Exploring the limits of limited-angle computed tomography complemented with surface data
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Abstract
Computed tomography investigations are challenging for planar objects, such as printed circuit boards (PCB), microprocessors packaging, etc. A conventional scan with a full rotation around a fixed axis may be impossible for such samples due to potential collisions with the system’s components or due to the limited penetration capability of the source and the finite dynamic range of the detector. Analytical reconstruction techniques are proved unsuitable given the incomplete projection data resulting from limited source trajectory coverage. Iterative algorithms with prior information offer a solution to this reconstruction problem. In this contribution, we demonstrate that, by incorporating surface data in the reconstruction, better contrast homogeneity and more reliable sample boundaries can be reconstructed. A maximum likelihood expectation maximization (MLEM) algorithm is employed in this work. The surface data of the object, which is preregistered to the reconstructed object, is added into the objective function of the reconstruction as prior information. The values of the pixels outside the object’s surface, extracted from the surface scanning, are penalized during the iteration process of the optimization. The proposed algorithm is applied to the limited angle projection data of simulated benchmark phantoms. The contrast of the reconstructed image, and the rooted mean square error between the phantom and the reconstructed image, are calculated to evaluate the performance of the proposed algorithm. Compared to the reconstruction without integrating the surface data, the reconstruction with complementary data gives an 80% higher homogeneity of the contrast, which is important to distinguish different materials and resolve fine features.

Keywords: Reconstruction algorithms, Surface data, Cone beam computed tomography, Multi-modality data fusion

1. Introduction

Computed tomography (CT) is an imaging technique to inspect the interior of a specimen by reconstructing the cross-sectional images from a series of transmission images of the fixed specimen acquired while the radiation source is travelling along a planned trajectory. As a non-destructive measurement technique, CT has been widely used in clinical applications and industrial inspection tasks. The trajectory employed in conventional acquisition protocols is a complete 360° circle or a 180° plus the fan beam opening angle for a short scan [1]. However, the requests from modern industry and research pose new challenges to the conventional acquisition protocols and the corresponding reconstruction algorithms. One of the challenges is the measurement of planar specimens at high spatial resolution under the constraints of acquisition time and inspection costs. A complete scan or a short scan for planar specimens is often prohibited due to collision risks and the finite penetration capability of the radiation beam. Therefore, only transmission images in a limited range of angles are available, resulting in a limited-angle CT reconstruction problem. A general solution to the exact reconstruction from limited-angle projections is impossible because of the violation of Tuy’s sufficiency condition [2]. In addition to the issue arising from the missing projections, the limited number of projections stemming from the acquisition time constraint will introduce severe streaking artifacts in the reconstruction. Another factor rendering the reconstruction more challenging is the noise in the measurement. To achieve sub-micrometer resolution, micro- or nano-focus X-ray sources with limited emitting power are often employed. The limited flux of these sources lead to photo starvation artifacts in the reconstruction.
The most elementary approach to reconstruct an object from limited-angle data is to add blank images to the set of projections for the missing projections and to apply the classical FKD reconstruction algorithm [3]. However, the result suffers from strong limited-angle artifacts. Alternatively, [4][5][6] incorporated a priori data, such as data sparsity and known CAD-models, into the reconstruction to reduce the artifacts.

In this paper, complementary surface data is integrated in a statistical reconstruction algorithm. The surface data is incorporated in the statistical model as a priori to restrict the solution of the reconstruction. Compared to [6], the prior information used in this paper, i.e. surface scanning data [7], is easier to acquire and is task-specific which is important to account for the deviation of the sample from the design. In [8], surface data is incorporated in the reconstruction, too, but the employed ART reconstruction may result in negative reconstruction values and generate streaking artifact.

2. Method

2.1 Problem statement and theoretical restrictions in limited angle reconstruction

The discussion in the section is restricted to CT systems with parallel beam and line detector. Let \( f(s, t) \) be the attenuation coefficient map of the cross section under investigation. The Radon transformation (1) calculates the line integral of the attenuation map along the ray.

\[
p(l, \theta) = \int_{-\infty}^{+\infty} f(l \cos \theta - \tau \sin \theta, l \sin \theta + \tau \cos \theta) d\tau
\]

The line is defined by the distance to the origin \( l \) and the projection angle \( \theta \). The integral of the attenuation, which is also referred as the projection, is denoted as \( p(l, \theta) \).

The reconstruction problem is the inverse operation of the Radon transformation. Louis et al. [9] have proved the reconstruction from complete Radon transformation is an ill-posed problem. Ill-posedness is a complementary concept to well-posedness. A well-posed problem has a unique solution and the solution is a continuous function of the input data or the model is well conditioned if it is discrete. Louis et al. also pointed out that the reconstruction from incomplete Radon transformation is more severely ill-posed in a sense that the solution is neither unique nor stable. The ill-posedness of limited-angle tomography reconstruction can be well understood in the Fourier reconstruction process. In Fourier reconstruction, the projection from each projection angle is transformed into Fourier space and the 1D Fourier transformation results are then transformed in polar coordinates (Fig. 1). As the projection from certain angles is missing, the Fourier space has a null space (4). The presence of the null space gives an uncertainty in the following inverse Fourier transformation (2).

\[
f(s, t) = \int_{-\infty}^{+\infty} P(u, v)e^{i2\pi(u\sin \theta + v\cos \theta)} dudv
\]

\[
tan \theta = \frac{v}{u}
\]

\[
P(u, v) = 0, \text{ if } \theta \in (\theta_1, \theta_2)
\]

In [10], the null space is filled with zeros and achieves a minimum norm solution, which contains strong limited angle artifact. Unless the null space can be estimated from complementary a priori information, the minimum norm solution is the best achievable
solution. In this paper, the surface information from optical measurement is used to explore the null space.

Figure 1. An illustration of the null space. The rearranged 1D Fourier transformation of the sonogram of a cross-like object is plotted in this figure. The area where there is no data is the null space.

2.2. New reconstruction procedure

In this model, the setup is composed of a monochromatic source, a photon counting detector and a 3D object, which is represented as a 3D array of voxels. The attenuation coefficients of the voxels are modelled as a random variable $x$ with $n$ elements, where $n$ equals the number of voxels in the object. The attenuation coefficients vector to be calculated from the reconstruction is a realization of random variable $x$, denoted as $x^*$.

The vector of the detector values, which are the line integral along each ray that arrives on a detector, is modelled as a random variable $y$ with $m$ entries, where $m$ equals the product of the number of projections and the number of pixels on the detector. $y^*$ is an observed value of $y$.

To simplify the model, the detector values, i.e. the entries in $y$, are considered statistically independent. The maximum likelihood expectation maximization (MLEM) reconstruction is a Bayesian process:

$$p_x(x^*|y^*) = \frac{p_y(y^*|x^*)p_x(x^*)}{p_y(y^*)}$$

where $x^* = \{x_1^*, ..., x_j^*, ..., x_n^*\}$

$$y^* = \{y_1^*, ..., y_l^*, ..., y_m^*\}$$

This implies that the probability of $x$ being equal to $x^*$, provided an observation of $y$ is given as $y^*$, is proportional to the product of the prior probability of $x$ being equal to $x^*$ and the likelihood of $y$ being equals to $y^*$ given that $x$ being equal to $x^*$. The $x_m^*$ that maximizes the $p_x(x^*|y^*)$ is the solution to the reconstruction problem.
The prior of $x^*$ is determined by the surface information, which eliminates the uncertainty of the estimation of random variables lying outside the surface. Assume $M$ is a subset of indices, whose spatial position is outside the observed surface, this set provides prior information.

$$x^*_M = \{ x^*_m = x^*_{ij}, i \neq j \}$$  \hspace{1cm} (8)

$$M = \{ m_1, m_2, ..., m_{n_M} \}, n_M \leq n$$  \hspace{1cm} (9)

$$P_z(x^* = x^*_M) = 1$$  \hspace{1cm} (10)

Therefore, the Bayesian estimation can be updated:

$$P_z(x^*_M | y^*) = \frac{P_y(y^* | x^*_M) P_z(x^*_M)}{P_y(y^*)}$$  \hspace{1cm} (11)

Given the current assumptions of the setup in this paper, i.e. monochromatic source, photon-counting detector, the photon emission, photon-matter interaction, photon detection is a cascaded Poisson process [11]. Let $n_{e,i}$ be the number of emitted photons from the source travelling along ray $i$ and the sum of the attenuation coefficient along the ray be $(Ax)_i$, $n_d$ be the number of photons detected by the detector. Based on the Beer-Lambert law of attenuation, the expected number of photons detected is given by eq. (12). Due to the Poisson nature, the probability of detecting $y^*_i$ photons given the expectation follows the probability density function (13). Therefore, the a posteriori estimation of $x^*_M$ is (15). Instead of trying to find the maximizer of (15), a logarithmic operation, which is strictly monotonic, is applied. The resulting function has the same maximizer as (15). The KKT conditions [12] are applied to the function to find the maximizer (17). The maximizer is calculated iteratively using a fixed-point iteration (18).

$$E(n_{d,i}) = n_{e,i} e^{-(Ax)_i}$$  \hspace{1cm} (12)

$$y^*_i = n_{d,i} \sim \text{Poisson}(n_{e,i} e^{-(Ax)_i})$$  \hspace{1cm} (13)

$$P_y(y^* | x^*_M) = \prod_{i=1}^{m} \left( \frac{n_{e,i} e^{-(Ax)_i} y^*_i e^{-n_{e,i} e^{-(Ax)_i}}}{y^*_i!} \right)$$  \hspace{1cm} (14)

$$P_z(x^*_M | y^*) = \frac{1}{P_y(y^*)} \prod_{i=1}^{m} \left( \frac{n_{e,i} e^{-(Ax)_i} y^*_i e^{-n_{e,i} e^{-(Ax)_i}}}{y^*_i!} \right)$$  \hspace{1cm} (15)

$$\ln(P_z(x^*_M | y^*)) = \sum_{i=1}^{m} \left( y^*_i \ln(n_{e,i}) - y^*_i (Ax)_i - n_{e,i} e^{-(Ax)_i} - \ln(y^*_i!) \right) - \ln(P_y(y^*))$$  \hspace{1cm} (16)

$$\frac{\partial \ln(P_y(y^* | x^*_M))}{\partial x_j} = x_{M,j} \sum_{i=1}^{m} \left( -y^*_i a_{i,j} + a_{i,j} n_{e,i} e^{-(Ax)_i} \right) = 0$$  \hspace{1cm} (17)

$$x_{M,j}^{k+1} = x_{M,j}^k \frac{\sum_{i=1}^{m} a_{i,j} n_{e,i} e^{-(Ax)_i}}{\sum_{i=1}^{m} y^*_i a_{i,j}}$$  \hspace{1cm} (18)

### 3. Simulations

#### 3.1 Simulation setup

A virtual cone beam CT setup is used in the simulation (Fig. 2). The X-ray source has a point-like focal spot and the photons generated from the source are assumed to be monochromatic. As for detector, a flat panel detector is used and aligned with the X-ray source, i.e. the central ray from the source hits the central pixel of the detector and is perpendicular to the detector...
surface. The sample is placed on a rotation stage, which is positioned between the source and the detector and is aligned to them such that a) the rotation axis is parallel to the detector, b) the axis intersects with the central ray from the source, and c) the axis is parallel to the pixel columns on the detector. The distance from the rotation axis to the source is 1000 mm while the distance to the detector is 500 mm. The source trajectory is a series of source positions with respect to the intersecting point of the central ray and the rotation axis. The trajectory is used to specify the acquisition geometry. In this experiment, an incomplete circular trajectory \((\theta_1, \theta_2)\) is used during the acquisition to generate limited angle scan data.

![Figure 2](image.png)

Figure 2. The schematic of the virtual CT setup used in the simulation. The source is aligned to the detector such that the central ray from the source is always perpendicular to the detector. The relative position of the source with respect to the detector is fixed during the scan. The object is placed between the source and detector. The relative position of the source with respect to the object forms the source trajectory. In the experiments, the angle range is always from \(\theta_1\) to \(\theta_2\).

### 3.2 Phantom

A simulated phantom with features to test the contrast at a given spatial resolution and uniformity of the reconstruction is used in the simulations and illustrated in Fig. 3a. The Phantom is placed in the described CT system with its geometric center overlapping with the intersecting point of the central ray and the rotation axis. The surface data in the experiment is the outer boundary of the phantom in Fig. 3a.

### 3.3 Simulation configurations

In the first simulation, the angle range during the acquisition is limited to 60 degree with increment of 1 degree, which is referred to as a fine scan in this paper. The acquisition result is the input of the reconstruction program. To illustrate the improvement by incorporating surface information, the reconstruction is carried out with and without surface information and for each case the reconstruction is stopped after 500 iterations. In a second simulation, the scan angle increment is set to 6 degree, which is denoted as a few-view scan in this paper. The same reconstructions are carried out for this experiment.
The central slice of the phantom and the central slices of the reconstruction results are illustrated in Fig. 3. The results shown in Fig. 3b and Fig. 3d are calculated without surface information. The projection values are backprojected to the outside of the object leading to unrecognizable boundaries and non-uniform reconstruction of the object. To describe the uniformity of the reconstruction, the standard deviation of the pixel values in area 1 as indicated in Fig. 3a is calculated for each of the reconstructed images. The results are shown in Table 1. The reconstruction of the parallel structures in the phantom is also affected by the limited scan angle and the number of projections. Fig. 3d shows the reconstruction of the phantom without surface data under the constraint of limited scan angle and limited number of projections. Denote the parallel structures in the phantom as a square wave, then the square wave response of the reconstruction is not homogenous resulting in reduced capability to differentiate materials. To assess the homogeneity, the contrast of the structures in area 2 as illustrated in Fig. 3a is calculated for all the reconstructed images and the standard deviation of the contrast within area 2 for each reconstruction is selected as a measure of the homogeneity. The contrast is calculated as (19)

$$c = \frac{v_{\text{local}}^{\text{max}} - v_{\text{local}}^{\text{min}}}{v_{\text{max}}^{\text{global}} + v_{\text{min}}^{\text{global}}}$$

(19)

Where $v_{\text{max}}^{\text{global}}$ is the maximum pixel value in area 2, and $v_{\text{max}}^{\text{local}}$ indicates the maximum pixel value at a local area. The same rule applies to the minimum values. The mean value of the contrasts at different locations within the area is plotted in Fig. 4 with the standard deviation shown as error bars. The contrast values at the end of the iterations are listed in Table 2.
Figure 4. a) shows a contrast assessment of the reconstruction from fine scan data. The traces show the mean value of the local contrast values with the standard deviation of the contrast values plotted as error bars. The data points represented by circles are from reconstructions without surface data and the points shown as crosses are from reconstructions with surface data; the data in b) is calculated from few-view data.

Table 1. The analysis of pixel values in area 1 after 500 iterations. The mean square error (MSE) between the reconstructed images and the phantom is calculated. The standard deviation of the reconstructed value is also calculated to quantify the uniformity of the reconstruction.

<table>
<thead>
<tr>
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<th>w/o surface data</th>
<th>with surface data</th>
<th>Improvement</th>
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<tbody>
<tr>
<td>Fine scan</td>
<td>MSE 0.077 std 0.2772</td>
<td>MSE 0.056 std 0.2362</td>
<td>Improvement MSE 27.27% std 14.79%</td>
</tr>
<tr>
<td>Few view</td>
<td>MSE 0.0516 std 0.2208</td>
<td>MSE 0.0382 std 0.1955</td>
<td>Improvement MSE 25.97% std 11.46%</td>
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Table 2. The analysis of pixel values in area 2 after 500 iterations. The mean value of the local contrast values and the standard deviation of the local contrast values are shown in this table as a measure of the homogeneousness of the square wave response.

<table>
<thead>
<tr>
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<th>with surface data</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine scan</td>
<td>mean 0.9181 std 0.037</td>
<td>mean 0.9402 std 0.0189</td>
<td>Improvement mean 2.41% std 48.92%</td>
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<tr>
<td>Few view</td>
<td>mean 0.609 std 0.2193</td>
<td>mean 0.8874 std 0.0283</td>
<td>Improvement mean 45.71% std 87.10%</td>
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4. Conclusions

Limited angle reconstruction is a typical problem in industrial CT measurements, especially when measuring planar objects which do not allow a conventional full circular scan trajectory. Meanwhile, other constraints such as limited number of projections, add more challenge to the reconstruction process. In this paper, surface data from optical measurements, which limits the image support, is used as a priori to constrain the solution of the reconstruction. An MLEM algorithm is developed to incorporate the surface data. The simulation results show promising improvement gained with the surface data. For the phantom used in this paper, under the conditions of the given trajectories and other scan parameters, the reconstruction with surface data leads to a better reconstruction of uniform areas and better resolved parallel structures. From the simulation results, the reconstruction of the uniform area with surface data achieved at least a 20% decrease in MSE and a 10% improvement in uniformity while the homogeneity for parallel structures is improved by 80% in the few-view configuration.
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6. Reference