Uncertainties when using EVA (extreme value analysis) in SHM

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Abstract
Wall thickness monitoring with permanently installed ultrasonic sensors to measure corrosion rates has become standard practice in industrial plants. However, the installed sensors only accurately measure the wall thickness at a single point. In order to assess much larger areas for the effects of corrosion one has to scan transducers over the surface (inspection), accept that the local measurement is a good representation of the rest of the structure or use a statistical extrapolation model. Extreme value analysis is a statistical method to estimate the minimum or maximum (extreme) value in the population of a random variable by combining the variations in the extremes of several smaller sample populations that have been gathered from the overall population. This technique is thought to have potential use in the assessment of general corrosion of engineering structures when wall thickness data is sampled from several sensor networks that have been permanently attached to the structure. Care is required in making sure that the spatial distribution of wall thickness loss makes it possible to apply this type of analysis. If the analysis can be applied then the errors associated with the assessment will depend on the ratio of the inspect area (i.e. the size of the area which is available to obtain data to sample to the population) and the overall area that is to be assessed (extrapolated to). This paper presents data from modelling studies that describe the expected uncertainties of this analysis approach (EVA) as a function of extrapolation ratio (assessed area/inspected area).

1 INTRODUCTION
While the techniques are much more widely applicable this paper is concerned with the analysis of wall thickness data from corroded components. One of the most commonly used non-destructive evaluation (NDE) techniques to assess components for corrosion damage is ultrasonic thickness testing. Testing can be carried out manually or automatically and report single thickness values or C-scan thicknesses maps. A key problem in practice is area coverage. In NDE there can be multiple reasons, such as cost or access limitations, as to why measurements cannot be made over the whole structure. Structural health monitoring (SHM) almost always has the same problem as only a limited number of sensors are deployed and they do not cover the full area of the structure. Nonetheless, one wants to assess the status of the whole structure from the measurements that are available. This can be achieved by extrapolation, which can yield useful results provided that the measurement data is representative of the whole structure (i.e. there are no local corrosion hot spots that are not captured).

There are several ways of presenting groups of wall thickness measurement data and for
extrapolating from it to larger areas. One method is to use the cumulative distribution function (CDF) of measured data [1] another is by means of using extreme value analysis. The CDF based methods can be dependent on the underlying thickness distributions whereas EVA takes into account statistical variation of the minimum thickness and can be applied to any underlying thickness distributions. In the authors eyes EVA forms a much stronger framework for carrying out extrapolations, however there is little published literature on the confidence intervals that are associated with an EVA extrapolation and in this paper we present simulation results that show how EVA predictions vary as the area to which one extrapolates is increased.

2 EXTREME VALUE ANALYSIS (EVA) OF WALL THICKNESS MEASUREMENTS

Consider a measured thickness map such as the one shown in figure 1. EVA says that the distribution of the minimum thickness in this area is given by:

\[
\Phi(x|\mu, \sigma, k) = 1 - \exp \left\{ - \left[ 1 + k \left( \frac{x - \mu}{\sigma} \right) \right]^{-1/k} \right\}
\]

where \(\mu\) is the location parameter, which determines the size of the minima; \(\sigma\) is the scale parameter, which determines the spread of the minima; and \(k\) is the shape parameter, which determines the shape of the distribution. All the parameters that determine \(\Phi\) can be estimated from a particular set of measurement data such as the thickness map that is shown in figure 1 by partitioning the large area into smaller blocks and extracting the minimum thickness of each block [2].

Figure 1: 200x200mm (dx,dy=1mm) colour coded thickness map as would be typically obtained during an ultrasonic C-scan. The map that is shown here is of Gaussian roughness, mean thickness = 10mm, RMS=0.1mm and correlation length 2.4mm.
Once $\Phi$ has been determined, its parameters can be used to calculate the $M^{th}$ return level:

$$r_M = \mu - \frac{\sigma}{k} \left[ 1 - \left\{ -\log \left( 1 - \frac{1}{M} \right) \right\}^{-k} \right]$$

which is the value that the minimum thickness will drop to at least once in $M$ blocks. $r_M$ therefore is an estimate of the smallest thickness one expects to find in an area the size of $M$ blocks. Based on the initial measurement data that was acquired on $N$ blocks, $r_M$ can be used to calculate the expected minimum thickness over a much larger area of $M$ blocks. This effectively allows to extrapolate from a small measurement area to a larger area whose condition is to be assessed (assuming that the larger area has the same properties, i.e. is from an identical thickness distribution as the measurements).

At this point it is convenient to define an extrapolation ratio, which is the ratio of the inspected to the extrapolated area:

$$ER = \frac{M}{N}.$$  

(3)

$ER < 1$ actually describes scenarios where areas smaller than the initially inspected area are investigated and any prediction should be very good for these values. As $ER$ becomes larger than 1 the actual extrapolation starts and the result is expected to become more uncertain.

3 CONFIDENCE INTERVAL OF THE RETURN LEVEL

EVA is mostly used to report a return level only. This however does not indicate the uncertainty with which the prediction is made. When taking decisions based on measurement data it is important to know the confidence intervals/uncertainty bounds with which a result has been obtained. This is especially important because the levels of uncertainty will change as the extrapolation ratio is increased and one tries to draw conclusions about populations that are much larger than the measurement sample size. It is therefore also important to report the confidence intervals of the estimated return level.

The 95% confidence intervals of $r_M$ can be determined using the profile likelihood method, the definition of a deviance function and the fact that the deviance function is chi-squared distributed [3] [publication pending].

4 SIMULATION RESULTS FOR GAUSSIAN SURFACES

Simulated data was used to test EVA extrapolations and predictions of the minimum thickness value of a large population based on the evaluation of smaller samples of the population. In order to do so, thickness measurements from 1000 Gaussian rough surfaces (mean $=10\text{mm}$ thickness, RMS$=0.1\text{mm}$ and correlation length $=2.4\text{mm}$) of $48\times48\text{mm}$ dimensions with $1\text{mm}$ steps were created using the same algorithm [4]. 25 of these surfaces were then used to construct an extreme value model. Using the model parameters and equations 2 and 3 the return level and its confidence intervals for different extrapolation ratios were predicted. Figure 2 shows the result of the return level predictions (back crosses, blue crosses indicate confidence intervals) as well as the actual distribution of minimum thicknesses of the 1000 surfaces in form of a box and whisker plot (where 50% of results are contained within the box and 90% within the whiskers).

When looking at the data in figure 2 it is important to note that for $ER < 1$, interpolation is
carried out and the predictions are very good. For example the return level prediction for ER=0.08 or 1 block is 9.66mm or just below the median return level (9.67mm) of all the 1000 simulated surfaces. For ER values larger than unity extrapolation is performed. The return level becomes closer and closer to the minimum thickness that is actually observed amongst the 1000 simulated surfaces and the 95% confidence interval range becomes larger and extends beyond the observed minimum thickness within the 1000 simulated surfaces.

Figure 2: Box and whisker plot showing the distribution (box 50%, whiskers 90% and red line median value) of thickness minima in 1000 simulated Gaussian rough surfaces patches (48x48mm) of mean thickness 10mm, RMS=0.1mm and 2.4mm correlation length. To the right of each box and whisker plot the return level values (black x) and upper and lower values of the 95% confidence intervals (blue o) as predicted by an EVA model based on minima from 25 of the 48x48mm patches for different extrapolation ratios (ER) are shown.

It is instructive to express the 95% confidence interval width as a percentage of the actual return level and to plot this as a function of extrapolation ratio. This was done several times in figure 3 for EVA models that are based on a different number of minima N (or different number of blocks that the data is split into for analysis). The figure shows that confidence intervals are very narrow for extrapolation ratios up to ER=1, beyond that they widen considerably and for ER>500 the confidence intervals width reaches about 30% of the return level value. An exponential rise in the confidence interval width as function of ER is observed and for ER values larger than 1000-5000 the confidence interval width will reach 50-100% of the return level. At this point the prediction will become not very useful. It becomes likely that in some realizations near through wall defects may be observed (i.e. the return level is 50-100% smaller than estimated).
8 CONCLUSIONS

EVA is a statistical tool that can be used to estimate the minimum thickness of a component based on measurement data. The return level is one of the key outputs of an extreme value analysis. It estimates the minimum thickness that is expected in an area that is larger than the measurement area from which samples were obtained. This paper investigated the uncertainty (95% confidence intervals) that are associate with the predicted return level as a function of the original sample size (number of minima) and the extrapolation ratio (the ratio of the area that is investigated compared to the area for which data is available). It was found that prediction up to ER~1000 result in confidence interval widths that are less than about 30% of the return level. For larger extrapolation ratios it is possible to have uncertainties in the return level that are as large as the return level itself. These predictions are not very useful for wall thickness assessment as they can represent through wall defects. Since EVA does not assume an underlying thickness distribution of the sampled surface, the curve that is provided in figure 3 can serve as a quick way to estimate the uncertainty that can be expected if EVA is carried out on inspection/monitored data (for components of roughly 10mm wall thickness).

REFERENCES


