DETERMINATION OF THE RESIDUAL STRESSES BY THE DYNAMIC INDENTATION METHOD

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Abstract
Analysis of the stress-strain state of the material during its straining by the spherical indenter at the transition from elastic to elastic-plastic strain has been conducted. Based on the Tresca criterion the influence of the surface uniaxial and biaxial residual stresses on the hardness value that is defined as the mean contact pressure under dynamic indentation was evaluated. It is shown that residual tensile stresses most severely influence the hardness value, while the uniaxial compressive stresses do not change the hardness values.

Keywords: indentation, impact, plastic yield, residual stress, steel.

1. Introduction

The residual stress generated in steels is considered an important cause of the fracture. Most conventional methods for measuring residual stress, such as hole-drilling, saw-cutting are not only destructive but also require difficult specimen preparation. The dynamic indentation has become the powerful means for in-field residual stress measurement because it is nondestructive and can measure localized properties. This paper devoted to the theory of stress measurement.

2. Theory

The process of the materials’ indentation starts with the elastic straining. Then local plastic yield takes place and results in elastic-plastic deformation. When the yield point is first exceeded the plastic zone is small and fully contained by material which remains elastic so that the plastic strains are of the same order of magnitude as the surrounding elastic strains. When the plastic deformation is severe so that the plastic strains are much larger as compared to the elastic strains, the elastic deformation may be neglected. At
very small loads, especially during the test of materials with high value of the yield stress the significant part of the indentation deformation is elastic, and only at the end of the deformation process it passes into elastoplastic.

To analyze the state of stress in the area of deformation (the transition from elastic to elastoplastic deformation - yield point), we can use the equations of the Hertz theory for elastic contact and criterion of yield under complex stress state. The simplest criterion for the onset of plastic yield is the Tresca criterion [1, 2], the calculation of which requires data about the value of the principal normal stresses. According to this criterion the start of the transition to the plastic state occurs when at any point of the strained material the shearing stress reaches the value \( \tau_T \):

\[
\tau_{\text{max}} = \frac{|\sigma_1-\sigma_3|}{2} = \tau_T = \frac{\sigma_T}{2}
\]

where \( \sigma_1 \) and \( \sigma_3 \) are the highest and lowest principal normal stresses in the state of complex stress, \( \tau_T \) and \( \sigma_T \) - values of the yield stress of the material in simple shear and simple tension (or compression) respectively.

At the spherical tip indentation into a homogeneous metal plate the principal stresses are normal \( \sigma_z \), radial \( \sigma_r \) and district \( \sigma_\theta \) stresses. In addition, due to the symmetry \( \sigma_\theta =\sigma_r \). According to the Hertz theory [1], the principle stresses on the surface in the contact point of the sphere with the plane in the elastic region will be equal to:

\[
\sigma_r = \sigma_\theta = -\frac{1+2\mu}{2} p_0 \]
\[
\sigma_z = -p_0, \quad \text{(3)}
\]

where, \( \mu \) is the Poisson ratio, \( p_0 \) is the pressure at the center of circular contact.

Both stresses \( \sigma_z \) and \( \sigma_\theta \) at the indentation are compressive, and half-difference of these stresses is equal to the maximum shearing stress \( \tau_{\text{max}} = \frac{1}{4}(1-2\mu) p_0 \). For steel with \( \mu = 0.3 \) \( \tau_{\text{max}} = 0.1 p_0 \).

Of greatest interest is the point located on the z axis at a depth 0.48\( a \), where \( a \) is the radius of the circular contact of a spherical tip and a metal plate. At this point, according to the Hertz theory the shearing stress \( \tau_{\text{max}} \) is at the maximum. It means that according to the Tresca criterion, a transition from elastic to plastic state does not begin at the contact surface but at a depth approximately equal to the radius of the...
contact. At this point \( \tau_{\text{max}} = 0.31p_0 \), i.e., the shearing stresses are three times larger as compared to the central contact point.

Value \( p_0 \) during the elastic deformation is related to the mean pressure \( p_m \) in the following equation:

\[
p_0 = 3/2 p_m, \tag{4}
\]

where \( p_m \) related with the hardness \( H \)

\[
p_m = H = \frac{p_{\text{max}}}{\pi a^2}, \tag{5}
\]

where \( p_{\text{max}} \) – is the maximum contact loading during dynamic indentation, \( a \) – is the radius of the circular contact corresponding to the beginning of the plastic yield.

Taking into account the equations (4) and (5) the maximum shearing stress \( \tau_{\text{max}} \) at a depth of \( 0.48a \) below the contact surface will be as follows:

\[
\tau_{\text{max}} = 0.46 H \tag{6}
\]

Equation (6) and Tresca criterion result in:

\[
H \approx 2.2 \tau_T \approx 1.1 \sigma_T \tag{7}
\]

It means that if there are no residual stresses in material, the plastic yield begins when the average pressure or hardness is approximately equal to the yield stress.

Let’s analyze the change of the yield stress and, consequently, of the hardness, if the residual stresses \( \sigma_{\text{sh}} \) appear in the material.

**Uniaxial tensile stress.** To determine the principal stresses with attention to the action \( \sigma_{\text{sh}} \) let us apply the superposition principle. In this case, the normal stress \( \sigma_z \) is the same as in the absence of \( \sigma_{\text{sh}} \) and radial stress \( \sigma_{\text{sh}} \) will be determined by algebraic sum of the radial compressive stress and residual tensile stress \( \sigma_{\text{sh}} \):

\[
\sigma_{\text{sh}} = \sigma_T - \sigma_{\text{sh}} \tag{8}
\]

Let’s assume that \( \sigma_{\text{sh}} \) is part of the yield stress of the material \( \sigma_{\text{sh}} = k \cdot \sigma_T \). In this case, the greatest principal stress will be \( \sigma_z \), and the smallest will be \( \sigma_T \). These stresses create the greatest shearing stress.

\[
\tau_{\text{max}} = \frac{\sigma_T - (\sigma_T - \sigma_{\text{sh}})}{2} = 0.31p_0 + \frac{1}{2} \sigma_{\text{sh}} \tag{9}
\]
or with the account of equation(6) and the value of $\sigma_{\text{BH}}$

$$\tau_{\text{max}}^{\text{H}} = 0.46 H + \frac{1}{2} k \sigma_T$$

Consequently, since the maximum shearing stress $\tau_{\text{max}}^{\text{H}}$ is larger in the absolute value of the stress $\tau_{\text{max}}$, plastic yield begins earlier than in the absence of the tensile residual stress, which would reduce the hardness values, that corresponds to the onset of yield during indentation.

According to the Tresca criterion, the plastic yield will start if the maximum shearing stress, $\tau_{\text{max}}^{\text{H}}$ reaches the value of yield stress at shear or half value of the tensile yield stress.

$$0.92H + k \sigma_T = \sigma_T$$

Hence, we obtain the expression for the normalized hardness that corresponds to the onset of plastic yield.

$$\frac{H}{\sigma_T} = 1.08(1 - k)$$

The figure shows the change in the normalized hardness value (line OA), which shows that the hardness corresponding to the yield strength can serve as an informative parameter for estimating the residual tensile stresses.
Figure 1. Dependence of the hardness at the yield point on the values of the tension and compression residual stresses. Line OA: tensile stress (solid line - uniaxial stress, dotted line - biaxial) Line OB: uniaxial compressive stress, OC - biaxial compressive stress

**Biaxial tensile stress.** In case of presence of residual stresses in the material in the form of uniform biaxial tensile stress the maximum shearing stress initiates plastic yield and will vary as well as in case of the acting uniaxial tensile stress, since the value of the maximum shearing stress does not changed.

On figure 1 this case is shown by the dotted line OA. However, during the transition into the plastic zone during increasing indentation load, the shearing deformation on additional slip plane is inclined at $45^\circ$ to the principal stresses $\sigma_z$ and $\sigma_\theta$. In comparison with uniaxial tension the value of the hardness will drop here more intensively.

**Uniaxial compressive stress.** In this case, the radial stress increases and approaches the value $\sigma_z$, and the maximum shearing stress beyond the indenter, at which the plastic yield begins, can be determined by a half-difference not between $\sigma_z$ and $\sigma_r$, but between $\sigma_z$ and $\sigma_\theta$. However, since on the axis of the indenter $\sigma_\theta=\sigma_r$, then the maximum shearing stress is determined by equation (6) as in the case of absence of residual stresses. It follows that the hardness under uniaxial compressive residual stress will not increase due to the fact that plastic deformation will occur on another slip plane formed by the stresses $\sigma_z$ and $\sigma_\theta$.

$$\tau_{\text{max}}^H = \frac{\sigma_z - \sigma_\theta}{2}$$  \hspace{1cm} (13)

The case of uniaxial compressive stress is displayed in the Figure by the OB line.

**Biaxial compressive stress.** Here, the maximum shearing stress will be determined by the same value of half-difference of the normal stresses: $\sigma_z - (\sigma_r + \sigma_{\text{nn}})$, and $\sigma_z - (\sigma_\theta + \sigma_{\text{nn}})$.

$$\tau_{\text{max}}^H = \frac{\sigma_z - (\sigma_r + \sigma_{\text{nn}})}{2} = \frac{\sigma_z - (\sigma_\theta + \sigma_{\text{nn}})}{2}$$  \hspace{1cm} (14)
In this case, the maximum shearing stress is smaller than their values would be in the absence of residual stresses. The plastic deformation in this case will be constrained and the hardness corresponding to the yield start will increase.

Through calculations similar to those made for the case of uniaxial tensile stress (equation (8)-(11)), we obtain:

$$\frac{H}{\sigma_y} = 1.08(1 + k) \quad (15)$$

This equation is shown by the direct line OC in the figure.

A good confirmation of the findings presented in the paper can be the experimental results obtained in [3-5] during the measurement of the hardness of high-carbon steel sample subjected to bending stresses. These stresses correspond to the uniaxial tension and compression stresses that are described in this paper. The studies have shown that the tensile stress reduces the hardness by 10%, while the compressive stress does not affect the value of the hardness (less than 1%).

References

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