Elastic guided wave based assessment of laminate composite material constants

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Abstract

In the paper the results of piezoelectrically induced elastic guided waves and laser Doppler vibrometry application to non-destructive estimation of laminate composite plate elastic properties are presented and discussed. The reconstruction procedure is based on the genetic algorithm minimization of discrepancies between the theoretically calculated and experimentally measured guided wave dispersion characteristics in the course of variation of input elastic constants in the corresponding computer model. Group velocities or wavelengths of fundamental Lamb waves measured at varying excitation frequencies and propagation directions serve as a data input to the identification algorithm. The proposed approach has been experimentally validated with unidirectional and cross-ply carbon-fiber reinforced plastic plate samples. An additional confirmation by conventional tensile tests has been also performed.

Keywords: guided waves (GW), layered composites, effective elastic properties, laser vibrometry

1. Introduction

Composite materials play an increasing role in critical engineering applications thus becoming one of the main NDT and SHM objects. As an indispensable part, elastic guided wave (GW) based inspection of composites assumes mathematical and computer modelling of wave processes in investigated structures to predict wave velocities and radiation diagrams, to locate defects based on signal’s time of flight (ToF), to tune optimal frequencies and allocation of piezoelectric active sensors, and so on. Adequate simulation of these phenomena requires a prior knowledge of elastic material constants. In contrast to isotropic materials, the composites represent a challenging task due to the lamination and anisotropy of elastic properties. Moreover, effective elastic moduli, required for adequate wave simulation, quite often cannot be obtained by simple averaging the properties of composite constituents and, thus, characterization methods are needed. One more reason for the development of methods for fast nonintrusive estimation of composite properties is their time-dependent degradation due to the accumulation of micro-defects, heat damage, humidity changes, etc. Therefore, regular in-situ monitoring of composite effective elastic moduli may serve as an independent integrity testing method.

The direct determination of elastic constants by the conventional mechanical testing techniques is complicated since these approaches are destructive in nature and incapable of reconstructing all the elastic moduli from a single test. Vibration-based and ultrasonic wave propagation approaches are more advantageous over the conventional techniques due to their nondestructive manner and a possibility of estimating several moduli simultaneously. Experimentally obtained vibration eigenfrequencies are used for the prediction of composite mechanical properties [1], while the ultrasonic techniques utilize either elastic bulk waves or guided waves [2,3]. The limitation, typical for both bulk wave and vibration-based methods, is that they require specially prepared samples and usually utilize massive and bulky equipment. At the same time, the GW-based methods may be directly implemented to the engineering structure under investigation.
due to the rapid development of modern light-weight excitation and sensing devices, such as piezoelectric active wafer sensors (PWAS) [4] or fiber-optic Bragg grating sensors (FBG) [5].

The widespread approach, which utilizes guided waves for material characterization, is yielded by the direct dependence of the GW dispersion characteristics, e.g., wavenumbers or group velocities, from composite effective elastic properties. Their identification is achieved through the minimization of the objective function, which is built on a metric between the theoretically calculated and experimentally measured dispersion curves, in the course of elastic constants variation. For the successful practical implementation of this technique, a precise GW dispersive data extraction is essential, as well as a fast and reliable forward models for GW dispersion curve computation for anisotropic layered composites is needed. Recent advances in signal processing tools for GW dispersion extraction [6,7] and development of efficient semi-analytical and FEM-based numerical algorithms for GW computation [8,9] yielded to the growth of such procedures. Another key-point of the inverse procedure is the choice of the proper optimization technique for the objective function minimization. In this context, various stochastic-based approaches, e.g., genetic algorithms (GA), are of great appeal due to the multimodal and dispersive nature of GW.

In this paper, the application of the aforementioned GW based approach for the identification of effective elastic constants of laminate composite plates made from transversely-isotropic carbon fibre-reinforced plastic prepregs is presented and discussed. The group velocities and/or wavelengths of fundamental antisymmetric and symmetric PWAS excited Lamb waves, acquired with a contactless laser vibrometer along the composite’s axes of symmetry at varying excitation central frequencies, serve as input data for the inverse restoration of specific elastic stiffness constants $C_{ij}$ of every sublayer. The theoretical modelling is performed in the context of general linear elastodynamics for 3D laminate anisotropic media and is based on the integral representations in terms of the Green’s matrix of the multilayered structure considered. The discrepancy minimization is achieved using a micro genetic algorithm.

The paper is structures as follows: after the introduction the semi-analytical mathematical model is briefly described, and then the experimental techniques applied for dispersion curves estimation are presented. After that, the results of the algorithm application to unidirectional and cross-ply laminates are shown and some conclusions are given.

2. Theoretical background

Time-harmonic oscillations $u(x,\omega)e^{-i\omega t}$, $u = (u_x, u_y, u_z) = (u_1, u_2, u_3)$, $x = (x, y, z) = (x_1, x_2, x_3)$ of a free layered anisotropic material of thickness $H$ excited by the surface load $q(x,\omega)e^{-i\omega t}$ localized in the area $\Omega$ at the top surface $z = 0$ are considered (the time-harmonic factor $e^{-i\omega t}$ is further omitted). The problem geometry is sketched in Figure 1. The technique described here is applicable to layered composite structures with general anisotropy and arbitrary thickness of each sublayer. Below it is assumed that the composite is fabricated from $M$ identical transversely-isotropic prepregs ideally bonded with each other. Therefore, the mechanical properties of the composite are characterized by the prepreg’s stiffness matrix $C_{ij}$, density $\rho$ and lamination scheme. At the same time transversely-isotropic material is described by five independent elastic moduli in the stiffness matrix or by five engineering constants, namely, two Young’s moduli $E_x, E_y$, two shear moduli $G_{yz}, G_{xy}$ and one Poisson coefficient $\nu_{xy}$. The direction of the axis $x = x_1$ coincides with the fiber orientation in the first layer.
Within the semi-analytical integral approach the displacement field \( \mathbf{u}(\mathbf{x}, \omega) \) is represented via the convolution of the Green’s matrix for the layered anisotropic structure considered \( k(\mathbf{x}, \omega) \) and the surface load vector-function \( \mathbf{q}(\mathbf{x}, \omega) \):

\[
\mathbf{u} = k * \mathbf{q} = \int_{\Omega} k(x - \xi, y - \eta, z) \mathbf{q}(\xi, \eta) d\xi d\eta
\]  

(1)

Applying the Fourier transform to the latter equation, which is possible due to the geometry of the problem considered, it might be rewritten in the following form:

\[
\mathbf{u}(\mathbf{x}) = F_{xy}^{-1}[K\mathbf{Q}] = \frac{1}{4\pi^2} \int_{\gamma}^{2\pi} K(\alpha, \gamma, z)\mathbf{Q}(\alpha, \gamma)e^{-i\alpha \cos(\gamma - \varphi)} d\gamma \, d\alpha
\]  

(2)

where \( K = F_{xy}[k], \mathbf{Q} = F_{xy}[\mathbf{q}] \) are Fourier symbols of the Green’s matrix and load vector \( \mathbf{q} \). In Eq. (2) the Cartesian variables \( \mathbf{x} \) and the Fourier parameters \( \alpha = (\alpha_1, \alpha_2) \) are taken in the cylindrical and polar coordinates \( (r, \varphi, z) \) and \( (\alpha, \gamma) \) respectively. Integration contour \( \Gamma^+ \) goes in the complex plane \( \alpha \) along the real semi-axis \( \Re \alpha \geq 0, \Im \alpha = 0 \), bypassing real poles \( \tilde{\zeta}_n = \zeta_n(\gamma) > 0 \) of the matrix \( K \) elements according to the principle of limiting absorption.

Utilizing the fast and reliable algorithms of Green’s matrix Fourier symbol calculation [10], the relations (2) allow efficient wavefield evaluation in the vicinity of the loading area \( \Omega \). However, with increasing distance \( r \), the computational expenses also increase due to the oscillating factor \( \exp(-i\alpha r \cos(\gamma - \varphi)) \) in the integrand. Therefore, the derivation of GW asymptotics from Eq. (2) is obtained via the application the residue theorem to the integral over path \( \Gamma^+ \) in combination with the stationary phase method for remaining integral over \( \gamma \). It brings the explicit integral representation for displacements \( \mathbf{u}(\mathbf{x}, \omega) \) to the asymptotic expansion [10]:

\[
\mathbf{u}(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{u}_n(\mathbf{x}) + O((\zeta r)^{-1}), \quad \mathbf{u}_n(\mathbf{x}) = \sum_{m=1}^{M_n} \mathbf{a}_{mn}(\varphi, z)e^{i\omega f}/\sqrt{\tilde{\zeta}_n r}, \quad \zeta r \rightarrow \infty
\]  

(3)
Here $N_r$ is the amount of real poles $\zeta_n$, $\gamma_m$ are stationary points of the oscillating exponentials (the roots of the equation $s_n' (\gamma) = 0$), $M_n$ is the number of roots $\gamma_m$ in the interval $0 < \gamma < \pi$. Each term in the second sum of the Eq. (3) is a cylindrical GW with the wavenumber $s_{nm}(\varphi)$ and group velocity

$$c_g(\varphi) = [ ds_{nm} / d\omega]^{-1}$$

(4)

Therefore, to determine dispersion characteristics of a layered anisotropic structure using the aforementioned relations one should first obtain real poles of the Green’s matrix $K$ and then solve the transcendental equation $s_n'(\gamma) = 0$ for each GW described by the second sum in Eq. (3). Along with the wavenumber and group velocity dispersion curve evaluation for the prescribed observation direction $\varphi$, the relations (3) and (4) allows obtaining so called steering angle which stands for the deviation between the GW propagation angle $\varphi$ and corresponding plane wave direction $\gamma_m$.

The developed mathematical model has been further applied for estimation of GW dispersion characteristics sensitivity to elastic moduli variation. The numerical studies have shown that for both unidirectional and cross-ply laminates group velocity of fundamental antisymmetric mode $A_0$ is sensitive to 4 from 5 elastic constants though in different frequency ranges. At low frequencies, as predicted with Kirchhoff plate theory, it is mainly affected by Young's moduli $E_x$ and $E_y$, while at middle and higher ones – by shear moduli $G_{yz}$ and $G_{xy}$. At the same time, symmetric mode $S_0$ is sensitive only to $E_x$ and $E_y$ parameters. In the frequency range considered, Poisson's ratio $\nu_{xy}$ has almost no effect on either $A_0$ or $S_0$ mode and, thus, its value cannot be reliably estimated. The obtained results in a good agreement with the available sensitivity studies performed for single frequency values from mid-frequency ranges [11]. It should also be noted, that the results remain qualitatively the same when wavenumbers/wavelengths or phase velocities of the aforementioned GW are considered.

3. Experimental setup and data extraction

In the experiments carbon-fiber-reinforced polymer (CFRP) plates with the lay-ups $[0^\circ]_4$ (unidirectional plate) and $[0^\circ, 90^\circ]_s$ (cross-ply laminate) made from unidirectional prepregs are used. The dimensions of the specimens are 1000x1000x1.15 mm$^3$. The elastic properties of the prepregs, except the density $\rho = 1482$ kg/m$^3$, are initially unknown. Vertically polarized thin piezoceramic actuators of circular ($a = 8$ mm) and square ($b = 10$ mm) shape adhesively bonded to the structure are utilized for guided wave excitation. The out-of-plane velocity field of propagating waves $u_z(x, y, t)$ is measured on the plate surface by means of a Polytec PSV-400 one-dimensional scanning laser vibrometer. The PWAS excitation is performed using a broadband square voltage pulse covering the frequency range up to 1 MHz. A sketch of the experimental setup is shown in Figure 1, a.

The experimental dependence of wavenumber from frequency at a fixed observation angle $\varphi$ is obtained using double Fourier transform over spatial and time variables [6]. The latter is applied to the velocities $\dot{u}_z(r, t)$ measured in a narrow and lengthy area along the radius-vector $r(\varphi)$ (shown by red dots in Figure 1, b):
\[
H(k, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{u}_n(r, t) e^{i(kx + 2\pi f t)} \, dr \, dt
\]  

(5)

Local maxima of \(|H(k, f)|\) function form the traces in wavenumber-frequency domain \((k, f)\), which give wavenumber dispersion curves. Along with the \(A_0\) and \(S_0\) wavenumbers, which values exhibit rather small absolute variation in the frequency range of interest, corresponding wavelengths \(\lambda = 2\pi/k\) are used as well.

To estimate group velocity dispersion curves at the observation angles another experimental procedure has been adopted [7]. For this purpose the out-of-plane velocities are measured at two points (A and B in Figure 1, b) along each trajectory. In order to extract group velocities from the obtained transient signals, they are processed with the Gabor wavelet transform. It has been shown that the arrival times of wave packages at each local frequency can be extracted by using the magnitude peaks of the wavelet coefficients [12]. The wave group velocities for each mode \(c_{g,n}^{(1)}\) and \(c_{g,n}^{(2)}\) are obtained at these points by taking the time of arrival of the corresponding wavelet-transform scalogram peaks at the various frequency values. Additionally, \(c_{g,n}^{(3)} = t_{n,B-A}/|B-A|\) has been calculated, where \(t_{n,B-A}\) is the time which takes the n-th mode to propagate from point A to point B. The experimental group velocity is finally obtained as an arithmetical mean of \(c_{g,n}^{(i)}, i = 1,2,3\).

![Graphs showing experimental group velocity (left subplots) and wavelength (right subplots) dispersion curves of \(A_0\) and \(S_0\) normal modes for the unidirectional laminate.](image)

Figure 2. Experimental group velocity (left subplots) and wavelength (right subplots) dispersion curves of \(A_0\) and \(S_0\) normal modes for the unidirectional laminate

An example of the described techniques application to experimental dispersion curves estimation is shown in Figure 2, where the dependencies of \(A_0\) and \(S_0\) group velocities (Figure 2, a) and wavelengths (Figure 2, b) from frequency for two observation directions \(\varphi = 0\) and \(\varphi = \pi/2\) in the unidirectional plate excited by the circular PWAS are depicted.
4. Identification procedure and verification

The elastic properties identification problem may be treated as the finite-dimensional optimization problem with the following objective function

\[
\text{ERR}(C) = \sum_{j=1}^{N} a_j^2 (d_j^m - d_j^2)^2
\]

(6)

Here C denotes the candidate stiffness matrix constrained by some initially prescribed boundary values \( C_{\text{min}} \leq C \leq C_{\text{max}} \); \( d_j^m \) and \( d_j^2 \) are measured and computed dispersion characteristics, e.g., wavelengths or group velocities, of \( A_0 \) and \( S_0 \) modes for the observation directions \( \varphi = 0 \) and \( \varphi = \pi/2 \) at which the maximal sensitivity of dispersion characteristics to elastic moduli variation is achieved; \( N \) is the total amount of measured values (both for \( A_0 \) and \( S_0 \) modes), \( a_j = d_{\text{max}}/d_j^m \) are normalizing coefficients, \( d_{\text{max}} = \max(d_j^m), j = 1,2, ..., N \).

Since the objective (6) is a sum of the least-square errors between the measured and calculated data, it has to be minimized. For this purpose, a real coded microgenetic algorithm (\( \mu \)GA) is applied. Every individual chromosome represents a candidate solution (matrix C). Thus, the sum \( \text{ERR}(C) \) is its fitness value. In contrast to the conventional GA, \( \mu \)GA starts with a very small population (five or six individuals) and has no mutation operator, which provides its very fast convergence to some solution [13]. After that the best individual is kept, while the remaining population is newly randomly generated. The process is repeated until the stopping criterion is met. In the current computer realization of the \( \mu \)GA tournament selection operator is used and simulated binary crossover (SBX) [14] is applied to model the single-point crossover, which is typical in binary-coded GA.

At first, the proposed reconstruction algorithm has been extensively tested on the unidirectional and cross-ply laminates with known elastic properties, where relations (3) and (4) have been used for the input wavelength or group velocity data generation. The approach has shown reliable results for both types of plates with noise-free and 5% Gauss noised data. After that, it has been applied to the investigated specimens and the following values of engineering elastic constants summarized in Table 1 have been obtained:

<table>
<thead>
<tr>
<th>Laminate type / input data type</th>
<th>( E_x )</th>
<th>( E_y )</th>
<th>( G_{yz} )</th>
<th>( G_{xy} )</th>
<th>( \nu_{xy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unidirectional plate [0°]₄ / group velocities</td>
<td>112.2</td>
<td>8.15</td>
<td>2.45</td>
<td>3.45</td>
<td>0.28</td>
</tr>
<tr>
<td>Unidirectional plate [0°]₄ / wavelengths</td>
<td>105.6</td>
<td>8.22</td>
<td>2.55</td>
<td>3.56</td>
<td>0.37</td>
</tr>
<tr>
<td>Cross-ply plate [0°,90°]₄ / wavelengths</td>
<td>101.4</td>
<td>7.63</td>
<td>2.37</td>
<td>3.91</td>
<td>0.34</td>
</tr>
<tr>
<td>Unidirectional plate [0°]₄ / tensile test</td>
<td>107.1</td>
<td>7.25</td>
<td>3.34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The values presented in the first three rows of Table 1 are obtained for the case of circular PWAS excitation after averaging the results of five \( \mu \)GA runs. Moreover, to decrease the influence of possible measurement errors at least five values of \( d_j^m \) have been taken for each normal mode in each observation direction \( \varphi = 0 \) and \( \varphi = \pi/2 \). The obtained results are in a good coincidence with each other and with the tensile test data: the relative variation is less than 10%.
With the reconstructed elastic moduli, it becomes possible, for example, to predict material anisotropy influence on PWAS generated GW directivity in the investigated composite plates. The plots of the normalized out-of-plane velocity magnitude $|v_z| = \omega|\mu_z|$ measured and computed at the points $C_1(25,0,0)$ and $C_2(0,25,0)$ on the top surface of the unidirectional plate are shown in Figure 3. Left subplots correspond to circular PWAS excitation, while the right ones are for the square piezoactuator. The theoretical results depicted by the dashed lines are obtained using equation (3) and the simplified pin-force approach is used to model PWAS-induce load function $q(x, \omega)$. The curves exhibit alternations of minima and maxima typical for dimensional sources, which are adequately predicted by the developed mathematical model.

5. Conclusions

The non-destructive guided-wave based approach for fiber-reinforced composite effective elastic moduli identification has been developed. It utilizes laser-vibrometry measured dispersion curves of PWAS excited fundamental symmetric and antisymmetric normal modes as an input data. The efficiency and reliability of the algorithm is confirmed by the conventional tensile test results as well as by the comparison of the predicted elastodynamic response with experimental measurements.

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