Quantitative evaluation of CT Images by means of Shannon Entropy

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Abstract We perform a quantitative evaluation of CT Images using Shannons Entropy (SE). The study based on a large set of simulated CT data, which were produced using a deterministic simulation tool. The data evaluation is done in two steps, first the SE is calculated directly from the voxelized data and second the SE is calculated from the partial derivative of the volumes. Comparing the two SE values leads to a clear mapping of the acquisition parameter space used in the simulation. Additionally, we compare SE and signal to noise ratio (SNR) for the same dataset.

Keywords: Image assessment, Shannon Entropy, X-ray imaging, image quality

1. Introduction

To assess the quality of CT images, a number of statistical measures are available. This work focuses on the Shannon Entropy (SE) as a quantitative measure for the quality of CT images of the kind produced in NDT applications. The aim of this study is to apply SE to simulated CT images which assumes the same geometry, components and photon statistics as in real measurements. By systematically changing the imaging parameters, i.e. camera exposure time, number of projections and X-ray spectrum, we are able to map the parameter space in terms of SE, thus revealing the influence of each parameter on the CT image quality individually, and to determine the optimum.

2. Definition of Shannon Entropy

Being $p_i$ with $i = 1 \ldots N$ the normalized Histogram values of a CT image with $N$ different grey values, the SE defines as

$$H := (-1) \sum_{i=1}^{N} p_i \log_2 p_i$$

Considering a CT image with equally distributed grey values we yield the maximum possible SE $H_{\text{max}} = \log_2 N$. Thus, we define a normalization of SE $H_{\text{norm}} = \frac{H}{H_{\text{max}}} \leq 1$.

We will call $H_{\text{norm}}$ the Shannon Entropy in the context of this paper [1]. This is just for the sake of more comparable numbers.

3. Simulation Setup and Data Evaluation

The X-ray-imaging simulation tool Scorpius XLab® (Fraunhofer EZRT), which was used to simulate the data for this work, obeys the deterministic simulation approach [3]. The specimen is a hole plate, see Bartscher et al. [4], placed in a 45° angle with respect to the rotation axis and consists of pure Aluminum. We changed the type of prefiltration, the tube acceleration voltage and the exposure time, in order to gather a large dataset with different acquisition parameters. The reconstruction algorithm was a filtered backprojection. The set of simulations is created from:

- eleven different acceleration voltages from 50 kV to 200 kV,
- three kinds of X-ray filters; no filter, 0.25 mm Aluminum and 0.3 mm Copper,
- four different detector exposure times from 250 ms to 2000 ms,
- in each simulation run a number of 200 projections was calculated for a 256x256 pixel detector and
reconstruction is performed with a state of the art filtered backprojection algorithm.

The pictures below show a schematic view of the simulated experiment as well as three example slice views of the reconstructed volumes with different acquisition parameters.

Figure 1 Schematic view of the acquisition geometry (left) as well as three example slice views (right).

Each of the 132 simulated CT volumes was evaluated by means of SE and SNR [2], whereby the SNR calculation was performed in a region of interest containing only Aluminum. In the case of SE an clear relationship to the prefiltration and tube voltage appears; not so for SNR. Since there is no clear SE mapping of the exposure time we introduced additional data treatment. Therefore we calculate the partial derivative of the voxel volume, in one arbitrarily chosen direction. This emphasizes noise and edges and can be considered as high-pass filtering.

4. Results and Interpretation

The expectation is, higher tube voltage and heavier prefiltration leads to less cupping artifact and in conclusion to a lower SE value. On the other hand lower photon statistics leads to more noisy images, thus to higher SE values. In summary: lower SE is expected to denote a higher image quality.

In Figure 2 four plots are shown. Two of them show the SE and SNR calculated from the raw volumes of each simulation. The second two plots show a comparison of the SE of the raw volumes to the SE of the partial derivative of the data, as described above. Especially in the case of the second plot one can clearly see a mapping of the acquisition parameters to the two dimensional SE-SE-Derivative area.

Having a look on the plot of the data simulated with Cu-filtering shows a difference of CNR and SE. The latter has values which map different Voltages while SNR does not change its value at all. However, SE calculated from the raw volumes does not clearly map different exposure times; this comes into account if one calculates SE of the partial derivative of the data.

5. Conclusion

Comparing SE to SNR shows: SE has clear advantages in terms of evaluating CT images and SE correlates with CT-artifacts in the simulated images. As far as we can tell from this study SE-calculation is independent of the object size and geometry.

6. Acknowledgements

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The first two plots show the dependence of SE and SNR of the acquisition parameters. The color labels the exposure time and the tick styles mark the simulated prefiltration.

The second two plots show the comparison of the SE directly calculated from the voxel data and the SE of the partial derivative of the data. Both plots show exactly the same data, while on the left hand side the color labels the exposure time and on the right the tube voltage.

References