8th European Workshop On Structural Health Monitoring (EWSHM 2016), 5-8 July 2016, Spain, Bilbao
www.ndt.net/app.EWSHM2016

SOLVING BAYESIAN MULTI-PARAMETER ESTIMATION PROBLEMS USING THE MECHANICAL EQUIVALENT OF LOGICAL INFERENCE

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Key words: COST Action 1402, Logical inference, Bayes' Theorem, Structural Health Monitoring, Value of Information.

Abstract

Structural Health Monitoring requires engineers to understand the state of a structure from its observed response. When this information in uncertain, Bayesian probability theory provides a consistent framework for making inferences. However, structural engineers are often unenthusiastic about Bayesian logic, finding its application complicated and onerous, and prefer to make inference using heuristics. Here, we propose a quantitative method for logical inference based on a formal analogy between linear elastic mechanics and Bayesian inference with linear Gaussian variables. To start, we investigate the case of single parameter estimation, where the analogy is stated as follows: the value of the parameter is represented by the position of a cursor bar with one degree of freedom; uncertain pieces of information on the parameter are modelled as linear elastic springs in series or parallel, connected to the bar and each with stiffness equal to its accuracy; the posterior mean value and the accuracy of the parameter correspond respectively to the position of the bar in equilibrium and to the resulting stiffness of the mechanical system composed of the bar and the set of springs.

Similarly, a multi-parameter estimation problem is reproduced by a mechanical system with as many degrees of freedom as the number of unknown parameters. In this case, the inverse covariance matrix of the parameters corresponds to the Hessian of the potential energy, while the posterior mean values of the parameters coincide with the equilibrium – or minimum potential energy - position of the mechanical system. We use the mechanical analogy to estimate, in the Bayesian sense, the drift of elongation of a bridge cable-stay undergoing continuous monitoring. We demonstrate how we can solve this in the same way as any other linear Bayesian inference problem, by simply expressing the potential energy of the equivalent mechanical system, with a few trivial algebraic steps and with the same methods of structural mechanics. We finally discuss the extension of the method to non-Gaussian estimation problems.

1 INTRODUCTION

Structural engineers usually have a solid background in mechanics, yet not always a good relationship with probability theory. In most cases, this is not that critical because code-based design is practically probability-free, with serious probabilistic analysis typically being confined to the most recondite annexes of the codes [1]. It is different for those engineers who grapple with structural health monitoring (SHM), an activity where the objective is to estimate the state of a structure from an uncertain batch of observations provided by different kind of sensors, such as strain gauge [2], fiber optic sensor [3] and accelerometers [4]. A consistent framework for making inferences from uncertain information is Bayesian probability theory [5].Yet structural engineers are often unenthusiastic about Bayesian formal logic, finding its application complicated and burdensome, and they prefer to make inference by using heuristics. In this contribution, we wish to help structural engineers reconcile with probabilistic logic [6] by suggesting a quantitative method for logical inference based on a formal analogy between mechanics and Bayesian probability. We will state the fundamentals of the analogy in the next section.

To start, we will limit the analogy to the case of linear Gaussian single-parameter estimation, which corresponds in the mechanical counterpart to mere linear elastic single-degree-of-freedom analysis: a cakewalk for structural engineers. In section 3, we apply this formal analogy to a classical inference problem: the estimation of the deformation of a cable belonging to a cable-stayed bridge, characterized by two independent parameters. We will carry out the simple problem of linear regression by solving the equivalent mechanical system of springs.

2 SINGLE-PARAMETER FORMULATION

In this section, we refer to the problem of logical inference of a single parameter based on uncertain information [7]. The goal is to estimate a parameter θ based on a set of uncertain information y_i. Further assumptions are that all the uncertain quantities have Gaussian distribution, and that the relationship between information and parameter is linear. When the problem is linear and Gaussian, in principle we can solve any logical inference problem using the following two fundamental rules.

First inference rule or inverse-variance weighting rule [8]. Given a set of n observations y_i of variance σ^2_i, the inverse of the variance σ^2_θ of the parameter is the sum of the inverse-variances of the observations, and the expected value of the parameter μ_θ is the inverse-variance weighted sum of the observations:

$$\frac{1}{\sigma_\theta^2}=\sum_{1=1}^{n}\frac{1}{\sigma_i^2},\qquad \mu_\theta=\frac{\sum_{1=1}^{n}\frac{y_i}{\sigma_i^2}}{\sum_{1=1}^{n}\frac{1}{\sigma_i^2}}. \tag{1a,b}$$

Second inference rule or linear propagation of uncertainties [9]. The indirect measurement $y=x_1+\ldots x_m$ being the sum of m different arguments x_j of variance σ^2_j, the variance of the observations is the sum of the variance of the arguments and the mean value of the indirect observation is the sum of the arguments:

$$\sigma_y^2=\sum_{j=1}^{m}\sigma_j^2,\qquad \mu_y=\sum_{j=1}^{m}x_j. \tag{2a,b}$$

Before proceeding, it is convenient, primarily to lighten notation, to introduce the quantity:

$$w = \sigma^{-2} = \frac{1}{\sigma^2}. \tag{3}$$

The quantity w is compatible with the official definition of *accuracy* [10] [11] and the word itself intuitively connects to the practical meaning of w: the higher the *accuracy* w of an observation is, the more accurate our knowledge about the parameter becomes. Therefore, in the rest of the paper we will refer to the inverse-variance w simply as *accuracy*. Based on that, we can reword and reformulate the two basic inference rules.

First inference rule. Given a set of n observations y_i with accuracy w_i, the accuracy w_θ of the parameter estimation is the sum of the accuracy of the observations, and the mean value of the parameter μ_θ is the sum of the observations weighted with their accuracy:

$$w_\theta = \sum_{1=1}^{n} w_i, \qquad \mu_\theta = \frac{\sum_{1=1}^{n} y_i w_i}{w_\theta}. \tag{4}$$

Second inference rule. The indirect measurement $y=x_1+\ldots x_m$ being the sum of m different arguments x_j of variance $\sigma^2{}_j$, the inverse accuracy of the observation is the sum of the inverse accuracy of the arguments and the mean value of the indirect observation is the sum of the arguments:

$$\frac{1}{w_y} = \sum_{j=1}^{m} \frac{1}{w_j}, \qquad \mu_y = \sum_{j=1}^{m} x_j. \tag{5}$$

At this point, it is not difficult for a structural engineer to spot in (5a) the same form of the expression that provides the stiffness of a set of springs in parallel; and similarly, (5b) recalls of the stiffness expression of a set of springs in series. This opens a door to an analogy between the world of logic and the world of mechanics. Particularly, the analogy statements are the following.

1. The value of the parameter is represented by the position of a rigid bar with one degree of freedom.
2. An uncertain piece of information on the parameter is modeled as a linear elastic spring fixed at one edge and connected at the other to the bar, with stiffness equal to its accuracy and pre-stretch equal to its mean value.
3. Multiple sources of uncertainties on the same information are modeled as serial springs, each with stiffness equal to its accuracy.
4. The posterior mean value of the parameter corresponds to the position of the bar, in equilibrium.
5. The posterior accuracy of the parameter corresponds to the resulting stiffness of the bar.

The basic elements of the analogy are summarized in Table 1. Figure 1 shows the mechanical representation of simple linear Gaussian inference problems.

Symbol	Logical meaning	Mechanical meaning
w, σ^{-2}	accuracy,inverse-variance	stiffness
σ^2	variance	flexibility
y	observation	pre-stretch
μ	expected value	equilibrium displacement

Table 1: Analogy between inference and mechanical models.

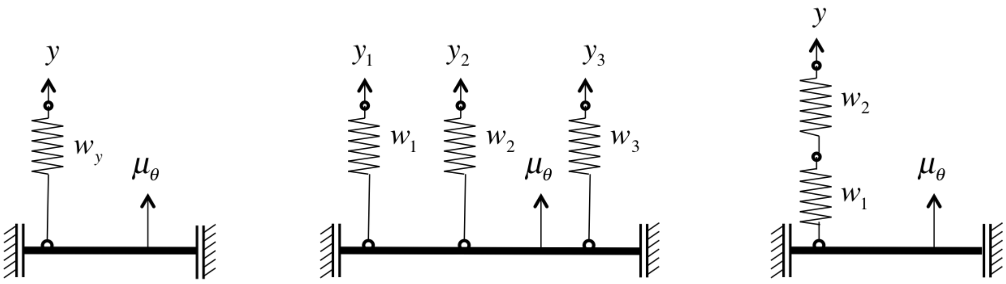

Figure 1: Mechanical analogy of simple linear Gaussian inference problems: parameter estimation based on one observation (a), three uncorrelated observations (b), one observation affected by three uncorrelated sources of uncertainty (c).

3 EXTENSION OF THE MECHANICAL ANALOGY TO N PARAMETERS

Now, we analyze a generic inference problem with N unknown parameters to estimate, represented by the vector $\boldsymbol{\theta}=(\theta_1, \ldots,\theta_N)^T$: we imagine that each parameter is characterized by a prior mean value $\mu_{\theta i}$ and a prior standard deviation $\sigma_{\theta i}$; the latter is linked by the equation $w_{\theta i} =\sigma^{-2}_{\theta i}$ to the i*th* accuracy, which in our mechanical analogy represents the stiffness of the spring associated to each single parameter. The multivariate Gaussian distribution [9], linked to the N-dimensional vector θ, takes the form:

$$\mathrm{N}(\boldsymbol{\mu},\Sigma;\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{N}{2}}}\frac{1}{|\Sigma|^{\frac{1}{2}}}\mathrm{e}^{\left\{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu})\right\}}, \tag{6}$$

where $\boldsymbol{\mu}$ is the N-dimensional mean vector, containing the N $\mu_{\theta i}$ values associated to each parameter, Σ is the NxN covariance matrix, and $|\Sigma|$ denotes the determinant of Σ.

We have to pay attention to the structure of the Gaussian distribution. We can notice that the exponent is characterized by a quadratic form that corresponds to the potential energy $E_p(\boldsymbol{\theta})$ of a mechanical system with N degrees of freedom, related to the inference problem in question. It takes the following mathematical form:

$$E_p(\boldsymbol{\theta}) = -\mathrm{In}(\mathrm{N}(\boldsymbol{\mu},\Sigma;\boldsymbol{\theta})) = \frac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\theta})^T\Sigma^{-1}(\boldsymbol{\mu}-\boldsymbol{\theta}). \tag{7}$$

We name here the inverse of the covariance matrix $\Lambda=\Sigma^{-1}$; this is also known as accuracy matrix [12]. Its diagonal terms represent the posterior stiffness $w_{\theta i|y}$ of each single parameter. Now, to obtain the N diagonal elements of Λ we must get the second derivative of $E_p(\boldsymbol{\theta})$ with respect to each of the parameters θ_i; the elements out of diagonal are instead obtained by calculating the mixed derivatives of each parameter with respect to all other parameters. To obtain the covariance matrix we simply make the inverse of Λ. The diagonal elements of Σ represent the posterior variance $\sigma^2_{\theta i|\mathbf{y}}$ of each single parameter θ_i.

The posterior mean values $\mu_{\theta i|\mathbf{y}}$ of each parameter θ_i correspond to those values that minimize the potential energy of our mechanical system. Therefore, to discover them, we have to resolve an algebraic system with *N* variables in which there are the partial derivatives of $E_p(\boldsymbol{\theta})$, each with respect to each parameter θ_i, set equal to zero.

4 A CASE STUDY: ELONGATION OF A CABLE BELONGING TO ADIGE BRIDGE

Adige Bridge [13][14] [15] was built in 2008, ten kilometers north of the city of Trento, Italy, and spans the Adige River. It is a two-span cable-stayed bridge with a steel-concrete composite deck 260 m long (Fig. 2). The composite deck is made from 4 "I"-section steel girders and a 25 cm cast-on-site concrete slab. The deck is also supported by 12 stay cables, 6 on each side, which have a diameter of 116 mm and 128 mm. Their operational design load varies from 5,000 kN to 8,000 kN. The cables are anchored to the bridge tower, consisting of four pylons and located in the middle of the bridge.

When the construction was completed, the Italian Autonomous Province of Trento, which owns and manages the bridge, decided to install a monitoring system to continuously record force and elongation of the stay cables. Elongations are recorded by 1 m long gauge sensors, placed on each of the 12 cables. These fiber-optical sensors (FOS) [16] are based on fiber Bragg gratings (FBG) [17] which rely on a principle similar to that of photonic crystals [18] [19] but provide better precision. These sensors also record local temperature for thermal compensation.

4.1 Two parameters to estimate

As an example, we use data acquired from October 12, 2011, to November 25, 2012, for cable 1TN, purified of the effect of temperature. We consider only one sample a day, recorded between 4 AM and 6 AM, as we assumed the temperature in this period to be constant.

We have discarded those days in which no samples were found in the time interval. Fig. 3 shows the data acquired, expressed in terms of difference of deformation and time:

$$\Delta y = y_i - y_1, \quad \Delta t = t_i - t_1. \tag{8}$$

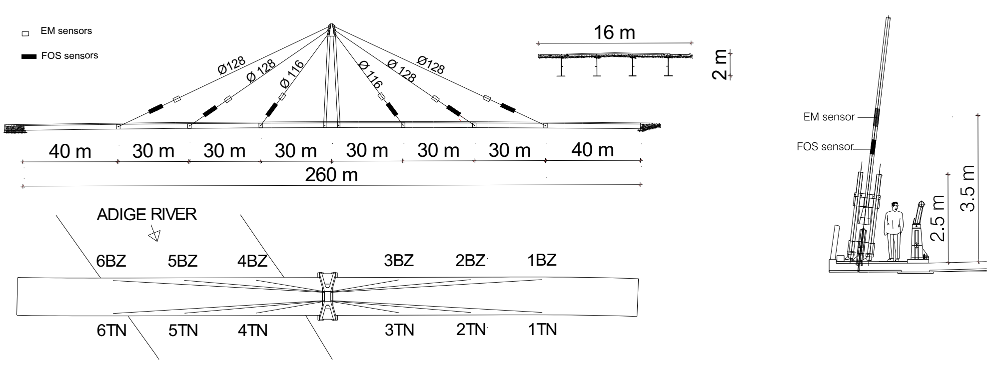

Figure 2: Longitudinal section of the bridge and sensor layout (upper left); plan view of the bridge (lower left); cross-section of the bridge (right).

During the analysis, 411 deformation measurements were recorded with an uncertainty for each measurement equal to w_y=0.0016 $\mu\varepsilon^{-2}$, i.e. σ_y=25 $\mu\varepsilon$.

This is clearly a classical problem of the linear regression. We have to estimate the two parameters that best characterize the straight line fitting our time-dependent data set. The function employed is:

$$y = y_0 + \varphi \cdot t, \tag{9}$$

where y_0 is the intercept and φ the slope of the straight line fitting our dataset. As we said before, the goal is to estimate the vector of the parameters $\boldsymbol{\theta} = (y_0, \varphi)^T$ that characterizes the parametric model resulting in the observations $\mathbf{y}=(y_1,y_2,\ldots,y_N)^T$, linearly dependent on the time t, as shown in Figure 4.

We can represent the problem as a bar with two degrees of freedom: vertical translation and rotation. According to the parametric model defined in (9), we consider the slope of the bar linked to the parameter φ, its length to the time t and its distance from the ground floor to the parameter y_0. Based on our experience, we assign to the two parameters φ and y_0 two prior Gaussian distributions that give us the initial information about the state of the bar. We connect the left-hand end of the rigid bar to a vertical linear elastic spring with flexibility equal to the standard deviation of the prior distribution associated to the parameter y_0 and pre-stretch equal to its mean value. We connect the same end to a torsion spring with flexibility and imposed rotation equal respectively to the standard deviation and the mean value of the prior distribution associated to the parameter φ, as shown in Figure 4. Finally, we introduce the measurements as a system of linear springs, each with flexibility and pre-stretch equal respectively to the standard deviation and value associated to a single measurement. Each spring is placed at a distance from the torsion spring equal to the corresponding interval of time t_i.

The elastic potential becomes:

$$\mathrm{E_p}(y_0,\varphi) = \frac{1}{2} w_{y_0}(y_0 - \mu_{y_0})^2 + \frac{1}{2} w_\varphi(\varphi - \mu_\varphi)^2 + \frac{1}{2} w_y \sum_{i=1}^{N} [(y_0 + \varphi t_i) - y_i]^2, \tag{10}$$

where $\Delta y_i = y_0 + \varphi \cdot t_i - y_i$ represents the elongation suffered by the N springs linked to the observations, due to a generic translation y_0 and a generic rotation φ imposed on the system. The accuracy matrix is simply the Hessian matrix of (10):

$$\boldsymbol{\Lambda} = \begin{bmatrix} \dfrac{\delta^2 \mathrm{E_p}(y_0,\varphi)}{\delta y_0^{\ 2}} & \dfrac{\delta^2 \mathrm{E_p}(y_0,\varphi)}{\delta y_0 \delta \varphi} \\ \dfrac{\delta^2 \mathrm{E_p}(y_0,\varphi)}{\delta \varphi \delta y_0} & \dfrac{\delta^2 \mathrm{E_p}(y_0,\varphi)}{\delta \varphi^2} \end{bmatrix}. \tag{11}$$

The inverse of the matrix (11) represents the covariance matrix Σ: the first term of its diagonal is the posterior variance associated to the parameter y_0 while the second term on the same diagonal is the posterior variance associated to the parameter φ. To identify instead the values $\mu_{y0|\mathbf{y}}$ and $\mu_{\varphi|\mathbf{y}}$, we must solve the system formed by the first derivative of (10) with respect to the parameter y_0 and the parameter φ, set equal to zero.

$$
\begin{cases}
\dfrac{\delta \mathrm{E_p}(\boldsymbol{\theta})}{\delta y_0} = w_{y_0}(y_0 - \mu_{y_0}) + w_y \sum_{i=1}^{N}[(y_0 + \varphi t_i) - y_i] = 0 \\
\dfrac{\delta \mathrm{E_p}(\boldsymbol{\theta})}{\delta \varphi} = w_\varphi(\varphi - \mu_\varphi) + w_y \sum_{i=1}^{N} t_i[(y_0 + \varphi t_i) - y_i] = 0
\end{cases}
. \qquad (12)
$$

The solutions of the system (12) give us the values of $\mu_{y0|y}$ and $\mu_{\varphi|y}$ that represent the posterior mean values associated respectively to the parameters y_0 and φ, and that minimize the potential $E_p(y_0,\varphi)$ of our mechanical system. We obtain the final outcomes reported in Table 2, compared with the prior values of the parameters. Figure 3 reports the two straight lines interpolating our dataset.

Parameter y_0				Parameter φ			
Prior distribution		Posterior distribution		Prior distribution		Posterior distribution	
$w_{y_0}\,[\mu\varepsilon^{-2}]$	0.0025	$w_{y_0}\,[\mu\varepsilon^{-2}]$	0.6601	$w_\varphi\,[\mu\varepsilon^{-2}\mathrm{day}^{-2}]$	1	$w_\varphi\,[\mu\varepsilon^{-2}\mathrm{day}^{-2}]$	36893
$\sigma_{y_0}\,[\mu\varepsilon]$	20.00	$\sigma_{y_0}\,[\mu\varepsilon]$	2.44	$\sigma_\varphi\,[\mu\varepsilon\,\mathrm{day}^{-1}]$	1.0000	$\sigma_\varphi\,[\mu\varepsilon\,\mathrm{day}^{-1}]$	0.0103
$\mu_{y_0}\,[\mu\varepsilon]$	0.00	$\mu_{y_0}\,[\mu\varepsilon]$	-49.07	$\mu_\varphi\,[\mu\varepsilon\,\mathrm{day}^{-1}]$	0.0000	$\mu_\varphi\,[\mu\varepsilon\,\mathrm{day}^{-1}]$	0.0473

Table 2: Prior and Posterior values of the parameters to estimate

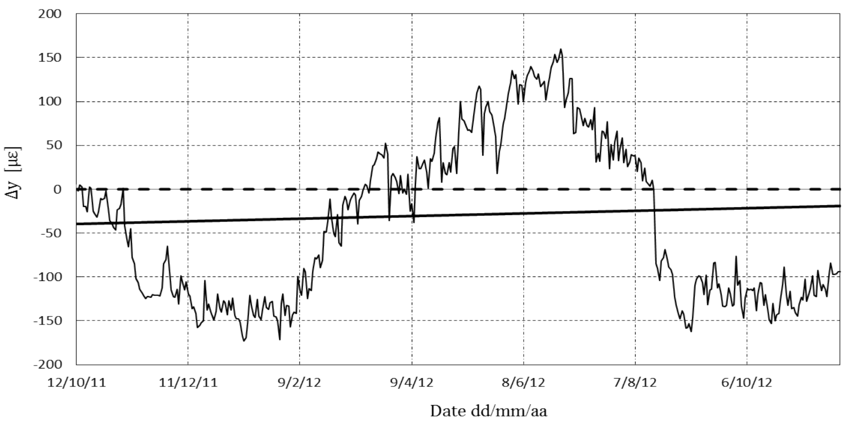

Figure 3: Relative strain of cable 1TN and interpolating lines.

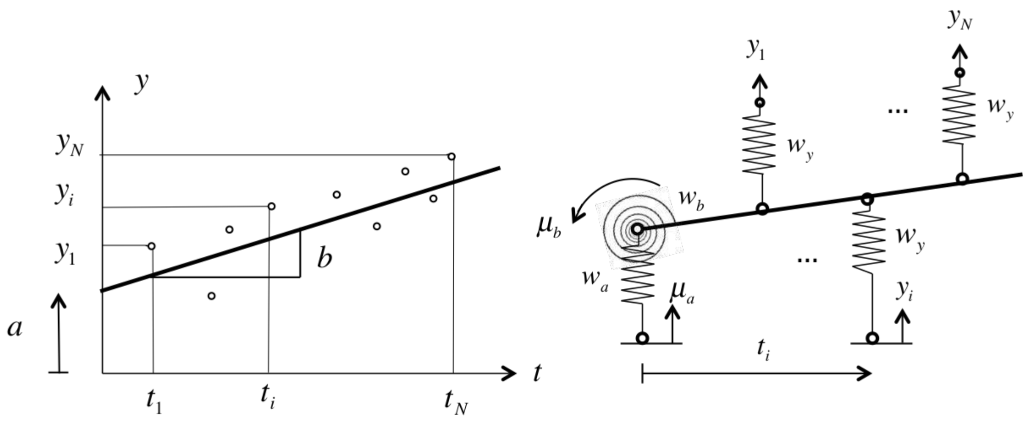

Figure 4: Representation of a linear regression problem in the world of Mechanics

4.2 Three parameters to estimate

We now extend the case of Adige Bridge, presented in the previous Section, by introducing the effect of temperature $\Delta\hat{T}$. Thus, we must estimate an additional parameter α and the model that fits our time dependent dataset becomes the following:

$$\Delta\hat{y} = y_0 + \alpha \cdot \Delta\hat{T} + \varphi \cdot \Delta\hat{t}, \tag{13}$$

In Figure 6, we can note the N translation springs linked to the different measurements with stiffness $w_{LH} = \sigma_{LH}{}^{-2} = 0.0016\,\mu\varepsilon^{-2}$ and the springs linked to the prior distribution: a translation spring associated to the parameter y_0, a rotational spring associated to α and a rotational spring associated to φ, whose their numerical values are the same as the case in Section 4.1. To determine the posterior standard deviation of the three parameters to estimate (y_0, α, φ), we have to express the potential energy $E_p(y_0,\varphi)$ of the mechanical system represented in Figure 6, as a function of the three unknown parameter. We can now obtain the accuracy matrix [redacted]simply by calculating the Hessian Matrix associated to $E_p(y_0,\varphi)$, and the covariance matrix from the inverse of [redacted]. To discover the values $\mu_{y_0|y}$, $\mu_{\alpha|y}$ and $\mu_{\varphi|y}$, which represent the posterior mean values associated respectively to the parameters y_0, α and φ, we must solve the system formed by the first derivative of the potential energy with respect to the three parameters, set equal to zero. Figure 7 shows the graphical representation, using the software Matlab, of the two surfaces fitting our data set. Finally Table 3 reports the numerical values obtained from the posterior distribution of the parameters.

5 CONCLUSIONS

We have stated an analogy between the world of logic and the world of mechanics, allowing us to solve, using the methods of classical structural engineering, any complex inference parameter estimation problem, in which the values of the parameters have to be estimated based on multiple Gaussian-distributed uncertain observations. By simply expressing the potential energy of the mechanical system associated to our inference scheme, we are able, with a few trivial algebraic steps, to determine the posterior mean values and standard deviations of the parameters to estimate. With the aid of real-life structural health monitoring cases, we have showed how our approach allows structural engineers to solve simply general problems of linear regression. Although the examples shown in this paper are incidentally all structural engineering cases, the scope of application of the method is evidently the most general, and we seek to demonstrate in the future its applicability to inference problem arising from various disciplinary fields, including cognitive science, economics and law.

Posterior distribution					
Parameter y0		Parameter φ		Parameter α	
$w_{y_0}\,[\mu\varepsilon^{-2}]$	0.6601	$w_{\varphi}\,[\mu\varepsilon^{-2}\text{day}^{-2}]$	36893	$w_{\alpha}\,[\mu\varepsilon^{-2}\,°\text{C}^{-2}]$	27.88
$\sigma_{y_0}\,[\mu\varepsilon]$	2.54	$\sigma_{\varphi}\,[\mu\varepsilon\,\text{day}^{-1}]$	0.0106	$\sigma_{\alpha}\,[\mu\varepsilon°\text{C}^{-1}]$	0.20
$\mu_{y_0}\,[\mu\varepsilon]$	0.48	$\mu_{\varphi}\,[\mu\varepsilon\,\text{day}^{-1}]$	-0.1209	$\mu_{\alpha}\,[\mu\varepsilon°\text{C}^{-1}]$	13.80

Table 3: Posterior values of the three parameters to estimate

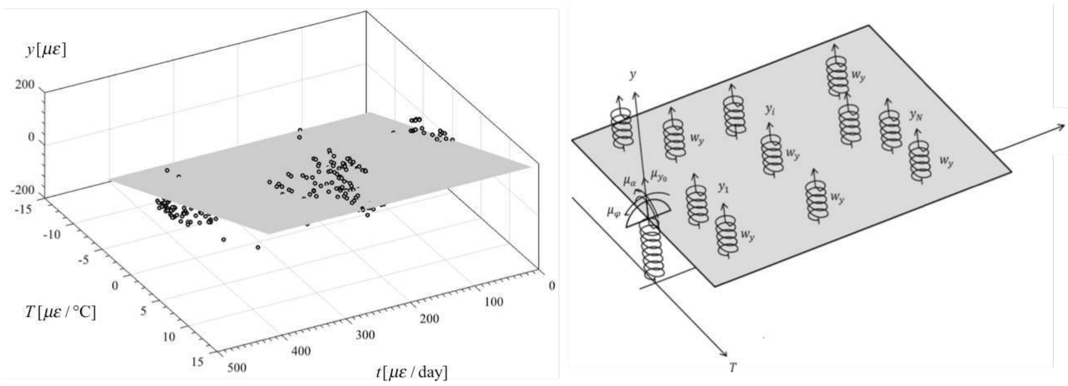

Figure 6: Mechanical representation of a linear regression problem with three parameters to estimate

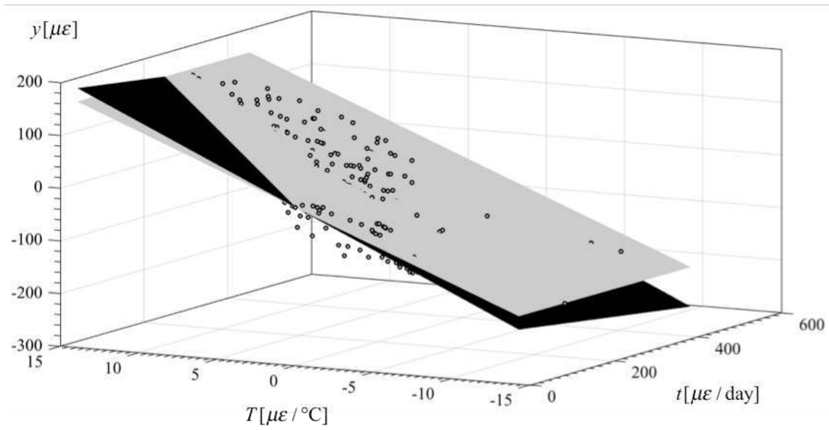

Figure 7: Representation of the two fitting surfaces associated to the prior parameters (gray) and to the posterior parameters (black).

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