8th European Workshop On Structural Health Monitoring (EWSHM 2016), 5-8 July 2016, Spain, Bilbao
www.ndt.net/app.EWSHM2016

Quantifying the value of seismic structural health monitoring of buildings

Piotr OMENZETTER[1], Maria Pina LIMONGELLI[2], Ufuk YAZGAN[3]

[1] The LRF Centre for Safety and Reliability Engineering, The University of Aberdeen, AB24 3UE, Aberdeen, UK piotr.omenzetter@abdn.ac.uk
[2] Politecnico di Milano, Milan, Italy mariagiuseppina.limongelli@polimi.it
[3] Istanbul Technical University, Istanbul, Turkey ufukyazgan@itu.edu.tr

Abstract
The decision to adopt a monitoring system for a structure should be based on sound appraisal of the expected economic benefits of such decisions. These benefits can be quantified in terms of the reduction of the risks posed by the failure of the structure versus the cost of monitoring. This paper discusses a framework for rationalising the adoption of monitoring for buildings subjected to seismic risks. The approach adopted is that of the pre-posterior decision analysis. Methods for automatic damage detection and joint utilisation of monitoring and visual inspection data are considered from a point of view of how they can be used in the pre-posterior analysis. Two short numerical examples are included to illustrate the proposed conceptual developments.

Keywords: Pre-posterior analysis, Seismic monitoring, Value of health monitoring.

1. INTRODUCTION

Structural health monitoring (SHM) techniques have now been a subject of extensive research and development for several decades. These efforts have led to significant progress in both hardware and software. Yet, the extent of adoption of the resulting technologies by different industries remains somehow underwhelming. One reason for this state of affairs can be that economic benefits of investing in SHM systems have rarely been quantified. Thus, various assertions are made about the usefulness of SHM but hard financial evidence is lacking. This paper contributes to addressing this gap in the current engineering practice by proposing a way of quantifying the value of SHM for buildings at risk from seismic damage using a pre-posterior decision analysis framework. Integrating automatic damage detection and visual inspections with the framework are also discussed.

An SHM system installed in a building cannot prevent seismic damage as such, but the types of seismic risk which it can help to reduce are related to the consequences of incorrect post-event decisions. For example, if the building sustains inconspicuous damage in the main shock, but people and valuable content are not evacuated, casualties and significant economic losses may ensue if aftershocks lead to further damage and partial or total collapse. On the other hand, evacuating a building which is structurally sound will entail unnecessary financial losses due to business and occupancy interruption. To manage such risks requires choosing between several competing alternatives. These include ‘do nothing’, invest in different types of SHM systems with varying performance characteristics, invest in enhanced pre/post-earthquake visual inspections (e.g. similar to San Francisco’s Building Occupancy Resumption Programme [1]), invest in seismic retrofit, and others; some choices may be combinations of the aforementioned. All such choices bring certain benefits but come at a cost; furthermore, the benefits and cost cannot usually be quantified purely deterministically.

2. PRE-POSTERIOR ANALYSIS FOR QUANTIFYING VALUE OF SEISMIC SHM

Suppose the stakeholders in the building are to decide if they want to adopt a monitoring system that will detect damage to the building struck by a strong earthquake. Depending on the output from the monitoring system (i.e. damage detection alarm), the building will be either evacuated or normal, uninterrupted occupancy will continue. The two different scenarios entail different consequences, expressed here as costs. We will use the pre-posterior analysis framework [2] to decide whether an SHM system should be used.

The possible decisions and their outcomes and states of nature for deciding whether to adopt SHM are shown in Table 1. The set of prior probabilities applicable to the problem are summarised in Tables 2 and 3. The prior probabilities of damage being actually sustained (Table 2) can be obtained via seismic hazard and structural vulnerability analysis [3]. The likelihoods of damage being detected (or not) if it actually occurred (or not) can be evaluated in many ways including numerical analysis, laboratory experimentation and full scale experimentation and observations (the latter option is rarely practical, due to the general dearth of monitoring data from full scale structures that have actually been damaged or tested to damage). The costs assumed in the decision making are summarised in Table 4.

Figure 1 shows a decision tree for the seismic SHM adoption analysis. In the decision tree, there are posterior probabilities, $p_{DSi|DDj}$, indicated for the actual damage sustained (DS_i; i=0 or 1, where i=0 corresponds to damage absence and i=1 to damage presence, respectively) given the damage detection (DD_j) outcome from the monitoring system (j=0 or 1, where j=0 corresponds to damage not being declared and j=1 to being declared, respectively). These probabilities can be calculated using Bayes' theorem as shown in Table 5. These calculations also yield the total probabilities of the monitoring system indicating damage p_{DDi} (i=0 or 1, where i=0 corresponds to damage being not declared and i=1 to being declared, respectively; note these declarations include false positives and negatives).

The optimal decision minimizes the overall expected cost C:

$$MO_{opt} = \min_{i=0,1} E_{DDj} \min_{k=0,1} E_{DSl|DDj} \left[C\left(MO_i, DD_j, EV_k, DS_l \right) \right], \quad i, j = 0,1 \tag{1}$$

over all possible sequences of choices and chance outcomes (MO_i, DD_j, EV_k and DS_l). In Eq. (1) E is the expected value operator. (We assume a risk neutral behaviour but by defining a different utility risk aversion or loving can easily be accommodated.)

2.1 Illustrative Example 1

This example uses the proposed framework to assess the benefits of using SHM systems with different performance and cost for a selection of buildings with different prior probabilities p_{DS1} of sustaining damage in an earthquake ranging from 1% through to 99%. The costs of different chance outcomes (expressed in non-dimensional units) are assumed to be C_{damage}=2×10^5, $C_{casualty}$=1×10^6 and $C_{interrupt}$=1×10^5. Several monitoring systems are considered, which have different likelihoods of correct damage detection as shown in Table 6, where the probabilities of correct indication, $p_{DD1|DS1}$ or $p_{DD0|DS0}$, are assumed to be equal and range between 50% and 99%. The former value represents a very poorly performing system which gives purely random indications, whereas the latter a system that is performing very well. It is assumed that the cost of monitoring system, C_{monit}, increases exponentially with the probability of correct indication, $p_{DD1|DS1}$ or $p_{DD0|DS0}$, as a proportion of C_{damage} starting with 0.1% for $p_{DD1|DS1}$=50% and ending with 5% for $p_{DD1|DS1}$=99%, respectively. This is shown in Fig. 2.

Figure 3 shows the results as the ratios of the total expected cost of decisions to use different monitoring systems to the expected cost of 'do nothing'. It can be seen that for buildings with low prior damage probabilities, e.g. 1%, the additional information provided by the monitoring system does not reduce the overall expected cost. This is because the monitoring information for this case will only provide some reassurance for the optimal prior decision that the optimal action is not to evacuate the building in the event of an earthquake (EV_0) (because of the low risk of casualties and high risk of unnecessary interruption to occupancy and business), while it will still cost to operate the SHM system. Similarly, for high prior damage probabilities, e.g. 99%, the additional information from monitoring does not change the prior optimal decision to evacuate (EV_1), because of the high risk of casualties. Where using monitoring makes economic sense is the intermediate range of prior damage probabilities. There, the additional information from monitoring can help to identify cheaper options. For example, for a prior damage probability of 20%, the additional information from monitoring systems with a likelihood of correct damage detection not less than 70% will enable making decisions about evacuation based on the outcomes of monitoring leading to the reduced overall expected costs. These conclusions follow the well-known fact that prior probabilities have a strong influence on the interpretation of additional data. All the observations and conclusions in this short illustrative example depend, as a matter of course, on the assumed costs of each decision and chance outcome.

3. METHODS FOR AUTOMATIC DAMAGE DETECTION

Damage detection systems are usually based on observing changes in a damage feature, i.e. a parameter which is sensitive to damage. An important family of damage features are these that can be retrieved from dynamic structural responses. A number of different vibration-based features have been proposed in literature in the last decades and surveys have been presented [4, 5]. Many of them are based on modal frequencies, modal or operational shapes and their derivatives (i.e. rotations and curvatures). Natural frequencies have been widely used for damage detection purposes, i.e. to assess the existence of damage, since they can be measured from a very limited number of sensors and are less contaminated by noise than modal or operational shapes. However, a major problems with the use of modal frequencies

Table 1: Decisions, their outcomes and states of nature.

Decision/random event	Decisions/outcomes/states of nature	Interpretation
Adopt monitoring system, *MO*	MO_0 MO_1	Do not adopt monitoring Adopt monitoring
Damage detected by monitoring system, *DD*	DD_0 DD_1	Damage not detected Damage detected
Evacuate building, *EV*	EV_0 EV_1	Do not evacuate Evacuate
Damage actually sustained, *DS*	DS_0 DS_1	Damage not sustained Damage sustained

Table 2: Prior probabilities of damage to be sustained by the building.

DS_0	DS_1
p_{DS0}	p_{DS1}

Table 3: Likelihoods of damage detection by a monitoring system.

	DD_0	DD_1
DS_0	$p_{DD0\|DS0}$	$p_{DD1\|DS0}$
DS_1	$p_{DD0\|DS1}$	$p_{DS1\|DS1}$

Table 4: Costs.

Cost type	Notation
Cost of monitoring system design, hardware, software, integration, installation, maintenance, data storage and data analysis	C_{monit}
Cost of structural, non-structural and content damage	C_{damage}
Costs as result of consequences to humans (casualties, injuries and trauma)	$C_{casualty}$
Cost of interruption to business and occupancy	$C_{interrupt}$

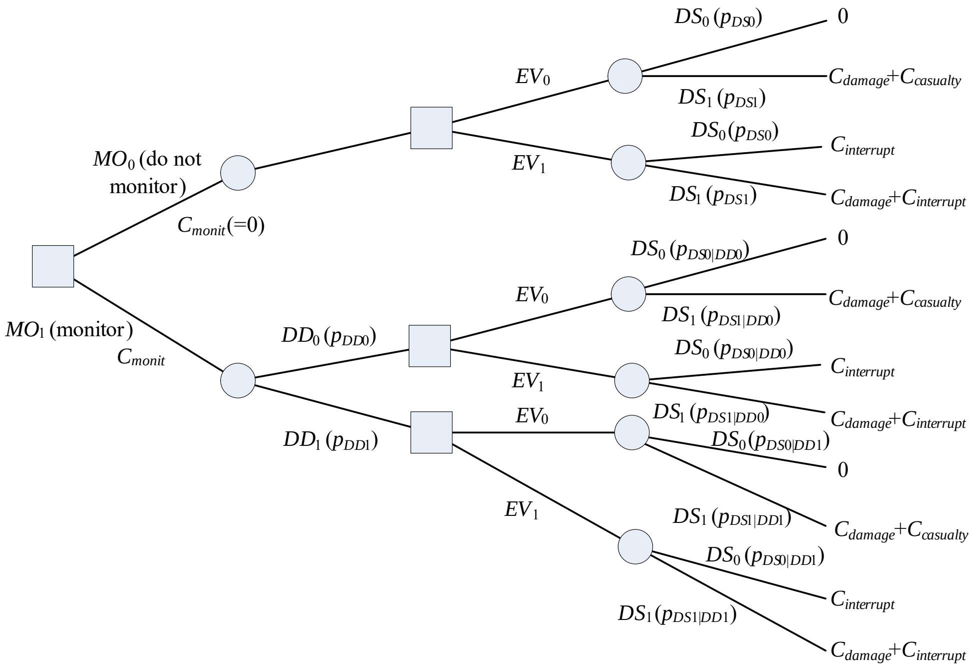

Figure 1: Decision tree for adoption of monitoring system.

Table 5: Calculation of posterior probabilities.

State of nature	Prior proba-bilities	Conditional likelihoods	Intersection probability	Posterior probability
DS_0	p_{DS0}	$p_{DD0\|DS0}$	$p_{DS0}\times p_{DD0\|DS0}$	$p_{DS0\|DD0} = p_{DS0}\times p_{DD0\|DS0}/p_{DD0}$
DS_1	p_{DS1}	$p_{DD0\|DS1}$	$p_{DS1}\times p_{DD0\|DS1}$	$p_{DS1\|DD0} = p_{DS1}\times p_{DD0\|DS1}/p_{DD0}$
			$p_{DD0} = p_{DS0}\times p_{DD0\|DS0}+p_{DS1}\times p_{DD0\|DS1}$	
DS_0	p_{DS0}	$p_{DD1\|DS0}$	$p_{DS0}\times p_{DD1\|DS0}$	$p_{DS0\|DD1} = p_{DS0}\times p_{DD1\|DS0}/p_{DD1}$
DS_1	p_{DS1}	$p_{DD1\|DS1}$	$p_{DS1}\times p_{DD1\|DS1}$	$p_{DS1\|DD1} = p_{DS1}\times p_{DD1\|DS1}/p_{DD1}$
			$p_{DD1} = p_{DS0}\times p_{DD1\|DS0}+p_{DS1}\times p_{DD1\|DS1}$	

Table 6: Assumed conditional likelihoods of damage detection by monitoring systems used in Example 1.

	DD_0	DD_1
DS_0	$p_{DD0\|DS0}= p_{DD1\|DS1}$	$p_{DD1\|DS0}$=1- $p_{DD0\|DS0}$
DS_1	$p_{DD0\|DS1}$=1- $p_{DD1\|DS1}$	$p_{DD1\|DS1}$ =$p_{DD0\|DS0}$

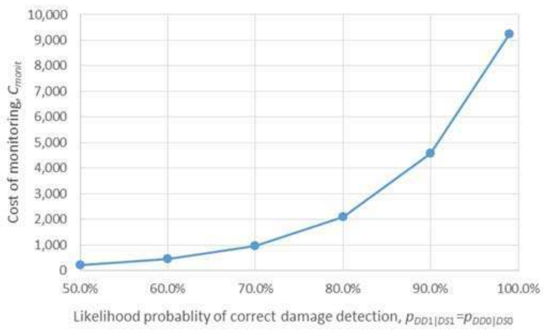

Figure 2: Cost of monitoring vs. likelihood of correct damage detection used in Example 1.

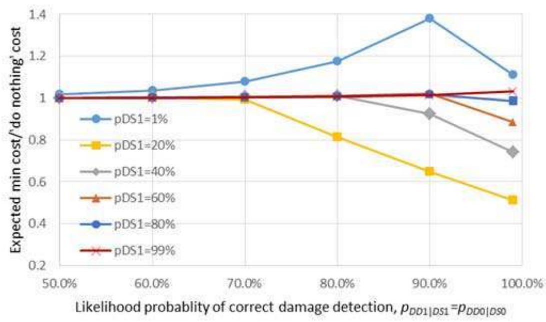

Figure 3: Ratio of total expected cost when using monitoring to 'do nothing' for buildings with varying prior damage probabilities in Example 1.

for damage detection is the possible marked influence of varying environmental and operational conditions [6]. Factors such as temperature, moisture, nonlinear behaviour, soil-structure interaction, noise in recorded data, unaccounted-for excitation sources such as traffic and others, can induce variations in the damage features even if no damage occurs leading to false alarms. Conversely, variations in the damage feature due to genuine damage may be erroneously attributed to the aforementioned sources leading to missing correct alarms. The variability of the damage features with environmental and operational conditions has thus to be taken into account in the damage assessment procedures. To this effect, two main approaches are proposed in literature. The first one is based on techniques which are able to remove the effects of the environment from the features chosen to indicate the existence and location of damage [7]. The second approach is based on the use of robust features that are hardly affected by changes in the environmental conditions but still sensitive to damage [8]. In any case, even when using the second type of features, the statistical variability of the considered feature must be properly taken into account in order to ensure damage assessment techniques are robust even for low extents of damage that may induce variations in the damage feature of the same order of magnitude than environmental and/or operational sources. This is particularly important for quantifying the performance of a monitoring system to assess if it is worth to install it on a structure.

In order to assess the performance of damage detection techniques in the probabilistic pre-posterior framework outlined in the previous section, the conditional and total probabilities included in the decision tree in Fig. 1 and in Tables 2 and 3 are defined herein using probability distribution functions of a damage detection feature. In Fig. 4, the distributions of the feature in the undamaged, f_{DS0}, and damaged, f_{DS1}, configurations are shown. In this case, it is assumed that damage shifts the distribution towards higher values, but an opposite behaviour may well be observed in practice.

Depending on the type of monitoring system installed, the following different cases may occur for estimating the feature distributions from measurement data:

- Permanent (long term) monitoring: a network of sensors is permanently installed on the structure allowing to estimate both f_{DS0} and f_{DS1}, assuming damaging events occur within the monitoring time window.
- Periodic monitoring: a network of sensors is deployed for a limited period on the healthy structure, allowing estimating f_{DS0}, and then removed. Further monitoring can be carried out in emergencies (e.g. after a possible damaging event), but does not allow full estimation of f_{DS1} because of limited data.
- Short term monitoring: A very limited number of tests (possibly just one in some cases) carried out in the reference and in a possible damaged configuration. In this case neither f_{DS0} nor f_{DS1} can be fully estimated.

However, most decisions on installing an SHM system on a structure will have to be under taken without access to any existing data from the structure. In such cases numerical models will have to be used. This is always a challenging task due to the difficulty in simulating reliably effects such as structural nonlinear behaviour and the variability of the damage features due the random sources such as temperature. This step is, however, indispensable if we are to quantify the value of future monitoring information.

It is noted that the outcomes DD_0 (no damage) and DD_1 (damage) from the monitoring system require that a threshold T is defined so that if the value of the damage feature is below the threshold the outcome will be DD_0, otherwise it will be DD_1 (see Fig. 4).

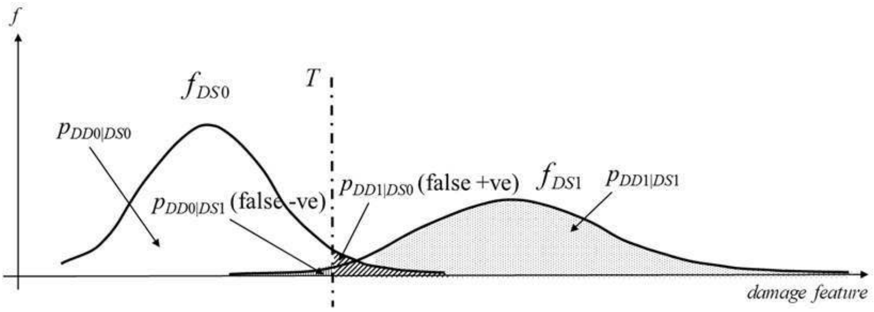

Figure 4: Probability distributions of damage feature in undamaged, f_{DS0}, and damaged, f_{DS1}, configuration.

The values of probabilities of true positives $p_{DD1|DS1}$, true negatives $p_{DD0|DS0}$, false positives $p_{DD1|DS0}$ and false negatives $p_{DD0|DS1}$ are all functions of the value chosen for the threshold T. The threshold value is usually defined based on a trade-off between the probabilities of false negatives and false positives, i.e. between the willingness to accept, on the one hand, the risk that for a damaged building a necessary evacuation is not carried out thus jeopardizing the lives of the building's users, $C_{casualty}$, and building contents, C_{damage}, and, on the other hand, the willingness to accept the risk that for an undamaged structure an unnecessary, but costly, evacuation is carried out resulting in interruption to occupancy and business, $C_{interrupt}$ (see Fig. 1).

In this paper, we consider a simplified situation that only two states of nature exist, i.e. DS_0 - the structure is not damaged, and DS_1 - the structure is damaged. However, damage existence is really a question of damage severity or extent. If damage extent is small we assume there is no damage and vice versa. Our simplification therefore assumes implicitly a damage extent threshold, albeit we do not explicitly set its value. With reference to the decision tree shown in Fig. 1 and Table 5, the prior probabilities p_{DS0} and p_{DS1} and the likelihoods of damage being detected or not if it actually occurred or not need to be estimated. The prior probabilities p_{DS1} (and p_{DS0}=1-p_{DS1}) can be estimated using fragility curves that express the probability that a structure will sustain a different degree of damage when subjected to different ground motion levels [3]. A fragility curve is the plot of the conditional probabilities $p[D>d_i|A=a_k]$ of the damage quantifying parameter D exceeding a given value d_i when the ground motion intensity A reaches a value of a_k. If, as in our assumption, we declare the structure damaged when a damage threshold is exceeded, the prior damage probability is:

$$p_{DS1} = \int_A p\left[D \geq d_i \middle| A = a_k\right] f_A\left(a_k\right) da_k \quad (2)$$

Several definitions have been proposed in literature for the parameter D. Most of them are based on ductility ratio and/or dissipated energy, such as the Park and Ang index [9], others are defined as a ratio of the repair cost to the replacement cost [3]. The distribution of the parameter A that describes the severity of the ground motion (e.g. peak ground acceleration), $f_A(a_k)$, can be obtained via site seismic hazard study [10].

3.1 Illustrative Example 2

This illustrative example explains the estimation of the probability p_{DD1} of the monitoring system issuing an alarm (the complementary probability of not issuing an alarm is of course p_{DD0}=1-p_{DD1}). Assume that the prior probabilities of damage being sustained by the building, retrieved from the fragility curves, are p_{DD1}=70% and p_{DD0}=30%, respectively. Using a numerical model, capable of simulating the non-linear behaviour of the structure and all the relevant sources of random variations of the damage feature, one can calculate the probability distributions of the damage feature f_{DS0} (for the undamaged structure) and f_{DS1} (for the structure damaged by an earthquake). Herein, for the sake of simplicity it will be assumed that the damage feature is the dominant fundamental period of the structure identified from seismic response records and that the two feature distributions are Gaussian. The means and standard deviations assumed for them are reported in Table 7.

Table 7: Parameters of damage feature (period) probability density functions used in Example 2.

	DS_0	DS_1
Mean	1.5 s	3.0 s
Standard deviation	0.4 s	1.0 s

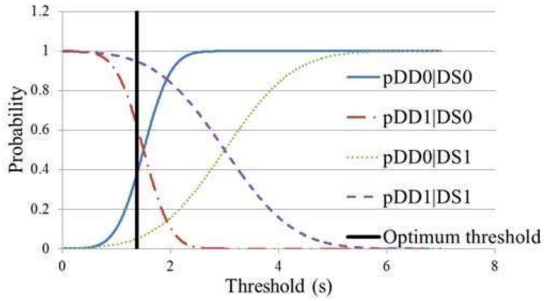

Figure 5 (right): Variation of conditional probabilities $p_{DD1|DS1}$, $p_{DD0|DS0}$, $p_{DD1|DS0}$ and $p_{DD0|DS1}$ with threshold *T*.

In Fig. 5, the variations of the conditional probabilities of damage detection with the value of the threshold *T* are reported. As already remarked, the probability of false and missing alarms follow opposite trends: at the increase of the threshold the $p_{DD1|DS0}$ decreases but $p_{DD0|DS1}$ increases.

A possible choice for the optimum threshold could be the value that results in equal risk for the false and missing alarms, i.e.:

$$p_{DD0|DS1} \times \left(C_{damage} + C_{casualty} \right) = p_{DD1|DS0} \times C_{interrupt} \tag{3}$$

Taking the same values for the costs as assumed in Example 1, i.e. C_{damage}=2×10^5, $C_{casualty}$=1×10^6 and $C_{interrupt}$=1×10^5, this choice of the threshold corresponds to the following ratio of $p_{DD0|DS1}$ to $p_{DD1|DS0}$:

$$p_{DD0|DS1} \big/ p_{DD1|DS0} = C_{interrupt} \big/ \left(C_{damage} + C_{casualty} \right) = 1\times10^5 \big/ \left(2\times10^5 + 1\times10^6 \right) = 0.0833 \tag{4}$$

The conditional probabilities corresponding to the so defined threshold are shown in Table 8.

Table 8: Conditional probabilities corresponding to the chosen threshold

$p_{DD0\|DS0}$	$p_{DD1\|DS0}$	$p_{DD0\|DS1}$	$p_{DD1\|DS1}$
37.7%	62.3%	5.3%	94.7%

Finally, the value of the probability of the system issuing an alarm, p_{DD1}, is:

$$p_{DD1} = p_{DD1|DS1} p_{DS1} + p_{DD1|DS0} p_{DS0} = 0.947\times0.7 + 0.622\times0.3 = 85\% \tag{5}$$

4. JOINT UTILIZATION OF INSPECTION AND HEALTH MONITORING DATA

The damage indications provided by automatic monitoring systems should be combined with information obtained from post-earthquake damage inspections for enhanced structural assessment. Furthermore, in order to predict the building performance with sufficient accuracy, a reliable analytical model of the structure needs to be developed and the expected seismic hazard at the building site needs to be estimated. This requires establishing a credible analytical model which captures the essential response characteristics of the considered building. In order to establish such an analytical model, accurate and precise information on the building of interest is needed. However, very often actual material properties cannot be determined precisely and they can only be estimated with significant uncertainty. Moreover, several idealizations and assumptions need to be introduced to reduce the problem to a manageable level of complexity. These simplifications may lead to considerable differences between the actual behaviour of the building and that obtained using the assumed analytical model. In order to represent the effects of uncertainties on the predicted performance, the expected variability of the performance of the structure must also be taken into account in the assessment of important buildings. This is usually achieved by taking into account potential variations in the model parameters and utilizing alternative modelling strategies. In the proposed framework, a set of alternative analytical models can be established for the building. Subsequently, from this entire model set the specific subset of models which better represent the actual seismic response characteristics of the structure are identified. Using the identified models, the expected performance of the building during its remaining service life can be predicted and the expected losses related to the structure can be reliably evaluated.

As the first step, the main uncertain parameters (P_1, P_2, …, P_{Np}) that influence risk estimation are identified. These parameters may be related to material strengths, strain limits, boundary conditions, etc. Probability distribution functions, f_{Pi}, are established for the considered N_p parameters using the available literature, e.g. [11]. For each parameter, N_m realizations are generated using the distributions f_{Pi}. As a result, N_m candidate models are generated. This step is similar to the plain Monte Carlo simulation. For an individual model (i.e. model-i) from amongst the entire set of generated models, all parameter values are contained in the vector $\mathbf{m}_i$. Before any inspection or monitoring data is considered, all models ($\mathbf{m}_1$, $\mathbf{m}_2$,…, $\mathbf{m}_{Nm}$) have an equal prior likelihood of being the most representative, $p[\mathbf{M}=\mathbf{m}_i]$ $=1/N_m$. Evaluation of model likelihood conditional on the inspection and monitoring data, is presented in the following.

Damage inspection involves identifying the damage grades of components and determining the related damage mechanisms. Damage grades are determined based on observed visual indicators (e.g. spalling of cover, cracking, and reinforcement bar rupture). When a structural component is observed to have sustained a specific grade of damage, it may be inferred that during the earthquake the component had deformed beyond the deformation limit, d_{ll}, which corresponds to the lower limit of the identified grade (Fig. 6a). Moreover, from the observation of the level of sustained damage it can also be inferred that the upper deformation limit, d_{ul}, for the identified damage grade was not exceeded during the earthquake. Limit state displacements corresponding to the lower and upper bounds can be estimated probabilistically for each model realization using the existing models for structural member performance prediction [12].

Taking into account an observation of component deformation D has exceeded the lower bound limit state displacement d_{ll}, the posterior likelihood $p[\mathbf{M}=\mathbf{m}_i|D\geq d_{ll}]$ for model $\mathbf{m}_i$ can be evaluated as:

$$p\left[\mathbf{M}=\mathbf{m}_i \left| D \geq d_{ll} \right.\right] = \left(p\left[D \geq d_{ll} \left| \mathbf{M}=\mathbf{m}_i \right.\right] p\left[\mathbf{M}=\mathbf{m}_i\right]\right) \Big/ \sum_{j=1}^{N_m} p\left[D \geq d_{ll} \left| \mathbf{M}=\mathbf{m}_j \right.\right] p\left[\mathbf{M}=\mathbf{m}_j\right] \tag{6}$$

Conditional probabilities $p[D\geq d_{ll}|\mathbf{M}=\mathbf{m}_i]$ can be evaluated using the component fragility models (Fig. 6b), which are frequently utilized in performance-based seismic design of structures. Subsequently, the resulting probabilities can be further updated by taking into account that the upper limit state deformation d_{ul} was not exceeded as follows:

$$p\left[\mathbf{M}=\mathbf{m}_i \left| D < d_{ul} \cap D \geq d_{ll} \right.\right] = \frac{p\left[D < d_{ul} \left| \mathbf{M}=\mathbf{m}_i \right.\right] P\left[\mathbf{M}=\mathbf{m}_i \left| D \geq d_{ll} \right.\right]}{\sum_{j=1}^{N_m} p\left[D < d_{ul} \left| \mathbf{M}=\mathbf{m}_j \right.\right] p\left[\mathbf{M}=\mathbf{m}_j \left| D \geq d_{ll} \right.\right]} \tag{7}$$

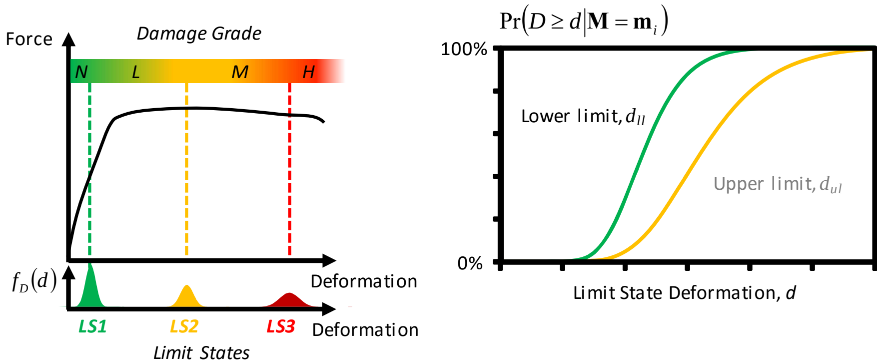

Figure 6: Damage grades and corresponding limit states: force-deformation behaviour (left), and cumulative probability distributions $p[D \geq d|\mathbf{M}=\mathbf{m}_i]$ of the limit state distributions conditional on model $\mathbf{m}_i$ (right).

The conditional probability $p[\mathbf{M}=\mathbf{m}_i|D \geq d_{ll} \cap D < d_{ul}]$ represents the likelihood estimated for model $\mathbf{m}_i$ by taking into account that damage grade observed for the structural component. Equations (6) and (7) correspond to the case of considering damage observed in a single component. In order to consider an entire set of inspected components, these equations may be evaluated repetitively. A joint set of all damage grade observations (e.g. $D_1 \geq d_{ll} \cap D_2 < d_{ul} \cap \ldots$) for the entire set of inspected components can be defined as the inspection event I formulated as follows:

$$I = \{D < d_{ul} \cap D \geq d_{ll} \cap \ldots\} \tag{8}$$

When the procedure presented above is applied, the posterior probability $p[\mathbf{M}=\mathbf{m}_i|I]$ for model $\mathbf{m}_i$ is obtained.

Seismic monitoring of a building may provide critical information about the stiffness, vibration mode shapes, and the damping characteristics. In the following, only the case for utilizing the stiffness value obtained from the monitoring data is presented but other types of information may be utilized through a straightforward modification of the equations. When the stiffness of the structure has been identified to be equal to a specific value k_h through the use of health monitoring data, the posterior probability $p[\mathbf{M}=\mathbf{m}_i|K=k_h \cap I]$ for model-i can be evaluated as follows:

$$p\left[\mathbf{M}=\mathbf{m}_i|K=k_h \cap I\right] = \frac{p\left[K=k_h|\mathbf{M}=\mathbf{m}_i\right]p\left[\mathbf{M}=\mathbf{m}_i|I\right]}{\sum_{j=1}^{N_m} p\left[K=k_h|\mathbf{M}=\mathbf{m}_j\right]p\left[\mathbf{M}=\mathbf{m}_j|I\right]} \tag{9}$$

In Eq. (9), K is the random variable representing the stiffness of the structure and k_h is the specific value of stiffness determined based on health monitoring data.

Similar to the joint event defined above for the inspection results, the joint set of observations (e.g. $K_1=k_{h1}$, $K_2=k_{h2}\ldots$) derived from structural health monitoring data can be represented as the event H as follows:

$$H = \{K_1 = k_{h1} \cap K_2 = k_{h2} \cap \ldots\} \tag{10}$$

From the sequential and repetitive evaluation of Eqs. (6), (7) and (9) the posterior probability $p[\mathbf{M}=\mathbf{m}_i|H \cap I]$ for model $\mathbf{m}_i$ which is jointly conditioned on the inspection data, I, and health monitoring data, H, can be evaluated.

5. CONCLUSIONS

This study has proposed a decision making framework for rationalising adoption of monitoring systems for buildings exposed to seismic risk. The benefits of monitoring are quantified in terms of the reduction of the risks posed by the failure of the structure to be monitored versus the cost of a monitoring system. The decision making framework is formulated as a pre-posterior decision problem. For the quick post-event condition assessment, it is proposed how methods for automatic damage detection and joint utilisation of monitoring and visual inspection data can be used in the pre-posterior analysis. Two short illustrative examples are provided to highlight some aspects of the conceptual framework developed.

ACKNOWLEDGEMENTS

Piotr Omenzetter's work within the Lloyd's Register Foundation Centre for Safety and Reliability Engineering at the University of Aberdeen is supported by Lloyd's Register Foundation. The Foundation helps to protect life and property by supporting engineering-related education, public engagement and the application of research. The COST Action TU1402 on Quantifying the Value of Structural Health Monitoring is gratefully acknowledged for networking support.

REFERENCES

[1] City and County of San Francisco, "*Building Occupancy Resumption Program (BORP): Guidelines for engineers*", http://sfdbi.org/borp-guidelines-engineer.

[2] H. Raiffa, R. Schlaifer, "*Applied statistical decision theory*", Harvard University, Boston, USA, (1961).

[3] A. J. Kappos, G. Panagopoulos, C. Panagiotopoulos, G. Penelis, A hybrid method for the vulnerability assessment of R/C and URM buildings, *Bulletin of Earthquake Engineering* **4**, 391–413, (2006).

[4] W. Fan, P. Qiao, Vibration-based damage identification methods: A review and comparative study, *Structural Health Monitoring* **10**, 83-29, (2011).

[5] E. P. Carden, P. Fanning, Vibration based condition monitoring: A review. *Structural Health Monitoring* **3**, 355–377, (2004).

[6] M. P. Limongelli, E. Chatzi, M. Dohler, G. Lombaert, Towards extraction of vibration-based damage indicators, in *Proc. 3rd Workshop of COST Action TU1402*, (2016) (to appear).

[7] B. Peeters, G. De Roeck, One year monitoring of the Z24 bridge: Environmental influences versus damage effects, in *Proc. IMAC-XVIII*, 1570–1576, (2000).

[8] M. P. Limongelli, Frequency response function interpolation for damage detection under changing environment, *Mechanical Systems and Signal Processing* **24**, 2898-2913, (2010).

[9] Y-J. Park, A. Ang, Mechanistic seismic damage model for reinforced concrete, *Journal of Structural Engineering, ASCE*, **111**, 722-739, (1985).

[10] J. W. Baker, "*An introduction to probabilistic seismic hazard analysis (PSHA)*", Stanford University, Stanford, USA, (2008).

[11] Joint Committee on Structural Safety, "*Probabilistic model code*", http://www.jcss.ethz.ch/, (2001).

[12] M. N. Fardis, D. E. Biskinis, Deformation capacity of RC members as controlled by flexure or shear, in *Proc. Int. Symp. on Performance-Based Engineering for Earthquake Resistant Reinforced Concrete Structures*, (2003).