IDENTIFICATION OF IMPACT EVENTS ON COMPOSITE PLATES USING WAVE CHARACTERISTICS AND IMPACT SIMILARITY LAWS

Theodosis C THEODOSIOU, Christoforos S REKATSINAS, Christos V NASTOS, Dimitris A SARAVANOS*
Dept. of Mechanical Engineering & Aeronautics, University of Patras, 26500 Rio-Patras, Greece

Manuel IGLESIAS VALLEJO, Jaime GARCIA ALONSO
Airbus Defence and Space, Getafe, Spain

Key words: Composite Plates, Low Energy Impact, Impact Characterization.

Abstract
This paper presents the development of a methodology for characterization of low energy impact events on composite plates. Event characterization accounts for identification of impact location, estimation of impactor velocity and estimation of impactor mass. Calculations exploit elastic wave characteristics and derive simple phenomenological formulations based on signal analysis and impact similarity laws. Results demonstrate excellent localization predictions and very reasonable impactor characterization.

1 INTRODUCTION

The purpose of this paper is to present a framework for characterization of low energy impact events on unstiffened composite plates. This includes three key tasks: (1) identification of impact location, (2) estimation of impactor velocity and (3) estimation of impactor mass. Numerous approaches may be found in literature either targeting individual aspects of impact events or as an integrated approach. Tobias et al. [1] employed an array of three sensors to capture the elastic waves created by impact, and the difference in the arrival time at each sensor was used as input to a geometric triangulation algorithm. Although this approach is rather simple and intuitive, it fails when more complex structures are concerned. The internal complexity of composites causes wave propagation through different phases and isotropic propagation is compromised. Jeong et al. [2] have performed wavelet-based time-frequency analysis on captured sensor signals; their work has been further extended by Kundu et al. [3], [4] who have exploited acoustic emission for impact localization on composite plates and performed nonlinear triangulation. Impact force reconstruction methods have also been proposed; these can be exploited not only for event localization, but also for further characterization of impact. Recent works include uncertainty models, as well, in force reconstruction methods [5]; these are based on frequency response matrices inversion and involve Monte Carlo Markov chain methods, which can be quite demanding. Time Reversal methods for impact investigations on plates have also been introduced [6]; these require efficient calibration of transfer functions. The number and variety of different approaches reveals the great interest on impact investigation and establishes a robust background for

* Corresponding Author, Professor. email: saravanos@mech.upatras.gr, tel: +30-2610-969437, fax: +30-2610-969714
further developments. Such approaches have proven to be very successful, but their underlying models induce a high computational cost. Furthermore, the number of experiments and/or simulations for confirmation of a model, is frequently prohibitive. To counteract these shortcomings, dimensional analysis and similarity laws are explored in this paper. Demonstration of reduced impact analysis within the context of similarity law has been successfully demonstrated [8].

In this paper we try to link this reduced impact model towards an effective impact detection and characterization framework for composites. The proposed methodology attempts to correlate the characteristics of an impact event with measurable quantities of the impact-generated guided elastic waves. The generated guided waves first propagate through the structure, subsequently reflect at the boundaries and ultimately create stationary waves that cause the structure to vibrate. This consideration roughly divides the response of the structure into a wave-dominated and a vibration dominated phase in the time domain (Figure 1). The envisioned methodology exploits the wave dominated response, in order to address three aspects of impact event characterization, namely: (1) localization of impact event, (2) estimation of impactor mass, and (3) estimation of impactor velocity.

![Figure 1. Sample response depicting the wave- and the vibration-dominated regions. Current methodology is focused on wave-dominated area.](image)

2 METHODOLOGY

The methodology is divided into three parts. First, the basic theory and formulation employed in simulations is presented. Proper references to numerical tools are provided, but some basic features are included for completeness. The second part includes impact localization on composite plates, i.e. identification of impact event location on a flat panel by using the wave-dominated response of the structure. Finally, estimation of impactor characteristics, namely mass and velocity, is pursued. Critical wave parameters uniquely describing each impact event are identified and dimensional analysis is conducted in order to extract formulation that correlate impactor characteristics with the wave response of the structure. Due to paper size limitation, various simplifications are necessary in order to provide a clear understanding of the envisioned methodology; these will be explicitly stated where needed.

2.1 Modeling of Impact and Wave Propagation in Laminated Structures

Impact of a spherical rigid impactor on a rectangular composite plate is considered in this work. Elastic response is assumed for the structure, i.e. no damage, thus, only low-energy impact is considered. When the impactor hits the composite plate, it induces a local indentation at the impacted area and eventually a global structural deformation. The exact response depends on the impactor mass and velocity, as well as, on the elastic properties of the both the
impactor and the plate. Christoforou et al. [8], [9] have derived a simplified, yet efficient, model to characterize the impact event. The physical system is replaced by an equivalent system (Figure 2) which includes the impactor dynamics and a local-indentation model expressed by the impactor mass \(m_I\) and velocity \(v_I\) and the contact stiffness \(k_y\), and a structural dynamics model represented by the fundamental modal mass \(m_s\) and the modal stiffness \(k_{st}\).

\[
\begin{align*}
\mu &= \frac{m_s}{m_I}, \\
\lambda &= \frac{k_{st}}{k_y}, \\
\zeta &= \sqrt{\frac{m_I \cdot k_y}{2 \cdot Z_s}}.
\end{align*}
\]

where, \(\mu\) is the ratio of modal mass over the impactor mass; \(\lambda\) is the relative stiffness; and \(\zeta\) is the loss factor; \(\rho\) is the plate density and \(h\) is the plate thickness. The structural impedance \(Z_s\) is defined in terms of material properties

\[
Z_s = 8\sqrt{\rho h D}, \quad D = \frac{1}{2} \left( D_{12} + 2D_{66} + \sqrt{D_{11}D_{22}} \right)
\]

where, \(D_{ij}\) are the plate stiffness parameters defined as usual. The contact stiffness \(k_y\) can be experimentally or analytically evaluated, and is assumed as a property of the composite material.

The axial and transverse displacement fields of the fundamental straight crested Lamb wave modes through the thickness of a plate are well-described as sinusoidal sums [12], which can be efficiently approximated using a third-order Taylor series expansion [13]. This has been used to recast an already confirmed layerwise model [14], and fine-tune the continuity and slopes through the thickness of laminated structures. This has led to the development of an innovative Time-Domain Spectral Finite Element (TDSFE) – fully described in [15]; the advantage of this approach is that collocation of nodes with Gauss-Lobatto-Legendre integration points yields diagonal or nearly diagonal element mass matrix, which boosts the speed of explicit time integration schemes and enhances the quality of approximation within the domain of the element. We have used the explicit TDFSE composite plate model with a contact law, to provide time simulations of the impact event and the propagating guided waves.
2.2 Localization of Impact Events

The methodology for impact event localization relies on determining characteristics of elastic waves propagation induced by the impact event. The localization process is outlined in the following steps.

**Step 1: Model development.**

A TDSFE assembly is developed to model the structure under investigation. Dimensions, materials and lamination need to be consistent with the physical structure. Ideally, the response of the structure should be monitored everywhere. However, due to the large amount of data produced, this is computationally prohibitive and a limited number of grid points are selected. For the needs of current investigations, a rectangular plate of dimensions 0.4x0.44x0.0021m$^3$ is considered. The plate is made of Carbon/Epoxy material with properties listed in Table 1; lamination is [0$^d$/90$^d$]$_S$. An equivalent TDSFE model has been developed using 120 elements.

<table>
<thead>
<tr>
<th>Property</th>
<th>$E_{11}$</th>
<th>$E_{22}$</th>
<th>$E_{33}$</th>
<th>$G_{12}$</th>
<th>$G_{13}$</th>
<th>$G_{23}$</th>
<th>$v_{12}$</th>
<th>$v_{13}$</th>
<th>$v_{23}$</th>
<th>$\rho$</th>
<th>$k_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>GPa</td>
<td>GPa</td>
<td>GPa</td>
<td>GPa</td>
<td>GPa</td>
<td>GPa</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>km.m$^{-3}$</td>
<td>MPa</td>
</tr>
<tr>
<td>Value</td>
<td>121.47</td>
<td>10.45</td>
<td>10.45</td>
<td>5.11</td>
<td>5.11</td>
<td>3.63</td>
<td>0.32</td>
<td>0.32</td>
<td>0.44</td>
<td>1578</td>
<td>12.77</td>
</tr>
</tbody>
</table>

A grid of 13x13=169 points, coinciding with TDSFE nodes is exploited for capturing the response of the structure (Figure 3).

![Figure 3. Conceptual view of the Grid. Dots designate Grid points, where the response of the structure is captured.](image)

**Step 2: Identification of impact event location.**

A geometrical optimization algorithm is employed. To introduce the localization method, a generalized plate is considered with $N$ sensors attached (Figure 5). The generalized i-th sensor is located at a fixed point $S_i(x_i, y_i)$. The plate is impacted at an arbitrary point $P(x_0, y_0)$ at an unknown time $t_0$. The impact event induces propagation of elastic waves along all directions which are captured by the sensors. The goal is to detect the impact location by analysis of the captured sensor signals. The Euclidean distance of the impact site from the i-th sensor is $s_i$:

$$s_i = \|P - S_i\| = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}, \quad i \in \{1, 2, \ldots, N\}$$

The elastic waves travel through the structure with a group velocity $c_g$ and trigger the sensor at time $t_i$ which is obtained from the data acquisition system. The Time of Flight (TOF) for each sensor is provided by

$$TOF_i = t_i - t_0$$

t_i can be determined by thresholding the signal; using 0.5% of maximum signal amplitude is a
typical choice. For the general case of an anisotropic plate, group velocity also depends on the propagation direction $\theta_i$ (Figure 4):

$$\theta_i = \tan^{-1}\left(\frac{y_i - y_0}{x_i - x_0}\right)$$

(5)

The fundamental modes travel faster along directions parallel to the fibers.

The group velocity, the time of flight and the distance from the impact site are by definition related by

$$s_i = c_g(\theta_i) \cdot T_i \Leftrightarrow \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} = c_g(\theta_i) \cdot (t_i - t_0), \quad i \in \{1, 2, \ldots, N\}$$

(6)

Eq. (6) actually represents a system of nonlinear equations with unknowns the impact location coordinates $(x_0, y_0)$, the impact time $t_0$ and the group velocity $c_g$. Due to its nonlinear nature, this system can be solved only numerically.

Up to this point, no assumptions have been made regarding the number and location of sensors. As intuitively expected, the more sensors, the more accurate the predictions. When numerical models are considered, this is generally acceptable. But when it comes to practical applications, only a limited number of sensors can be monitored simultaneously. To assess the efficiency of sensor number and arrangement, numerous configurations have been assessed using a narrowband force excitation and employing the introduced localization algorithm. Force excitation was preferred because it provides ideal conditions for signal detection and
analysis, thus, investigation can be focused on the assessment of sensor configurations; in following parts of the paper, the algorithm will be extended to actual impact events. Various configurations have been examined employing up to 5 sensors; some example configurations along with their respective predictions for the same event are shown in Figure 6.

![Figure 6](image)

Figure 6. Prediction of excitation location using sample configurations of (a) three sensors, (b) four sensors and (c) five sensors. Prediction (circle) and actual event sites (x) are designated for validation purposes.

The Euclidean distance of prediction from the actual site has been used to assess the quality of prediction. The prediction quality is quantified by averaging predictions for excitation localization in all 169 grid points shown in Figure 3. Figure 7 demonstrates that the use of five sensors limits prediction error.

![Figure 7](image)

Figure 7. Prediction error using various numbers of sensors. Values have been obtained by averaging prediction errors from multiple impact simulations.

### 2.3 Estimation of Impactor Characteristics

The characteristics of the elastic waves generated by an impact event depend on quite many factors, including the mass, velocity and geometry of the impactor, as well as the geometry, material properties and lamination of the impacted structure. In order to extract a phenomenological model to correlate the impactor characteristics with the characteristics of guided waves, a thorough investigation of all key quantities is need, which requires a tremendous number of simulations and experiments. Such an approach is computationally and financially prohibitive and lacks transferability to other systems. In order to avoid these shortcomings, dimensional analysis through Buckingham’s π-theorem [7] is employed. To this cause, the work of Christoforou et al. is exploited for characterization of impact [8], [10]. These models consider impact of a spherical impactor on rectangular composite plates and derive a series of dimensionless parameters for characterizing impact – shown in Eq. (1). The envisioned approach is outlined in the following steps:

**Step 1: Identification of dependent and independent variables.**

The first step is to identify all key-parameters that distinguish one impact event from
another. Preliminary results have shown that impactor mass and velocity affect the transient response of a structure; there is evidence that impactor characteristics can be correlated to (a) the amplitude $A$ of the elastic wave and (b) the frequency $f$ generated by the impact event. These quantities can be regarded as two dependent variables, thus, a tuple of equations is pursued to correlate them to a set of independent variables. A reasonable choice for independent variables, compatible with the work of Christoforou et al., includes impactor mass ($m_i$), the modal mass ($m_s$), impactor velocity ($v_i$), contact stiffness ($k_y$), modal stiffness ($k_{st}$) and structural impedance ($Z_s$). Additional parameters may be accounted for, but at the cost of computational efficiency and complexity.

**Step 2: Casting all variables in non-dimensional form**

All identified variables need to be analyzed into their fundamental dimensions, namely Mass (M), Length (L) and Time (T). The dimensional analysis of incorporated variables is shown in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimensions</th>
<th>Variable</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impactor mass</td>
<td>$[m_i]$</td>
<td>Modal Stiffness</td>
<td>$[k_{st}]$</td>
</tr>
<tr>
<td>Modal mass</td>
<td>$[m_s]$</td>
<td>Structural Impedance</td>
<td>$[Z_s]$</td>
</tr>
<tr>
<td>Impactor velocity</td>
<td>$[v_i]$</td>
<td>Maximum amplitude</td>
<td>$[A]$</td>
</tr>
<tr>
<td>Contact Stiffness</td>
<td>$[k_y]$</td>
<td>Excited frequency</td>
<td>$[F]$</td>
</tr>
</tbody>
</table>

The three considered fundamental dimensions are expressed in terms of the identified variables. An admissible choice is the following

$$M = m_i, \quad L = v_i \cdot \sqrt{\frac{m_i}{k_y}}, \quad T = \sqrt{\frac{m_i}{k_y}} \quad (7)$$

The expressions derived in Eq. (7) can be exploited to produce proper multiplication factors for casting variables of Table 2 into dimensionless form – denoted by overbar:

$$\bar{m}_s = \frac{m_s}{m_i} \equiv \mu, \quad \bar{k}_{st} = \frac{k_{st}}{k_y} \equiv \lambda, \quad \bar{Z}_s = \frac{Z_s}{\sqrt{m_i k_y}}$$

$$\bar{A} = \frac{A}{v_i} \cdot \sqrt{\frac{k_y}{m_i}}, \quad \bar{f} = f \cdot \sqrt{\frac{k_y}{m_i}} \quad (8)$$

Exploiting Eq. (8) within Buckingham’s $\pi$-theorem, leads to existence of a tuple of equations connecting the dimensionless variables:

$$\bar{f} = \phi_1(\mu, \lambda, \zeta)$$

$$\bar{A} = \phi_2(\mu, \lambda, \zeta) \quad (9)$$

Functions $\phi$ need to be determined by fitting simulation or experimental results; $\mu, \lambda, \zeta$ are the dimensionless quantities introduced in Eq. (1). As shown in following, calculation of only one of those is adequate for preliminary impactor characterization.

**Step 4: Estimation of $\phi$ functions**

A series of impact simulations have been performed using the TDSFE impact model on as described in Step1, in order to extract the form of $\phi$ functions involved in Eq.(9). Results are
Figure 8. Polynomial fitting for extracting the relation between dimensionless Frequency and Amplitude vs. $\mu$ parameter.

For the sake of clarity and simplicity, the following simplifying assumptions have been made: (1) Only impact at the center of the plate is considered; future integration with the localization algorithm will eliminate this restriction; (2) Calculations include only the signal of one sensor (sensor at the bottom of the configuration employed for localization in Figure 6) – using more sensors would increase confidence, but also processing time. Signal from any sensor should yield similar results; (3) In order to reduce the dimensionality of the derived expressions, only the most significant parameters have been preserved – this enables derivation of simple univariate functions, but at the expense of accuracy. In fact it is evident that there are some overlapping points in Figure 8a, thus frequency (and related) calculations are not expected to be precise. At current development stage, this is considered as an acceptable assumption. Following these assumptions, polynomial fitting of the derived values yields the following equations:

$$f = \varphi_1(\mu) = (0.0001\mu^3 - 0.0087\mu^2 + 0.4691\mu + 2.0474) \cdot 10^8$$  \hspace{1cm} (10) \\
$$A = \varphi_2(\mu) = -3.2 \cdot 10^{-5} \mu^2 + 0.0047\mu + 0.0519$$  \hspace{1cm} (11)

After capturing and post processing the signal, the excited frequency can be calculated. This should correspond to a dimensionless frequency value that satisfies both Eq. (8) and (10). The residual between calculated and measured dimensionless frequency is defined as

$$\Re = \varphi_1(\mu) - \bar{f}_{\text{measured}} = 0$$  \hspace{1cm} (12)

Setting this to zero yields the optimal value of $\mu$, and therefore $m_i$. Alternatively, Eq. (11) could have been employed, but since the dimensionless amplitude involves both impacter mass and velocity, numerical solution becomes complicated. Then, the estimation for $m_i$ is provided to the formulation of dimensionless amplitude in Eq. (8), which can now be solved for $v_i$.

3 VALIDATIONS

A series of validation cases have been performed using the structure described in Table 1. The plate was impacted at arbitrary locations using a rigid impacter with mass and velocity within the range investigated. Regarding localization, each signal is post-processed as previously described for the case of induced excitation, and the geometrical optimization algorithm is employed. Localization results are shown in Table 3 and visualized in Figure 9.
Table 3. Validation Results using the introduced localization methodology.

<table>
<thead>
<tr>
<th>Case</th>
<th>Impact Site</th>
<th>Case</th>
<th>Impact Site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Predicted</td>
<td>Actual</td>
</tr>
<tr>
<td>1</td>
<td>(0.13,0.25)</td>
<td>4</td>
<td>(0.10,0.09)</td>
</tr>
<tr>
<td></td>
<td>(0.14,0.29)</td>
<td></td>
<td>(0.06,0.11)</td>
</tr>
<tr>
<td>2</td>
<td>(0.32,0.40)</td>
<td>5</td>
<td>(0.25,0.05)</td>
</tr>
<tr>
<td></td>
<td>(0.28,0.33)</td>
<td></td>
<td>(0.24,0.04)</td>
</tr>
<tr>
<td>3</td>
<td>(0.37,0.24)</td>
<td>6</td>
<td>(0.23,0.18)</td>
</tr>
<tr>
<td></td>
<td>(0.32,0.20)</td>
<td></td>
<td>(0.24,0.17)</td>
</tr>
</tbody>
</table>

When impact takes place near the border edges of the plate, reflections partially compromise calculations, however, predictions are still valid and reasonable.

![Figure 9. Demonstration of localization for arbitrary low-energy impact events.](image)

Similarly, six impact cases at the center of the plate have been analyzed using the proposed methodology. Results are summarized in Table 4. Due to the assumptions made for univariate formulation, predictions are not as good as in the case of localization, but still they are quite reasonable; it has to be noted that no strict assumptions (e.g. mass value is not bounded) have been applied to solution of Eq. (12).

Table 4. Validation Results using the introduced impactor characterization methodology.

<table>
<thead>
<tr>
<th>Case</th>
<th>Impactor Mass (g)</th>
<th>Impactor Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Predicted</td>
</tr>
<tr>
<td>1</td>
<td>2.10</td>
<td>3.44</td>
</tr>
<tr>
<td>2</td>
<td>4.60</td>
<td>1.80</td>
</tr>
<tr>
<td>3</td>
<td>1.71</td>
<td>2.50</td>
</tr>
<tr>
<td>4</td>
<td>3.50</td>
<td>4.20</td>
</tr>
<tr>
<td>5</td>
<td>4.20</td>
<td>3.10</td>
</tr>
<tr>
<td>6</td>
<td>3.40</td>
<td>2.50</td>
</tr>
</tbody>
</table>

4 SUMMARY

A methodology has been presented for characterization of low-energy impact events on rectangular composite plates. The presented approach aims to provide reasonable estimates for
the impact location, as well as the mass and velocity of the impactor, using a set of five sensors. First, impact localization takes place by analyzing the time of flight of captured signals in a geometrical optimization algorithm. Then, impactor mass and velocity are correlated to wave characteristics. A series of simulations have been conducted to extract such relations in dimensionless form; this enables adaptation of methodology to dimensionally similar structures and significantly reduces the number of simulations that need to be performed. Results demonstrate excellent localization performance and reasonably well impactor characterization. Future work will involve integration of the localization procedure, in order to perform impactor characterization are arbitrary locations.

ACKNOWLEDGEMENT

Parts of this work were financially supported by EU JTI-CleanSky Project “Flexible Sensor Cooperation for Structural Health Diagnosis/Prognosis: Wireless-FlexSense. Grant Agreement/Project No: 632506.

REFERENCES

[14] T. S. Plagianakos and D. A. Saravanos, “Higher-order layerwise laminate theory for the