Simulation of Nonlinear Time Reversal wave propagation in carbon fibre reinforced polymer

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Abstract
Wave propagation in carbon fibre reinforced polymer is analysed using numerical simulations and compared physical experiment of a real material. The effect of input wave shape on the propagation properties is analysed for use in nonlinear elastic wave spectroscopy. Response to positive and negative excitations is compared to examine the nonlinearity of the material in simulation. The use of solitary waves in nondestructive testing of nonlinear materials is considered. It is found that in these kind of materials, approximated by nonlinear laminate, a narrow input wave should give best results for TR-NEWS. The approximate nonlinearity of an actual carbon fibre reinforced polymer block is analysed for use with a simplest nonlinear model, based on perturbation from Hooke’s law.

Keywords: Nonlinear elastic wave spectroscopy (NEWS), ultrasonics, carbon fibre reinforced polymer (CFRP), solitary waves

1. Introduction

Recent ten years have seen considerable development of optimized signal processing methods for improving nonlinear NDT methods derived from Nonlinear Elastic Wave Spectroscopy (NEWS). Using symmetry invariance and Time Reversal (TR), the classical NEWS methods are supplemented and improved by new excitations having the intrinsic property of enlarging time-frequency scales. This is the case of TR-NEWS, now recognized as a useful tool for microcrack detection of various complex samples [1], but also for nonlinear scatterers localization in the wide sense [2]. TR-NEWS signal processing is performed using symmetrisation of coded-excitation practically realized using pulse-inversion methods. Response to positive and negative excitations allows the extraction of nonlinear signature of the sample under test. Among these family of “pulse coded excitation”, solitonic coding constitutes a new scheme in the sense that solitary waves are the best candidates for pulse propagation in nonlinear and dispersive media. Their robustness during propagation could inform aeronautic end-users during monitoring process of layered; granular, lightweight or functionally graded materials. One of the main advantages of this approach is the possibility of taking into account intrinsic space scales, namely, the size of the grains or the distance between microcracks. It has been proved [3] that in such a medium dispersion and nonlinearity could be combined in a way that solitonic propagation could be observed experimentally [4].

The purpose of simulating wave propagation with direction perpendicular to the layers is to: i) try to characterise the change of input wave shape for use with TR-NEWS method; ii) evaluate the nonlinearity of the whole material using simulations; iii) find out if material nonlinearity and dispersion from the laminated material can sustain solitary waves in this experiment.
2. Test sample and physical experiments

The preliminary physical experiments on a test sample of a CFRP block were conducted in Blois INSA lab. These include Pulse Inversion (PI) (Fig. 1) and Time Reversal (TR) (Fig. 2) experiments. The direction of wave propagation is in plane with the CF laminate (Fig. 3). The test results show that the material is nonlinear to the wave propagation.

![Figure 1: PI test of the CFRP sample](image1)

![Figure 2: TR-NEWS test of the CFRP sample](image2)

![Figure 3: The CFRP block under testing](image3)

3. Model

There are several material models available with different assumptions. The CFRP has several microstructures: it consists of CF weaved fabric, which consists of CF yarns, which in turn consists of individual CF fibres. This is all set in polymer matrix. It is assumed that the yarn of the fibres can be homogenized, because the scale of the used wavelengths are much smaller than the size of individual fibres. At the same time, the individual yarns cannot be homogenized, because their dimensions can be comparable to the ultrasonic wavelengths used. While the individual fibres affect the homogenisation of the material to some level, they have no effect on the ultrasonic wave passing through, due to their small size. The wave propagation, however, is affected by the geometry of yarns and fabric.
Modelling a thick layered composite is computationally quite expensive, therefore the model must be sufficiently simple but detailed enough. The material is modelled as laminate, where different CF fabric layers are modelled as orthotropic material. It is assumed that the shape of the yarn has little effect. The effect of yarn undulations are discarded. The sample is modelled as a periodic laminate of polymer and polymer/carbon fibre mix. If the yarn is an ellipse (Fig. 4) with axes $a = 0.75$ mm and $b = 0.13$ mm, then the averaged thickness of the yarn is $0.21$ mm and pure polymer is $0.05$ mm (Fig. 5).

![Figure 4: Shape of a single yarn in the fabric of the BMT-2 sample](image)

![Figure 5: A representative cell of CFRP yarn, modelled by two parallel plates of CFRP yarn and matrix material](image)

4. **Mathematical model**

The wave propagation in composite is simulated by direct use of laws of motion and constitutive equations of elasticity. The system of first-order partial differential equations is transformed to system of ordinary differential equations by use of Chebyshev pseudospectral method [5]. This system is then solved using standard solvers (SciPy ODE solver with “vode” method’ [6]).
4.1 One-dimensional simulation

In 1D case, the material is assumed to consist of alternating layers of pure polymer (isotropic) and polymer/fibre composite (transversely isotropic). Firstly we consider the 1D case where the wave propagation is always perpendicular to the fibre direction, giving us the simplest model. Obviously, the arrangement of yarns in the composite is not periodic, but rather stochastic. Nevertheless, it is assumed that there are enough layers to allow dealing only with statistical average thickness of a yarn.

As the 1D wave propagation could only apply to wave propagation perpendicular to the layers, the orientation of the yarns are not important, as the CF fabric is here transversely isotropic. Data sheet gives the material parameters for unidirectional carbon fibre (UD CF) composite and pure matrix. Phase velocity \( c \) is found by assuming that \( E \) represents the effective elastic constant, resulting \( c = \sqrt{E/\rho} \), representing longitudinal waves in a rod. The relevant material properties are in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>UD CF 90°</th>
<th>polymer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E ) (GPa)</td>
<td>10</td>
<td>3.7</td>
</tr>
<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>1600</td>
<td>1200</td>
</tr>
<tr>
<td>( c ) (m/s)</td>
<td>2500</td>
<td>1800</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

The model for 1D wave propagation in layered laminate model has laws of motion

\[
\begin{align*}
\rho v_t &= \sigma_x \\
\varepsilon_t &= v_x
\end{align*}
\]

where \( v \) is velocity, \( \sigma \) is stress and \( \varepsilon \) is strain. The constitutive equation is nonlinear: \( \sigma = \rho c^2 \varepsilon (1 + \beta \varepsilon) \) and has proven to sustain solitonic waves [7]. In practice this simulation uses only the velocities and stresses. Only the longitudinal wave is considered, as the transverse waves are usually much slower.

In each layer, the \( \sigma \) and \( v \) are saved in Chebyshev nodes. Neighboring layers’ endpoint nodes are overlapping. The stress and velocity are propagated into a layer by asserting that the stress at the current layer’s first point is the same as in the left neighbors last point and the velocity at the left neighbors last point is the same as in the current layer’s first point (Fig. 6). This forces the continuity requirements on boundary between layers \( \sigma_{\text{boundary}} = \text{const.} \) and \( v_{\text{boundary}} = \text{const.} \).

The boundary conditions of the whole material can be specified as needed. This is all done before the pseudospectral differentiation, between the time steps of the numerical integration.

The input wave for solitonic analysis is inserted into the left boundary of the material by a half-wave of cosine [8].

\[
\sigma(0, t) = \begin{cases} \\
\frac{A}{2} \left( 1 + \cos \left( \frac{\pi (t-w/2)}{w/2} \right) \right) , & \text{if } t \leq w \\
0 , & \text{if } t > w
\end{cases}
\]

where \( w \) is width and \( A \) the amplitude of the input pulse.
The input wave for PI analysis is signal:

$$\sigma(0, t) = \begin{cases} \frac{\lambda}{2} \sin (2\pi f(t)t), & \text{if } t \leq T \\ 0, & \text{if } t > T \end{cases}$$

where $T$ is duration of the signal, and frequency changes linearly $f(t) = f_{\text{start}} + t(f_{\text{end}} - f_{\text{start}})/T$

### 4.2 Two-dimensional simulation

The simulation is conducted as a case of plane deformation, simulating an sufficiently thick specimen in $x_3$ direction (Fig 7). The deformations in $x_3$ are taken zero and stress $\sigma_{33} \neq 0$ is not of interest at this moment. For orthotropic material, this results in Hooke’s law in 2D in form:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}$$  \hspace{1cm} (2)

The nonlinear constitutive equation is obtained from Hooke’s law by perturbation of strains

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} + \beta_1 \varepsilon_{11}^2 \\ \varepsilon_{22} + \beta_2 \varepsilon_{22}^2 \\ 2\varepsilon_{12} + 8\beta_3 \varepsilon_{12}^3 \end{bmatrix}$$  \hspace{1cm} (3)

The 2D simulation is modelled as a laminate of 3 different laminae (Fig. 7):

1. pure polymer layer,
2. composite with fabric at $0/90^\circ$ direction,
3. composite with fabric at $45^\circ$ direction.
The stiffness matrix for polymer is isotropic:

\[
C = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix}
1 - \nu & \nu & 0 \\
\nu & 1 - \nu & 0 \\
0 & 0 & \frac{1-2\nu}{2}
\end{bmatrix}
\] (4)

The stiffness matrices for composite phases are here simulated as orthotropic materials [9]

\[
C = S^{-1}, \quad S = \begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix},
\] (5)

where

\[
S_{11} = \frac{1}{E_1}, \quad S_{22} = \frac{1}{E_2}, \quad S_{12} = -\frac{\nu_{12}}{E_1}, \quad S_{66} = \frac{1}{G_{12}}.
\]

The Cauchy laws of motion can be written in form

\[
\begin{cases}
\sigma_{kl} + \rho(f_l + a_l) = 0, \\
\sigma_{kl} = \sigma_{lk},
\end{cases}
\]

where \(f_l\) is force and \(a_l\) is acceleration. Due to the plane strain, \(\sigma_{33} = \text{const.}\), therefore laws of motion become

\[
\begin{cases}
\frac{\partial v_1}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} \right), \\
\frac{\partial v_2}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} \right),
\end{cases}
\] (6)

Additional equations come from differentiating Cauchy relations

\[
\frac{\partial v_1}{\partial t} = \frac{\partial v_1}{\partial x_1}, \quad \frac{\partial v_2}{\partial t} = \frac{\partial v_2}{\partial x_2},
\] (7)

and then using constitutive equations to get stresses. Shear strain is found from compatibility relation

\[
\frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} = 2 \frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2}.
\] (8)
5. Simulation results

5.1 1D solitary wave analysis

Boundary conditions of different width were used to analyse the effect of the wave shape on the propagation characteristics. The simulation scheme was verified by running the simulation the second time, doubling the number of spatial grid points and comparing the results. In case of little or no change in results, the scheme is suitable.

Shown in Figs. 8 and 9 are the results of the simulation where the initial input wave is very short in time (duration 0.4 µs). The wave changes shape in large extent during the propagation. The oscillations that form in the tail of the main pulse are caused by the dispersion. In propagation of solitary waves, this also happens when the initial pulse is “too narrow” for the material parameters. If the pulse would be “too wide” for the material in a medium that supports solitary waves, the wave would steepen due to the nonlinearity and eventually decompose into narrower waves. As this cannot be seen from the results of wider initial input (Fig. 10, where the duration of input is 2 µs), the wave is either acting as a solitary wave that has a shape almost suitable for material parameters, or is propagating nearly linearly. Figure 11 shows the wave in the beginning of the simulation and again after two reflections from end surfaces. The wave shape shows minimal change, even when nonlinearity would be \( \beta = 1000 \). This supports the conclusion of the almost linear propagation. The solitary wave propagation might need more material length, different input wave, different model or parameters.

![Figure 8](image.png)

Figure 8: Narrow half-cosine wave showing oscillations developing behind the main pulse (\( \beta_{\text{polymer}} = 5 \)).

As the TR-NEWS relies on time reversal, more information is gathered when the wave changes a lot during the propagation and is sensitive to changing material properties. For this purpose, the results advocate the use of narrow pulses, which generate an oscillatory tail due to the dispersion.
5.2 1D Pulse Inversion

The excitation with linearly changing frequency was applied as input boundary condition, with both positive and negative amplitudes. In the opposite side of the block, the simulation displayed a definite nonlinear effect when compared to linear simulation. Figure 12 shows the amplitude of the nonlinear part. The positive and negative responses are not plotted because maximum nonlinearity amplitude is 0.1% of the maximum amplitude of responses and normalization is done with respect to amplitude of nonlinearity. The PI nonlinear part in the simulation with $\beta = 5$ (Fig. 12) is not nearly as large as in the physical experiment shown in Fig. 1. If nonlinearity $\beta = 1000$, however, the result of simulation (Fig. 13) is comparable with result of physical experiment (maximum of nonlinearity is 27% of the maximum of responses, normalization is with respect to responses from positive and negative inputs), indicating that the nonlinearities are quite large in the material.
Figure 11: Wider half-cosine wave in the beginning and end of the simulation ($\beta_{\text{polymer}} = 5$).

Figure 12: Pulse Inversion simulation of the CFRP sample, showing nonlinearities only ($\beta_{\text{polymer}} = 5$)

Figure 13: Pulse Inversion simulation of the CFRP sample for CF yarn ($\beta_{\text{CF}} = 1000$).

6. Conclusions

The main purposes of 1D simulations were:

1. Characterisation of the change of input wave shape for use with TR-NEWS and PI method.
2. Evaluating the nonlinearity of the real material, using simulations with this simple model.
3. Determining if material nonlinearity and dispersion from the laminated material can sustain solitary waves for this material and inputs.

From the experiments with half-cosine results it is apparent that narrower pulses are more suitable for nonlinear spectroscopy purposes, as they change more during the propagation through the material. The CFRP block appears to have large nonlinearity when comparing the PI results of the simulation to the physical experiment. The nonlinearity parameter of the 1D simulation should be quite large to obtain pulse inversion nonlinearities of the physical experiment. The experiments with different nonlinearity $\beta$ also show that this simplest nonlinear relation is promising in giving results that can coincide with the physical experiments. Further verifica-
tion needs to be done, however, to ascertain that the simulation reflects the reality accurately enough. It must be noted, however, that 2D results are needed for better comparison, because the physical experiment had the waves propagating in plane with layers, while 1D simulation considered waves propagating perpendicular to them.

The results also indicate that the simple solitonic propagation is not as sensitive in measuring the magnitude of nonlinearities, as is the pulse inversion technique when using high-frequency input oscillation. Determining solitonic propagation’s sensitivity to the dispersion should be done in future work. The solitonic wave propagation would aid mostly in generating a more stable wave propagation, but it is not immediately apparent with the inputs and simulation configuration used here.

7. Acknowledgments

This research was supported by European Social Fund’s Doctoral Studies and Internationalisation Programme DoRa carried out by Foundation Archimedes, Estonian Science Foundation grant ETF865 and European Regional Development Fund Project TK124 (CENS).

References