Abstract

Recent analytical eddy current models, which correctly account for each of the electromagnetic coupling effects arising in inductive circuits that contain a conducting and even ferromagnetic sample, have been used to develop algorithms for the simultaneous evaluation of material parameters. In particular, the electrical conductivity and magnetic permiability of different material and thickness tubes have been extracted from single transient eddy current signals. Simple Laplace and Fourier transform rules are used to generate an arbitrary number of independent equations relating experimental data and theory. These equations are simultaneously solved to determine the material parameters of interest. Furthermore, results computed from these equations can be tabulated and implemented in a lookup-table scheme for an ultra-fast multi-parameter characterization of material properties.

Forward Problem

In driver-pickup pulsed eddy current applications, a square wave excitation with amplitude \(v_0\) and period \(P\) is applied to a driver coil. The general solution for the transient current response of the pickup coil is [2] written as

\[
\int_0^T (i(t) \cos \omega t) dt = \frac{M + \mathcal{M}(s)}{s} \left[ R_1 + s(I_1 + L_1(s)) \right],
\]

where \(L_1\) and \(L_2\) are the coils' self-inductances, \(R_1\) and \(R_2\) are the total driver and pickup circuits' resistances, and \(M\) is a mutual inductance coefficient. Additionally, \(L_1\) and \(L_2\) are complex inductances arising from the self-coupling of the driver and pickup coils through the sample, \(M\) is a complex mutual inductance coefficient arising from cross-coupling between the coils through the sample, and \(s\) the Laplace transform parameter, is a complex frequency.

For a coaxial driver-pickup arrangement encircling a tube, the complex inductances are:

\[
\begin{align*}
L_j(s) &= 2L_j^0 \int_0^L (x) \cos \omega t_j(x) dx_j, \\
M(s) &= 2L_j^0 \int_0^L \cos \omega t_1(x) \sin \omega t_2(x) dx_1 dx_2,
\end{align*}
\]

where \(x\) is the Fourier cosine transform parameter and \(\mu_0\) is the permeability of free space. Driver and pickup coil functions \(x_1(x)\) and \(x_2(x)\) are defined as

\[
x_j(x) = \frac{2\sin \frac{\omega x}{2}}{\omega} \int_0^L \cos \frac{\omega x_j t}{2} \cos \frac{\omega x_j x}{2} dx_j,
\]

with the following definitions:

\[
\begin{align*}
\mu(x_1, x_2) &= \mu_0 \left[ \mu_1 \left( x_1 \right) \left( x_2 \right) - \mu_2 \left( x_1 \right) \left( x_2 \right) - \mu_3 \left( x_1 \right) \left( x_2 \right) \right] \left( \mu_1 \left( x_1 \right) \left( x_2 \right) + \mu_2 \left( x_1 \right) \left( x_2 \right) \right), \\
\chi(x_1, x_2) &= \mu_0 \left[ \chi_1 \left( x_1 \right) \left( x_2 \right) - \chi_2 \left( x_1 \right) \left( x_2 \right) - \chi_3 \left( x_1 \right) \left( x_2 \right) \right] \left( \chi_1 \left( x_1 \right) \left( x_2 \right) + \chi_2 \left( x_1 \right) \left( x_2 \right) \right), \\
A &= \frac{1}{\mu_0} \left( \chi - \mu \right).
\end{align*}
\]

Thus, all quantities are described and the solution can be plotted for tubes of different \(\sigma\), \(\mu_0\) and wall thickness.

Inverse Problem

The frequency-domain differentiation property of the Laplace transform [2], written as

\[
\int \omega^p e^{\omega t} d\omega = (-1)^p \frac{d^p}{dt^p} F(\omega),
\]

provides a simple method of generating linearly independent correspondences between theory and experiment. Multiple relationships can be drawn from a single measured TEC signal. A first equation, \(n = 0\) and \(n = 1\), relates the area under the pickup current signal and theory

\[
A_0 \equiv \int_0^T (i(t) \cos \omega t) dt = \frac{M + \mathcal{M}(s)}{s} R_1 R_2.
\]

Using a measured value for \(A_0\), the equation may be solved numerically for \(\mu_0\). A second equation, \(n = 2\) and \(n = 1\), relates the area under the pickup current signal scaled by \(t\) and theory

\[
A_2 \equiv \int_0^T (t \cdot i(t) \cos \omega t) dt = \frac{1}{2} \frac{M + \mathcal{M}(s)}{s} R_1 R_2.
\]

where the prime denotes the derivative of the function \(M(s)\) with respect to complex frequency \(s\). This second equation, together with a measured value of \(A_0\), may be analytically solved for \(\sigma\).

For example, consider a carbon steel tube with an outer radius of 3/8" and a wall thickness of 0.125". The area under the measured transient pickup signal \((t)\) is found to be 36,078 \(\Omega^\cdot s\), whereas the area under the same signal scaled by \(t\), \(t \cdot (i(t))\), is measured to be 6,573 \(\Omega^\cdot s\). Properties \(\mu_0\) and \(\sigma\) are evaluated as 138.3 and 6.57 MSm\(^{-1}\), respectively.

Experiment

The circuit diagram of the experimental apparatus, and the geometrical and electrical properties of the coaxial driver-pickup probe, are presented below:

![Experiment Diagram]

Summary

A novel approach for the simultaneous characterization of a material's magnetic permeability and electrical conductivity has been developed. The inverse method was demonstrated on tubes, but is applicable to different geometries such as multilayer tubular or plate structures, provided that the expressions for the complex self- and mutual inductances can be obtained. A model-based approach, such as this, is useful for the interpretation of multivariant transient eddy current inspection data.

References