New damage detection and localization strategy from numerical library

Roger SERRA¹,*, Juan Pablo PIATTI¹, Guillaume GAUTIER¹, Serge DOS SANTOS²

¹INSA Centre Val de Loire, LMR E.A. 2640, 3 rue de la chocolaterie, 41000 Blois, France *roger.serra@insa-cvl.fr
²INSA Centre Val de Loire, U930 "Imagerie et Cerveau" Inserm-Université de Tours, 3 rue de la Chocolaterie, 41000 Blois, France

Highlights:
- Detection and localization of damage
- Comparison of indicators features used in vibration based condition monitoring
- Numerical library of damage (level, position and type)
- Comparison of numerical / experimental results

1. Introduction

Today the knowledge of the structural modifications is very important to monitor and detect defects in the initial stage to avoid unexpected breakdowns. It is often difficult to achieve a correct and complete scan an entire structure. There are various techniques but they all have their flaws. In fact, the defect identification techniques are of great importance in the field of nondestructive Testing (NDT). Thanks to the exact determination of position, we are able to decide whether the source represents the threat of damage in the device or material considered. Unfortunately, in most cases, the received signal is influenced by passing through the device itself and also by sensors. Therefore, the classification of defects into groups according to their origin is a very difficult task and sometimes equivocal. To solve the problem, some experimental deconvolution was suggested, but some disadvantages were reported. In fact, the initial source position has to be localized beforehand because we require the measurements to be done at the initial source position. The second strategy could be done by using statistical classification of defects. In this case, for any classification two main tasks are important: classification parameters and classification method. The appropriate choice of parameters is crucial for success of method. So, parameters have to be chosen very carefully. This is one of the reasons why the Vibration Based Structural Health Monitoring has an essential role. There are four main stages in this study:
- detection
- localization
- estimation
- prediction

In this paper the first two stages will be highlighted. Specifically a beam will be considered, to which the defects will be tested in different sectors of its length. The objective is to compare the results obtained by damage indicators, and then assess its reliability along the beam in the experimental and numerical cases.

2. Model and Methods

The model under study is a fixed-free beam divided into ten elements as it can be seen in fig.1, which has allowed only two degrees of freedom, a vertical translation “y” and a rotation “θz”.

The dynamic equation of motion can be written:

\[ M \ddot{X} + C \dot{X} + K X = F, \]

(1)
whose terms are: “$M$” (mass matrix), “$C$” (damping matrix), “$K$” (stiffness matrix), “$X$” (displacement vector), and “$F$” (force vector).

Fig.1 Model of fixed-free beam with two degrees of freedom.

Canceling the damping term and the force term, and considering the following equation:

$$\ddot{X} = -\omega^2 X,$$

and by rearranging the equation, eigenvalues (frequencies) and eigenvectors (displacements) can be extracted from:

$$(K - \omega^2 M) X = 0.$$

For a matrix based solution, we need to define the following matrices where: “$K_e$” (elementary stiffness matrix), “$M_e$” (elementary mass matrix), “$E$” (Young’s modulus), “$I$” (quadratic moment of the beam), “$h$” (length of each element), “$\rho$” (volume weight), “$S$” (area).

$$K_e = \frac{E.I}{h^3} \times \begin{bmatrix} 12 & 6h & -12 & 6h \\ 6h & 4h^2 & -6h & 2h^2 \\ -12 & -6h & 12 & -6h \\ 6h & 2h^2 & -6h & 4h^2 \end{bmatrix} \quad \text{And} \quad M_e = \frac{\rho.S.h}{420} \times \begin{bmatrix} 156 & 22h & 54 & -13h \\ 22h & 2h^2 & 13h & -3h^2 \\ 54 & 13h & 156 & -22h \\ -13h & -3h^2 & -22h & 4h^2 \end{bmatrix}$$

2.1. Modal Assurance Criterion (MAC)

It allows comparison of the eigenmodes of healthy beam and damaged beam, where “$\Phi$” and “$\Phi^*$” are the healthy and damaged natural mode shapes, “i” and “j” are the numbers of the mode shapes. If modes are identical (same state), it returns a value of 1, otherwise less than 1.

$$MAC(\phi_i, \phi_j) = \frac{|\phi_i^T \phi_j|}{(\phi_i^T \phi_i) (\phi_j^* \phi_j^*)}$$

2.2. Coordinated Modal Assurance Criterion (COMAC)

COMAC use the MAC but has the advantage of checking each element of the structure. Where “$\Phi^a$” and “$\Phi^b$” are the healthy and damaged natural mode shapes, “i” is the node indice, “j” is the mode indice, and “$n_m$” is the total number of modes considered.

$$COMAC(i) = \frac{\sum_{j=1}^{n_m} (\phi_i^a \phi_j^b)}{(\sum_{j=1}^{n_m} |\phi_i^a|^2) \times (\sum_{j=1}^{n_m} |\phi_j^b|^2)}$$

2.3. Modal Shape Curvature (MSC)

MSC detects the change of curvature. Where “$\Phi^a$” and “$\Phi^b$” are the curvatures of the mode shapes of healthy and damaged beams respectively. The higher the index is, the greater is the damage.

$$MSC = \sum_{j=1}^{n_m} |\phi_j^a - \phi_j^b|$$
2.4. Local Frequencies Change Ratio (LFCR)

Compare local natural frequencies on each element of the beam. Where “\(LF_{ej} \)” and “\(LF_{sj} \)” are the frequencies of damaged and healthy elements. The damage is greater when the indicator grows.

\[
LFCR_{ji} = \frac{|LF_{ej} - LF_{sj}|}{LF_{sj}} 
\]  

(8)

2.5. Energy of Deformation (ED)

This indicator compares the energies of deformation before and after being damaged, where the lower value corresponds to the location of the damage.

\[
ED = \sum_{j=1}^{n_m} \frac{J_{sain}}{J_{endo}} 
\]  

(9)

2.6. Damage Index (DI)

This indicator utilizes characteristics of mode shape curvature for a beam-like structure as the main variable in the derived damage localization algorithm based on the relative differences in modal strain energy before and after damage. Variables “\(\varepsilon\)” and “\(\varepsilon^*\)” are the second derivatives of the healthy and damaged Eigen modes, “\(j\)” is the number of modes and “\(i\)” is the number of elements, “\(Nt\)” is the total number of modes.

\[
\beta_{ji} = \frac{\left[ (\varepsilon_{ij})^2 + \sum_1^{Nt} (\varepsilon_{ij}^*)^2 \right] \sum_1^{Nt} (\varepsilon_{ij})^2}{\left[ (\varepsilon_{ij})^2 + \sum_1^{Nt} (\varepsilon_{ij}^*)^2 \right] \sum_1^{Nt} (\varepsilon_{ij}^*)^2} 
\]  

(10)

Transforming the damage indicator values into the standard normal space, normalized damage index “\(Z_i\)” is obtained where “\(\mu_{\beta i}\)” is the mean of \(\beta i\) values and “\(\sigma_{\beta i}\)” is the standard deviation of \(\beta i\) for all “\(i\)” elements.

\[
Z_i = \frac{\beta_i - \mu_{\beta i}}{\sigma_{\beta i}} 
\]  

(11)

3. Results

![Graphs and diagrams related to the results of the analysis.](image)
The case study in this figure (Fig.2) is the simulation of a pore Ø1mm on the neutral axis of the beam to 46mm of the embedment. This pore size is equivalent to a specific reduction of Young's modulus of 0.08% on the second element.

4. Conclusions

Damage cases were studied in different sections of the entire beam, for ten and twenty elements. The results obtained for the study with ten elements demonstrate that the "DI" indicator works correctly on the entire length of the beam to the second degree of freedom but only in the vicinity of embeddedness for the first; the "LFCR" indicator works correctly in entire length of the beam; the "MAC" indicator correctly detected over the entire length on both degrees of freedom but it is better for the second; the "COMAC" indicator only works for defects next to the fixed end to the second degree of freedom; the "MSC" indicator does not work properly on the free end of the beam to the second degree of freedom and does not work in any section to the first degree of freedom; the "ED" indicator only works correctly in full length for first grade of freedom.

While the study with twenty elements best results are obtained, the "DI" indicator now works correctly for two degrees of freedom throughout the length of the beam, the "COMAC" indicator gets a little improvement but not very important, while the "ED" indicator works correctly for both degrees of freedom throughout the length of the beam but does not work on the free end to the first degree of freedom.

References


5. F.C. Choi, J.Li, B. Samali, K. Crews, Application of the Modified Damage Index Method to Timber Beams, University of Technology Sydney, 2007.
