Simulation of Nonlinear Time Reversal wave propagation in carbon fibre reinforced polymer

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Figure: Carbon Fibre Reinforced Polymer (CFRP) block of 144 layers
Goals

1. Simulating Carbon Fibre Reinforced Polymer (CFRP) as a partially continuous material
2. Determining if the material nonlinearity and dispersion could support solitary wave formation and propagation

**The solitary wave**

- High stability in propagation due to the balance between dispersion and nonlinearity
- Elastic interactions
- **Sensitivity of its shape and amplitude to material characteristics**

3. Simulating the focusing of the energy with nonlinear Time Reversal (TR) on the defect to excite the nonlinearity in medium
Structure of the test object and model

Two microstructures:

**Homogeneous** Carbon Fibre (CF) structure in individual yarns

**Approximated by layers** Individual yarns in CF fabric

![Figure: Close-up image of the structure of the test object](image)

![Figure: The material is modelled as a laminate, with individual laminae having thickness proportional to the materials cross-sectional area](image)
Laminate model

- Material consists of homogeneous layers
- Each layer has its own elasticity properties
- Dispersion due to the discontinuity of the properties

Figure: Waves in $X_1$ direction for 1D case; $X_1$-$X_3$ plane strain in 2D case
Known to support solitary waves

The simulations by LeVeque [1] show that solitary waves could arise in layered media with nonlinearity $\sigma(\varepsilon) = K\varepsilon(1 + \beta K\varepsilon)$.

Figure: Stress at time $t=80$. Simulation of layered nonlinear media by LeVeque [1] with Finite Volume Method.

Figure: Stress at time $t=80$. Simulation of layered nonlinear media by Chebyshev Pseudospectral method.

Known to support solitary waves

The simulations by LeVeque [1] show that solitary waves could arise in layered media with nonlinearity \( \sigma(\varepsilon) = K\varepsilon(1 + \beta K \varepsilon) \).

Figure: Stress at time \( t=160 \). Simulation of layered nonlinear media by LeVeque [1] with Finite Volume Method.

Figure: Stress at time \( t=160 \). Simulation of layered nonlinear media by Chebyshev Pseudospectral method.

Known to support solitary waves

The simulations by LeVeque [1] show that solitary waves could arise in layered media with nonlinearity $\sigma(\varepsilon) = K\varepsilon(1 + \beta K\varepsilon)$.

Figure: Stress at time $t=240$. Simulation of layered nonlinear media by LeVeque [1] with Finite Volume Method.

Figure: Stress at time $t=240$. Simulation of layered nonlinear media by Chebyshev Pseudospectral method.

Mathematical model

- Cauchy’s laws of motion
  \[
  \begin{align*}
  \sigma_{kl,k} + \rho \ddot{u}_l &= 0 \\
  \sigma_{kl} &= \sigma_{lk}
  \end{align*}
  \]

- Small deformations \( \varepsilon_{kl} = \frac{1}{2} (u_{k,l} + u_{l,k}) \)

- Weak nonlinearity in constitutive equation.
  - 1D case
    \[
    \sigma = K \left( \varepsilon + \text{sgn}(\varepsilon) \beta \varepsilon^2 \right)
    \]
  - 2D case
    \[
    \begin{bmatrix}
    \sigma_{11} \\
    \sigma_{33} \\
    \sigma_{13}
    \end{bmatrix} =
    \begin{bmatrix}
    C_{1111} & C_{1133} & 0 \\
    C_{1133} & C_{3333} & 0 \\
    0 & 0 & C_{3131}
    \end{bmatrix}
    \cdot
    \begin{bmatrix}
    \varepsilon_{11} + \text{sgn}(\varepsilon) \beta_1 \varepsilon_{11}^2 \\
    \varepsilon_{22} + \text{sgn}(\varepsilon) \beta_2 \varepsilon_{22}^2 \\
    2\varepsilon_{12} + 4\text{sgn}(\varepsilon) \beta_3 \varepsilon_{12}^2
    \end{bmatrix}
    \]
Nonlinear parameter

\[ \sigma = K \epsilon (1 + \text{sgn}(\epsilon) \beta \epsilon) \]

- \( K = 135.7 \text{ GPa} \)
- \( \beta = -17 \)

Figure: Stress-strain curve of Narmco 5605 unidirectional graphite-epoxy [2]

Nonlinear parameter

\[ \sigma = K \varepsilon (1 + \text{sgn}(\varepsilon) \beta \varepsilon) \]
\[ K = 11.2 \text{ GPa} \]
\[ \beta = -8 \]

Figure: Stress-strain curve of Narmco 5605 unidirectional graphite-epoxy [2]

Nonlinear parameter

\[ \sigma = K\gamma(1 + \text{sgn}(\gamma)\beta\gamma) \]

- \( K = 6.2 \text{ GPa} \)
- \( \beta = -15 \)

Figure: Stress-strain curve of Narmco 5605 unidirectional graphite-epoxy [2]

1D elasticity moduli

For strain-stress relation in 1D constrained wave propagation

\[ \sigma_{33} = M \varepsilon_{33} \]

Longitudinal wave modulus for isotropic material

\[ M = \frac{E(\nu - 1)}{(2\nu - 1)(\nu + 1)} \]

Longitudinal wave modulus for transversely isotropic material \((E_2 = E_3, \nu_{12} = \nu_{13})\) (Fig. 9)

\[ M = \frac{(E_2 \nu_{12}^2 - E_1)E_2}{(2E_2 \nu_{12}^2 + E_1 \nu_{23} - E_1)(\nu_{23} + 1)} \]

Figure: The cylinder of interest in 1D constrained longitudinal wave propagation through unidirectional CFRP
2D parameters for $\sigma = C \varepsilon$

Epoxy (isotropic)

$$C = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1 - 2\nu}{2} \end{bmatrix}$$

Composite phases as orthotropic materials, 2D constrained elasticity:

$$C = \begin{bmatrix} c_{1111} & c_{1133} & 0 \\ c_{1133} & c_{3333} & 0 \\ 0 & 0 & c_{3131} \end{bmatrix}$$

where

$$c_{1111} = \frac{(E_3 \nu_{23}^2 - E_2) E_1^2}{2 E_2 E_3 \nu_{12} \nu_{13} \nu_{23} + E_2^2 \nu_{12}^2 + E_2 E_3 \nu_{13}^2 + E_1 E_3 \nu_{23}^2 - E_1 E_2}$$

$$c_{1133} = -\frac{(E_3 \nu_{23}^2 + \nu_{12}^2 + \nu_{13}^2) E_1 E_2 E_3}{2 E_2 E_3 \nu_{12} \nu_{13} \nu_{23} + E_2^2 \nu_{12}^2 + E_2 E_3 \nu_{13}^2 + E_1 E_3 \nu_{23}^2 - E_1 E_2}$$

$$c_{3333} = \frac{E_3 \nu_{23}^2 + \nu_{12}^2 + \nu_{13}^2}{2 E_2 E_3 \nu_{12} \nu_{13} \nu_{23} + E_2^2 \nu_{12}^2 + E_2 E_3 \nu_{13}^2 + E_1 E_3 \nu_{23}^2 - E_1 E_2}$$

$$c_{3131} = G_{13}$$

Figure: 2D $x_1 - x_3$ plane strain problem in layered material
Chebyshev collocation method

Figure: Interpolation in case of equi-spaced points
[3]

Figure: Interpolation in case of Chebyshev points
[3]

Connecting the layers

The connection between the layers is described in [1], but the following simple specification appears to work well.

1. Spatial differentiation and calculations in each layer separately
2. The layers are interconnected by carrying over the stress and velocity [5] Laws of motion in 1D, knowing the $\sigma = \sigma(\varepsilon)$

\[
\begin{align*}
\rho_v t &= \sigma_x \\
\varepsilon_t &= v_x
\end{align*}
\]

3. Boundary conditions can be specified according to the problem (usually mixed boundary conditions)

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Wide initial pulse

Figure: Propagation of a half-cosine initial pulse with width of 2ms. Material $\beta = -15$. 
Wide initial pulse

Figure: Propagation of a half-cosine initial pulse with width of 2ms. Material $\beta = -15$. 
Narrow initial pulse

Figure: Propagation of a half-cosine 0.4ms initial pulse. Material $\beta = -15$. 
Effect of nonlinearity

Figure: Propagation of a half-cosine 0.4ms initial pulse. High amplitude is normalised by initial amplitude, but is 14 times higher than shown in y-axis.
Simple relation for the defective layer

Figure: Material with tensile strength 70% of compressive strength
Pulse inversion in simulation

Figure: Velocity responses at the far end of the test object
TR wave focusing

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**Figure:** Velocity responses at the far end of the test object
Future work: 2D

Figure: 2D wave propagation in layered media, normal to the layers, $t = 1\mu s$
Future work: 2D

Figure: 2D wave propagation in layered media, normal to the layers, $t = 2\mu s$
Future work: 2D

Figure: 2D wave propagation in layered media, normal to the layers, $t = 3 \mu s$
Future work: 2D

Figure: 2D wave propagation in layered media, normal to the layers, $t = 4 \mu s$
Chebyshev collocation method can be used for simulating a layered, anisotropic media in 1D and 2D.

The weak material nonlinearity in simple laminate model is not enough for formation of solitary waves. Other effects and sources of nonlinearity must be investigated.

The nonlinear TR will create focusing on the nonlinearly defective layer in the simulation.
Conclusions

- Chebyshev collocation method can be used for simulating a layered, anisotropic media in 1D and 2D

- The weak material nonlinearity in simple laminate model is not enough for formation of solitary waves. Other effects and sources of nonlinearity must be investigated.

- The nonlinear TR will create focusing on the nonlinearly defective layer in the simulation

*Thank you for your attention!*