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A Proposal of Mode Shape Estimation Method Using Pseudo-Modal Response : Applied to Steel Bridge in Building

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Abstract

Studies of system identification (SI)-based structural health monitoring (SHM) are being actively conducted to ensure structural safety. Recently, many SI techniques have been developed using the output-only SI paradigm for estimating the modal parameters. The features of these output-only SI methods are obtained using frequency domain decomposition (FDD) and stochastic subspace identification (SSI), both of which involve the use of algorithms based on orthogonal decomposition, such as singular value decomposition (SVD). However, the SVD leads to a high level of computational complexity to estimate modal parameters. This paper proposes a technique to estimate the mode shape at lower computational cost. This technique shows pseudo-modal responses (PMR) through a bandpass filter and suggests a time-history modal assurance criterion (MAC). Finally, the mode shape is estimated from the PMR and the time history MAC. Experimental tests of the vibration measurement were performed, and the results of mode shape and computation time between a representative SI method and the proposed method were compared.

1 INTRODUCTION

In a variety of fields, including civil and architectural engineering as well as mechanical and aerospace engineering, structural health monitoring (SHM) and damage detection techniques have been actively developed over the past several decades for evaluation of the safety of structures[1-4]. The process of estimating structural modal parameters is referred to as system identification (SI)[5]. Recently, SI techniques have been rapidly developed based on the output-only SI paradigm[6-8]. The modal parameters from SI methods, such as natural frequency, mode shape and modal damping ratio, present structural dynamic properties that condensing the time history vibration measurement data.

The features of these SI methods are obtained using frequency domain decomposition (FDD)[9] and stochastic subspace identification (SSI)[10]; both of these approaches involve the use of algorithms based on orthogonal decomposition, such as singular value decomposition (SVD), because the applications of the algorithms have been validated in various fields[11,12]. Therefore, orthogonal decomposition algorithms are used in one part of the SI methods to identify the modal parameters of structures from vibration measurements based on successfully applied examples. Despite the broad use of SVD for estimating modal parameters, the SVD leads to a high level of computational complexity when performing SI[13]. SVD starts with bidiagonalizing the target matrix [14]. Next, the bidiagonalized matrices are decomposed into singular values and vectors. Modified procedures of decomposition and new algorithms are proposed to reduce the computational cost. For target matrices that are square matrices, several studies[15,16] developed SVD with less computation time using the bisection method, one of the optimization algorithms. The computation time is such a critical issue when estimating the modal parameters because both FDD and SSI (one of the most popular SI methods lately) involve iterative SVDs. As the building structures continue to become higher, larger and more complex, the computational complexity and the time required to perform SI will be very important issues. As a result, SI methods for estimating the modal parameters must be considered to reduce computational costs.

This paper proposes a method for visually estimating the mode shape using a band-pass filter and the historical responses of the structure. This proposed method does not require the use of the SVD algorithm, which causes high computational complexity. As a result, the proposed method requires a reduced amount of computation time compared to the conventional methods. Experimental tests of a steel bridge in a building are performed to verify this proposed method. Finally, the results involving the mode shape and the computation time required for the representative of SI method, the FDD, and the proposed method are compared.

2 METHOD FOR ESTIMATION

2.1 Target frequency

The most important aspect of SHM is performing vibration measurements of a structure. The measured responses of structure at a specific time t are presented as $u_i(t)$. i is the number of the sensor locations, ranging from $1 \text{ to } M$; M is the total number of sensors to measure. To set the target frequency of the bandpass filter, measured responses are transformed into the frequency domain through the use of a fast Fourier transform (FFT) using the measured responses.

$$U_i(\omega) = \mathrm{FFT}[u_i(t)], \quad i = 1 \text{ to } M \tag{1}$$

$U_i(\omega)$ is the frequency domain response of the structure at the ith sensor location. The peaks of the frequency domain response curve are set as the target frequency of the bandpass filter.

2.2 Pseudo-modal response

There are various filter types currently in use, e.g., Butterworth, Chebyshev and elliptic. The Butterworth type filter has the advantages of constant gain level on bandwidth compared to the other filters. Thus, the Butterworth filter is used in this proposed method.

The measured responses can be divided into separate responses of each mode through the use of a bandpass filter. In this paper, the separate responses obtained using a filter are called the pseudo-modal response (PMR). The bandpass filter is designed with an appropriate range (half of bandwidth) and order based on the target frequency. The designed transfer function (TF) of the filter and the frequency domain response (expressed as $H^j(\omega)$ and $U_i(\omega)$, respectively) are used to produce the composite function. $H^j(\omega)$ is the TF that has a target frequency equal to the j th peak frequency of the frequency domain response. Finally, inverse Fourier transform (IFT) of the composite function comprised of the TF and the frequency domain response are calculated to obtain the PMR at each mode and sensor location.

$$PMR_i^j(t) = IFT[H^j(\omega) \cdot U_i(\omega)], \quad i = 1 \text{ to } M, \quad j = 1 \text{ to } N \tag{2}$$

where $PMR_i^j(t)$ denotes the pseudo-modal response of the i th sensor location and j th mode at the specific time t. M is the maximum number of sensors, and N is the number of interesting modes. One of the practical decisions regarding how to set the number of interesting modes was the use of the sum of the effective modal mass ratios up to 90%, based on a previous analytical model.

2.3 Components of mode shape

A single column matrix is formed by combining the pseudo-modal responses of a specific mode. In this paper, this single column matrix is called the components of mode shape (CoMS) because of its similarity between the mode shape and the combined pseudo-modal responses. CoMS can be expressed as follows:

$$\phi_c^j(t) = \left\{PMR_1^j(t), \ \cdots, \ PMR_M^j(t)\right\}^T \tag{3}$$

The letter c in CoMS is used to distinguish the term from mode shape. CoMS at time t, that is $\phi_c^j(t)$, presents the global movements of the structure related to the j th mode. Thus, the j th mode shape is observed through CoMS at some specific time steps.

2.4 Time-history Modal Assurance Criterion

This paper presents a proposed method of identifying the mode shape from CoMS. Assuming that CoMS in every time step is the mode shape, the modal assurance criterion (MAC) can be calculated in every time step. Practically, the MAC value is used to analyze the correlation between mode shapes. However, the time-history MAC value can be obtained using the time-history CoMS instead of the mode shape by assumption. If the MAC value between the a th and b th mode is close to 1, then the mode shapes exhibit high correlation. In contrast, if the MAC value is close to 0, then the mode shapes are found to exhibit low correlation and are nearly orthogonal to each other.

$$MAC^{a,b}(t) = \frac{\left|\left\{\phi_c^a(t)\right\}^T \left\{\phi_c^b(t)\right\}\right|^2}{\left\{\phi_c^a(t)\right\}^T \left\{\phi_c^a(t)\right\} \cdot \left\{\phi_c^b(t)\right\}^T \left\{\phi_c^b(t)\right\}} \tag{4}$$

$$SUMAC(t) = \sum_{a=2}^{N} \sum_{b=1}^{a-1} MAC^{a,b}(t) \tag{5}$$

$SUMAC(t)$ is time-history summation of the MAC values at time t, and N is the number of interesting modes. A smaller value of $SUMAC(t)$ indicates that CoMS of each mode satisfies orthogonality as a principle vector. Thus, the proposed method regards the CoMS at a specific time that minimizes $SUMAC$ as a part of the mode shape.

2.5 Mode shape

The time related to the k th smaller $\mathrm{SUMAC(t)}$ that is defined as the minimum time (MTime) is expressed as t_k^{min}. For example, the first MTime is t_1^{min}, and the second one is t_2^{min}. The mode shape can be estimated using a group of CoMS values at MTime.

$$\phi^j = \sum_{k=1}^{K} \phi_c^j (t_k^{min}) \tag{6}$$

ϕ^j denotes the j th unnormalized mode shape, and K is a certain ratio of the number of used CoMS to the number of measured data samples. The estimated mode shapes reflect the amplitude of the CoMS. Therefore, if CoMS at MTime exhibits good orthogonality and low amplitude, it will be a bad proportion of mode shape; if not, then CoMS affects a great proportion of the mode shape because of high amplitude. Finally, the obtained unnormalized mode shape can be normalized by using one of a variety of approaches. The proposed method for estimating the mode shape does not require SVD, an approach that introduces a high level of computational complexity and is broadly used in previous SI techniques. Thus, it is expected that the computation time will be reduced when estimating mode shape using the proposed method.

3 EXPERIMENTAL APPLICATION

3.1 Experimental set-up

The target structure is a steel bridge in a building; the dimensions of the bridge are 16 m in length and 4 m in width. The target structure is used as a passage. Two girders of 16 m in length were constructed, and three beams were connected with almost pinned joints. Steel deck plates were located above the girders, and concrete slabs that are 15 cm in thickness were placed on the deck. Three piezoelectric accelerometers were attached at 1/4, 2/4, 3/4 of the target structure to measure the vertical vibration. During operational modal analysis (OMA), an important assumption is that the loads on the structure are regarded as following white noise of Gaussian distribution. To satisfy this assumption, the ambient vibration of structure should be measured. Thus, the measurement was performed without any artificial excitation. The measuring time was approximately 128 seconds at a sampling rate of 128 Hz. The measured data sample equals to 16384. The accelerometers have resolution of 0.00007 g and a frequency range of 0.5 to 1000 Hz.

3.2 Target frequency

FFT was conducted using data from the measurement of ambient vibration. The natural frequency was approximately estimated by observing the FFT curve, as shown in below. The results establish the target frequency of filter according to the peaks.

ACC1, 2 and 3 are located at 1/4, 2/4 and 3/4 of the bridge length. The target frequencies obtained of 7.1 Hz, 19.3 Hz and 24.4 Hz are related to the first, the second and the third mode, respectively. The narrow peaks are not considered as structural modes.

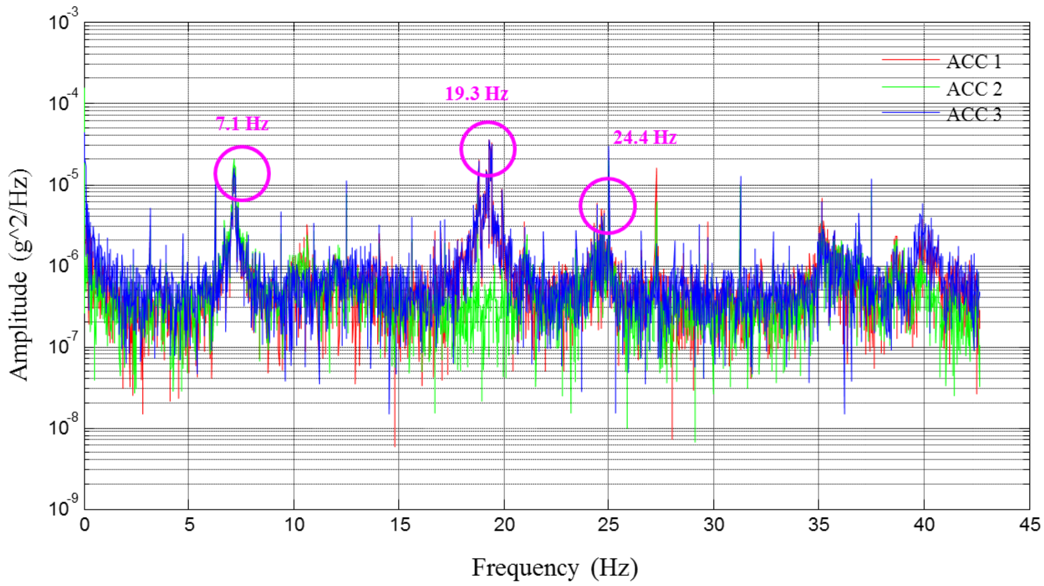

Figure 1: Results of FFT.

3.3 Components of mode shape

The CoMS could be obtained using equations (2) and (3). The bandpass filter was designed as Butterworth type. The filter is of 5th order and has a range of 1.0 Hz (half of bandwidth). The filtered data known as the pseudo-modal response are combined to make the vector, CoMS.

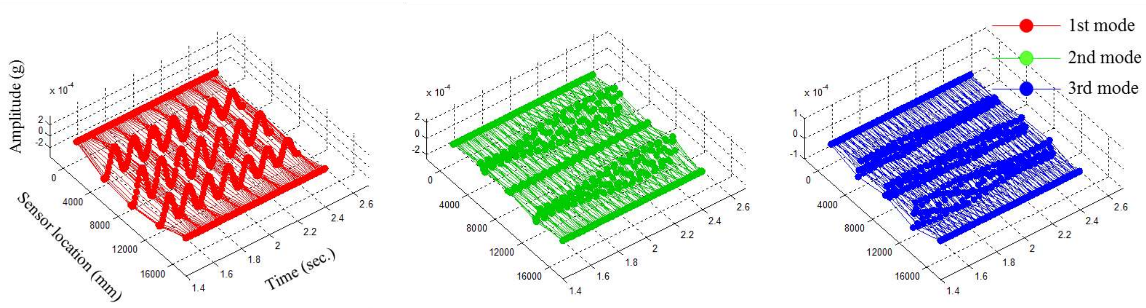

Figure 2: CoMS of each mode.

Figure 2 shows the three dimensional plots of the CoMS of each mode during 1 second. The similarity between the mode shape and the CoMS can be confirmed visually in Figure 2.

3.4 Time-history MAC

The time-history MAC value and SUMAC were calculated using equations (4) and (5). For this experimental test, the number of sensors or the number of interesting modes can be three. Thus, the SUMAC sets the range of 0 to 3 as going from the minimum to the maximum values.

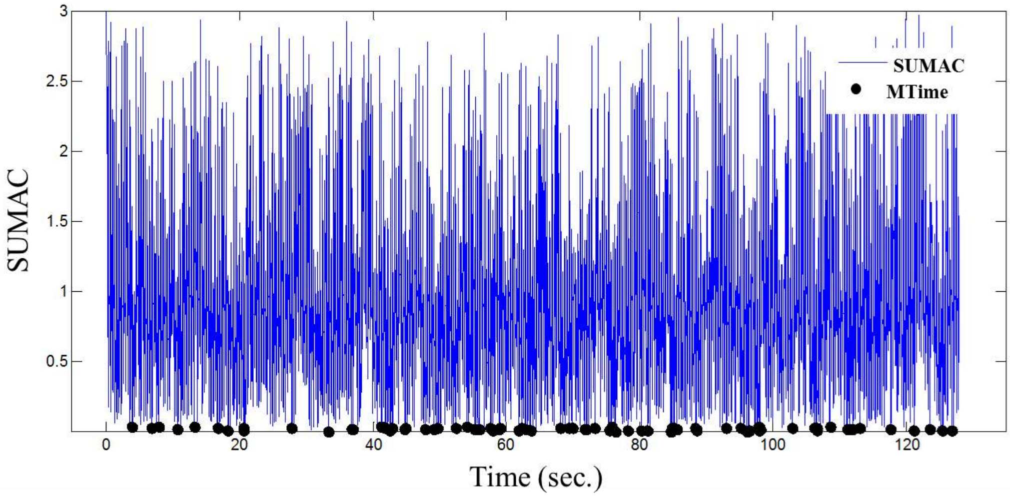

Figure 3: Time-history MAC value.

The blue line presents the time-history MAC, and the black markers present the MTime expressed in equation (6). The number of MTime, K, was 81, i.e., 0.5% of the total number of measured data samples of 16384.

3.5 Mode shape

The unnormalized mode shape can finally be estimated using equation (6). The CoMs of 81 contributing mode vectors have measurable amplitudes. Although the selected CoMS have a lower SUMAC, if its amplitude is also small, then the contribution of the selected CoMS for estimating the mode vector is small. Thus, the proposed method considers both the orthogonality and the amplitude of the CoMS.

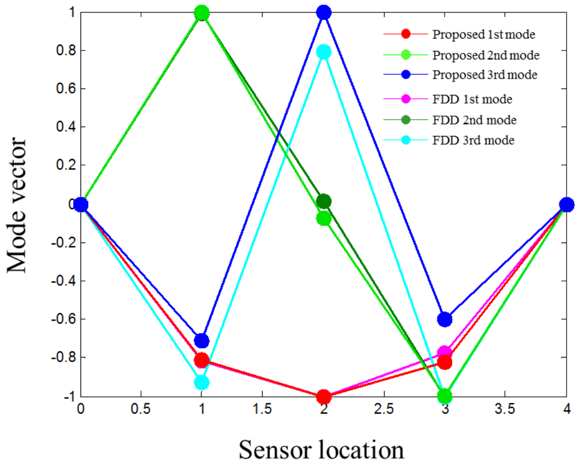

Figure 4: Mode shape.

The mode shapes of the proposed method and of the previous SI method, FDD, were compared. The normalization was performed in the manner that has the maximum value of the mode vector be equal to 1. The first mode shapes showed good agreement among each other; however, the mode shapes of higher order did not exhibit good agreement among each other. As a result, the MAC values were calculated to check the orthogonality, as described below.

	FDD			Proposed		
	1st	2nd	3rd	1st	2nd	3rd
1st	1.0000	0.0005	0.0956	1.0000	0.0001	0.0003
2nd	0.0005	1.0000	0.0018	0.0001	1.0000	0.0005
3rd	0.0956	0.0018	1.0000	0.0003	0.0005	1.0000

Table 1: Comparison of MAC value

FDD was performed under the following conditions: 8192 NFFT, Hanning type window and 50% averaging ratio. The mode shape from FDD showed a high correlation between the first and the third modes, in contrast to the proposed method. In addition, orthogonality of the mode shapes from FDD was not exactly satisfied. However, such a result is typical for FDD because the responses are mixed and affected by contiguous mode. Thus, the proposed method was confirmed to be able to separate the principal vector of the structural responses with good orthogonality and to overcome the weakness of FDD.

3.6 Computation time

	Data length (NFFT)	FDD	Proposed
Computation time (sec.)	4096	2.642	1.413
	8192	4.692	1.843
	12288	6.791	2.295
	16384	8.870	2.921

Table 2: Computation time for estimating mode shape

Finally, the computation times for the different methods were calculated and compared. The proposed method was confirmed to reduce the computation time required to estimate the mode shape. Practically, to identify a high-rise building or large spatial structures, long-term measurements and tests have been performed. However, the more the measurement is conducted over a long period, the more estimation of the mode shape makes the computation complicated. The reason why real-time SI has problems is the increments of the computation time due to the long measuring time. Therefore, if the proposed method in this paper is applied to higher and larger structures with long measurement times, it will be more useful to conduct SHM and SI and even to perform damage detection.

4 CONCLUSIONS

In this paper, the method for visually estimating the mode shape was proposed and applied to the experimental test to overcome the high computational complexity of SVD, the method that is used frequently in the previous SI methods. The proposed method was applied to a steel bridge in a building, and the measurement of the ambient vibration was used to perform

OMA. Both the computation time and the mode shape for the previous SI method, FDD, and the proposed method are compared. The mode shapes of low order showed good agreement; in contrast, the higher order mode shapes showed poor agreement. However, the modal orthogonality from the proposed method was found to be exactly satisfied, in contrast to the result of FDD. In addition, the proposed method was confirmed to reduce the computation time required to estimate the mode shape. Therefore, if the proposed method were used, the real-time SHM or SI, even damage detection, using the mode shape are apparently possible to perform, with reduced computation time.

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