coupling surfaces in the EN ISO 16811:2012 can be neglected as well.

Even for complex coupling geometries, such as e.g. for a saddle, trueDGS® (phased array) probes can be calculated compensating the effects resulting from the coupling surface such as focussing or defocussing. The only precondition is that the coupling surface can be described mathematically.
Figure 11 shows an example for a solid rail axle from Deutsche Bahn.

**Figure 11: Complex coupling geometry and resulting trueDGS® transducer shape**

4. Bandwidth dependent DAC curves

4.1 Introduction

Bandwidth dependent DAC curves for side drilled holes are only dependent on the probe used: near field length, delay line length and probe sensitivity (figure 12). The diameter of the side drilled holes changes only the sensitivity of the echo response. Based on these facts a general bandwidth dependent DAC curve can be calculated if trueDGS® phased array angle beam probes are used (equation (3)).

![General DAC Diagram](image)

- $z$: Sound path
- $\bar{p}_A(z)$: Sound pressure on the acoustic axis at sound path $z$
- $D$: Transducer diameter
- $A$: Defines the bandwidth
- $\omega$: Angular velocity
- $k$: Angular wavenumber

$$p_{sdh}(z, t) \propto \bar{p}_A(z) \int_0^D e^{A(2z-\sqrt{x^2+4z^2})^2} \frac{x}{\sqrt{x^2+z^2}} \cos \left( \omega t - k \sqrt{x^2+4z^2} \right) \, dx \quad (3)$$

**Figure 12: Calculation of a bandwidth dependent DAC curve for side drilled holes**

4.2 DAC recording

For DAC recording first a single angle of the phased array probe used is selected. For this angle the DAC curve is recorded the same way as using a conventional single element angle beam probe. The pre-calculated curve is only dependent on the probe sensitivity resulting in a vertical shift of the calculated curve and the sound attenuation in the reference block used.
When at least two echoes are recorded the measurements are fit to the curve using the *least squares method* to calculate these two parameters. With each recorded echo the *least squares method* is repeated for all so far recorded measurement values minimizing the deviations between measurement values and calculated curve. Alternatively to the DAC curve time corrected gain (TCG) can be used (figure 13).

![Flaw Detector Display Curve: 4 MHz, 53°](image)

![TCG Setting: 4 MHz, 45°](image)

**Figure 13:** Calculated DAC curve with measurement values, alternatively TCG can be used

### 4.2 Calculated DAC curves and validation

After recording the DAC curve for one single angle, the DAC curve for all other angles can be calculated with a high accuracy as shown in figure 14:

![FD Display Curve Side Drilled Holes: 4 MHz, 45°, 9.5 dB/m](image)

![FD Display Curve Side Drilled Holes: 4 MHz, 53°, 9.5 dB/m](image)

![FD Display Curve Side Drilled Holes: 4 MHz, 60°, 9.5 dB/m](image)

![FD Display Curve Side Drilled Holes: 4 MHz, 70°, 9.5 dB/m](image)

**Figure 14:** Calculated DAC curves with measurement data for validation
For validation of the calculated DAC curves echoes were recorded additionally for 45°, 60° and 70°. These measurement values are displayed in figure 14 together with the calculated DAC curves showing minimal deviations to the curves. The resulting deviations in dB are shown in the following tables:

<table>
<thead>
<tr>
<th>Recorded Angle:</th>
<th>Sound Path/mm</th>
<th>ΔG/dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>18.5</td>
<td>0.23</td>
</tr>
<tr>
<td>60°</td>
<td>48.5</td>
<td>0.13</td>
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<tr>
<td>70°</td>
<td>81.0</td>
<td>-0.68</td>
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<tr>
<td>53°</td>
<td>114.0</td>
<td>0.44</td>
</tr>
<tr>
<td>60°</td>
<td>148.0</td>
<td>-0.13</td>
</tr>
<tr>
<td>70°</td>
<td>180.0</td>
<td>-0.33</td>
</tr>
<tr>
<td>95°</td>
<td>213.5</td>
<td>0.35</td>
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<table>
<thead>
<tr>
<th>Sound Path/mm</th>
<th>ΔG/dB</th>
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</thead>
<tbody>
<tr>
<td>15.0</td>
<td>1.08</td>
</tr>
<tr>
<td>42.0</td>
<td>-0.20</td>
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<tr>
<td>70.0</td>
<td>0.30</td>
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<td>97.0</td>
<td>0.23</td>
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<tr>
<td>127.0</td>
<td>1.14</td>
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<td>153.0</td>
<td>0.77</td>
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<tr>
<td>183.0</td>
<td>0.15</td>
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<table>
<thead>
<tr>
<th>Sound Path/mm</th>
<th>ΔG/dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.5</td>
<td>-1.15</td>
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<tr>
<td>58.0</td>
<td>0.30</td>
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<tr>
<td>97.0</td>
<td>0.06</td>
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<td>138.0</td>
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<tr>
<td>175.0</td>
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<tr>
<td>216.0</td>
<td>1.49</td>
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<tr>
<td>254.0</td>
<td>-0.16</td>
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<table>
<thead>
<tr>
<th>Sound Path/mm</th>
<th>ΔG/dB</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.38</td>
</tr>
<tr>
<td>85.0</td>
<td>1.18</td>
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<tr>
<td>139.0</td>
<td>0.17</td>
</tr>
<tr>
<td>196.0</td>
<td>0.46</td>
</tr>
</tbody>
</table>

5. Conclusions

trueDGS® technology:
- Rotational symmetric sound fields
- High precision of DGS evaluation for all angles within the restriction of sound paths > 0.7 N in the EN ISO 16811:2012 using the General DGS Diagram
- High DGS accuracy even having curved coupling surfaces
- The restrictions using DGS with curved coupling surfaces in the EN ISO 16811:2012 can be neglected.

Bandwidth dependent DGS diagrams:
- High DGS accuracy for the entire range of sound paths and for all angles
- The restriction of sound paths > 0.7 N in the EN ISO 16811:2012 can be neglected.

Calculated bandwidth dependent DAC curves:
- By this approach defect sizing using phased array probes is as easy as using a single element angle beam probe.
- This new approach results in high accuracy and a huge productivity gain.

Patents pending: trueDGS technology and bandwidth dependent DGS curves for flat bottom and side drilled holes

References

3. EN ISO 16811:2012, *Non-destructive testing - Ultrasonic testing - Sensitivity and range setting*