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Evaluation of the Probability of Failure using Bayesian Theorem for Real-time Condition Monitoring

Atsushi IWASAKI[1]
[1] Dpt Mechanical System Engineering, Gunma University
1-5-1 Tenjin-cho, Kiryu, Gunma 376-8515, JAPAN
aiwasaki@gunma-u.ac.jp

Abstract
This paper proposes a method for the numerical evaluation of probability of failure (PoF) using the Bayesian theorem and employing diagnostic results of real-time condition monitoring. When performing maintenance based upon the results of real-time monitoring, it is ideal to obtain diagnostic results without inspection error. However, failure does not occur if the monitoring method overestimates sufficiently small damage; indeed, failure does not occur even if there is a slight underestimation of a large damage. To reduce the PoF, it is important to accurately estimate the specific damage. This study proposes a method for diagnosing the specific damage level with improved accuracy; this improved accuracy is achieved by using a weight function to control the sampling ratio of the training data for learning. The consequences of overestimation and underestimation of damage differ. The risk caused by the underestimation is called failure risk, and that caused by the overestimation is called economic risk. This paper discusses the shape of the weight function used to reduce the economic risk. The proposed method is validated by applying it to the delamination identification problem of a CFRP beam using the electric potential method.

Keywords: Statistical Analysis, Probability of Detection, Risk Analysis, Probability of Failure, Bayesian Theorem

1. INTRODUCTION

An inverse problem for damage or fault identification is an optimization problem for minimizing estimation error. To identify the relationship between sensor measurement and damage properties, several statistical analysis methods, such as multiple regression, response surface, generalized regression model, discriminant analysis, and spatial statistics[1-5], or optimization methods, such as neural network, and genetic algorithm, are used[6-9].
The magnitude of the estimated error is reduced when the method is advanced. However, to evaluate the reliability of a structure based upon diagnostic results, it is important to accurately evaluate the probability of underestimation of severe damage, which causes structural failure. Hence, it is necessary to evaluate the probability distribution of the occurrence of true damage size at arbitrary estimated result. Therefore, this paper proposes a method for estimating occurrence probability of true damage vs arbitrary estimated results, using occurrence probability estimation of the cause events via the Bayesian theorem. Using this method, the residual strength distribution from the acquired distribution and the probability of failure (PoF) are estimated. The proposed method is validated by applying it to the delamination identification problem of a CFRP beam using the electric potential method.

2. ESTIMATION OF OCCURRENCE PROBABILITY OF EACH DAMAGE SIZE USING ESTIMATION OF OCCURRENCE PROBABILITY OF THE CAUSE EVENT VIA BAYESIAN THEOREM

In general, for the damage identification problem, the maximum or average estimation error is mainly discussed for the evaluation of each method. Hence, the distribution of the estimated value vs arbitrary true value is mainly discussed. However, when evaluating the reliability of the structure from the diagnostic result, it is important to consider the distribution of the true value vs arbitrary estimated value of the damage. The distribution of the true value vs arbitrary estimated value approximates that of the estimated value vs arbitrary true value when (1) each damage size correlates to a uniform probability distribution of occurrence and (2) the distributions of the regression error for each damage size are equivalent. However for the damage identification problem, since a large damage is caused by the accumulation or growth of a small damage, the occurrence probability of the large damage is smaller than that of the small damage, and thus, the above assumption fails. Moreover, the range wherein damage evaluation necessitates a greater accuracy is the range where the failure to detect the occurrence of damage is critical. In contrast, it is not important to accurately estimate small damage and the identification accuracy need not be uniform over the size range of damage. In summary, this paper describes the use of the Bayesian theorem to estimate the probability distribution of true damage size in comparison with that of the damage size estimated from the regression error.

2.1 Estimation of distribution of occurrence probability of true damage size vs arbitrary diagnostic result using Bayesian theorem

Bayesian theorem[10-13] is a simple mathematical theorem used for calculating conditional probabilities. In the field of engineering, it is mainly used for the structural reliability assessment from a small sampling[14-18] (Ex. risk based maintenance[19, 20]).
In this study, the theorem is used to estimate the occurrence probability of true damage according to an estimated damage size. The Bayesian theorem is generally expressed as

$$P(E_i \mid F) = \frac{P(E_i)P(F \mid E_i)}{\sum_j P(E_j)P(F \mid E_j)} \quad (\text{i, j} = 1, 2, \cdots) \qquad (1)$$

where $P(E_i)$ is the probability of occurrence of event E_i; $P(F \mid E_i)$ is the conditional probability that event F causes under event E_i; and $P(E_i \mid F)$ is the "posterior probability," which is the conditional probability that event E_i causes under event F. For identifying damage sizes, damage occurrence is the event that generates the arbitrarily estimated results and $P(a_i)$ is the occurrence probability of the true damage size a_i. $P(EstA_k \mid a_i)$ is the probability that the occurrence of damage a_i causes the estimated results $EstA_k$. In this case, Eq. (1) can be modified as follows.

$$P(a_i \mid EstA_k) = \frac{P(a_i)P(EstA_k \mid a_i)}{\sum_j P(a_j)P(EstA_k \mid a_j)} \qquad (2)$$

where the left side is the posterior probability. Hence, $P(a_i \mid EstA_k)$ is the occurrence probability of $EstA_k$ because of the true damage size a_i. By estimating the occurrence probability for all a_i values, the occurrence probability distribution is determined for the true damage parameter and the estimated value.

2.2 Procedure for estimating PoF at arbitrary estimated result

PoF is estimated by the following formula using the limit state function method:

$$Pof = P[g(R-S) < 0] \quad (3)$$

where R is the strength, S is the applied force, and g is the limit state function. Figure 1 illustrates the procedure for estimating the probability distribution of failure and the estimated result. First, the occurrence probability distribution of the true damage size vs arbitrary estimated size is deduced by the procedure given in paragraph 2.1. Residual strength is a function of damage properties. In this paper, the buckling failure caused by delamination cracking is assumed and the distribution of residual buckling strength is calculated by the proposed method.

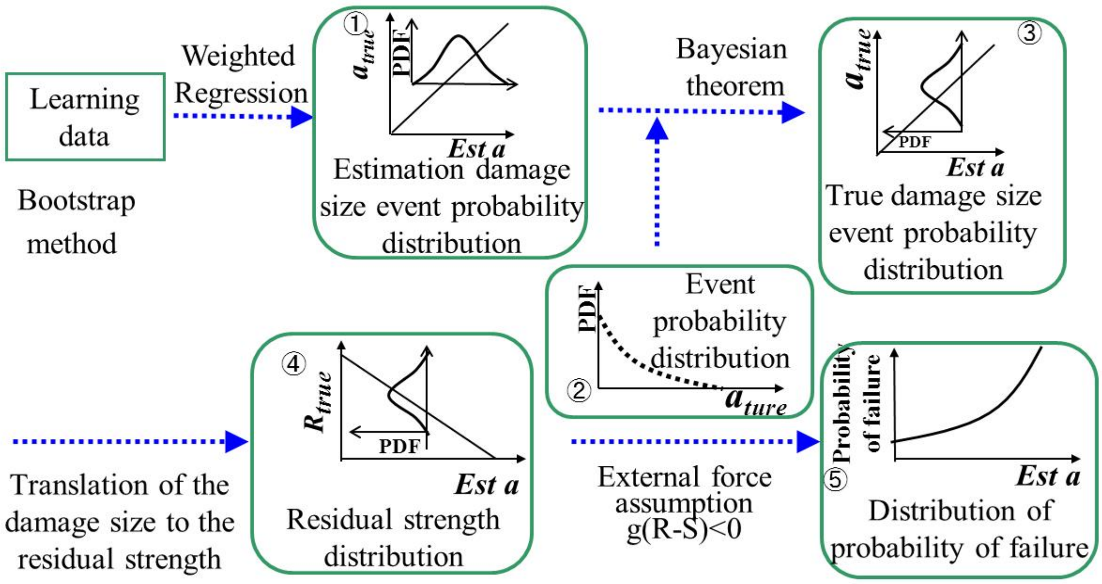


Figure 1. Procedure for estimating PoF

2.3 Evaluation of accidental and economic risks

Figure 2 shows the accidental and economic risks evaluated using the proposed method. The accidental risk is defined as the risk of failure caused by an underestimation of damage size, while the economic risk is defined as the risk of unnecessary maintenance costs caused by an overestimation of damage size. The the adjusted PoF (PDF of POF vs PDF of occurrence of estimated damage size: PoO) is plotted on the vertical axis and the estimated size is plotted on the horizontal axis. The accidental risk is evaluated by the area surrounded by the adjusted PoF and the threshold for maintenance operation. The threshold is set as the accidental risk and exhibits a constant value (0.03). The economic risk is evaluated by the area surrounded by the adjusted PoF, the PoO, and the threshold. This study aims to reduce the economic risk.

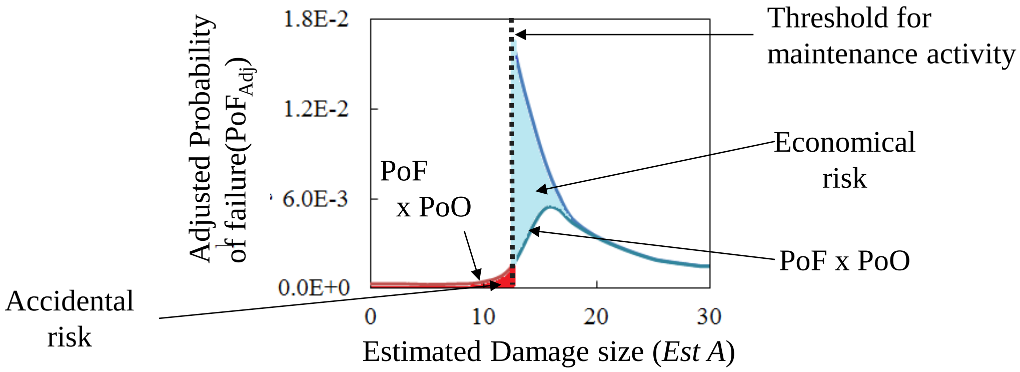


Figure 2. Accidental and economic risks from evaluated PoF

3. APPLICATION OF THE METHOD TO DAMAGE IDENTIFICATION USING ELECTRIC POTENTIAL CHANGE METHOD

3.1 Electric potential change method

As mentioned above, the proposed method for estimating PoF is applied to the identification of delamination in a CFRP beam using the electric potential method [1, 21-23] via regression analysis. FEM analysis is employed in this study, which is detailed in our previous studies [1,]. Specimen configuration is shown in Figure 3. The specimen is a CFRP beam with a thickness of 2 mm and a stacking sequence of $[0_2/90_2]s$. To measure the change in electric potential caused by a delamination crack, seven electrodes (each with 10 mm length) are mounted on one side of the specimen. FEM analysis is performed using the commercially available FEM tool (ANSYS). Four-node-rectangular elements (each with 0.125 mm × 0.125 mm size) are adopted for the analysis. A delamination crack is modeled by the release of a nodal point of the element. The electric conductance ratio is experimentally obtained from a CFRP laminate with a volume fraction of 62%, as follows:

$\sigma_{90}/\sigma_0=3.7*10^{-2}$
$\sigma_t/\sigma_0=3.8*10^{-2}$

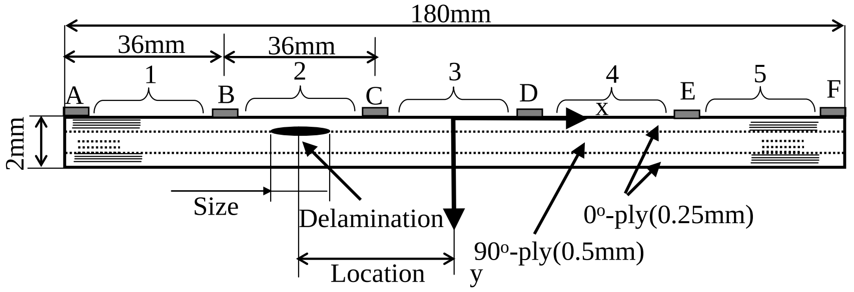


Figure 3. Model of Specimen

3.2 Identification of delamination crack

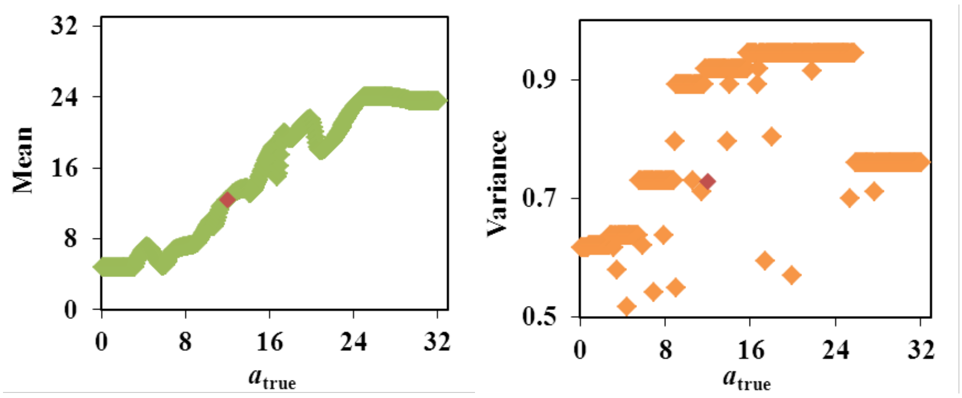
I The delamination crack is identified by analyzing the change in electric potential caused by the crack. The electric potential change of each region is defined as v_i (i=1 to 5). For the diagnosis, a vector length Z and standardized potential changes V_i are used as the parameters.

$$Z = \sqrt{\sum_{i=1}^{5} v_i^2}, \quad V_i = {}^{v_i}\!/_{Z} \; (i = 1 \cdots 5) \tag{4}$$

The delamination size is identified using the following linear polynomial:

$$y = \beta_0 + \sum_1^5 \beta_i V_i + \beta_6 Z \tag{5}$$

where y is the predictor variable (in this case, the damage size) and β_i represents the regression coefficients. The number of datasets for the regression is 74. The mean and distribution of the identification error are plotted against the true value in Figures 4 (a) and 4 (b), respectively. The horizontal axes in both figures show the true damage size and vertical axis of (a) shows the mean and that of (b) shows the variance. In these examples, the regression accuracies differ largely at each damage size.



(a) Mean (b) Variance

Figure 4. Mean and variance of the estimation

3.3 Estimation of the distribution of true damage size vs arbitrary estimated result using Bayesian theorem

By assuming the prior occurrence distribution of the true damage size, we estimate the true damage size vs arbitrary estimated result by the Bayesian theorem. The exponential distribution shown in Figure 5 is used as prior distribution; an example of the result is shown in Figure 6. The abscissa corresponds to the true value of the delamination length, and the y-axis shows the PDF when estimated as 12 mm or 15 mm using the damage diagnostic method. As shown in the figure, occurrence probability of the damage size around the estimated size is the highest; it is asymmetrical because of the prior distribution of the occurrence probability.

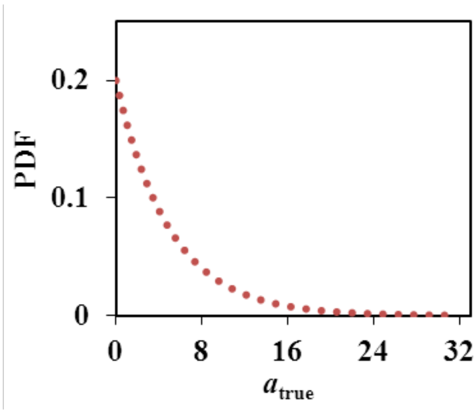


Figure 5. Prior distribution

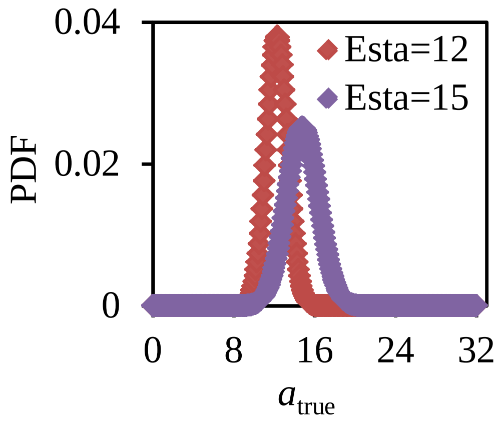


Figure 6. Posterior distribution

3.4 Estimation of distribution of buckling strength

A delamination crack deteriorates compressive strength. In this study, we consider the buckling failure of the surface layer caused by delamination. Buckling strength is calculated as follows:

$$p_w = \frac{4\pi^2 EI}{a^2} \tag{6}$$

where p_w is the buckling strength, E is the stiffness, and I is the moment of inertia of areas. Figure 7 illustrates the estimated results of strength distribution for delamination sizes of 12 mm and 15 mm. The abscissa of the figure represents the buckling strength. As shown in the figure, the strength decreases with increasing delamination size.

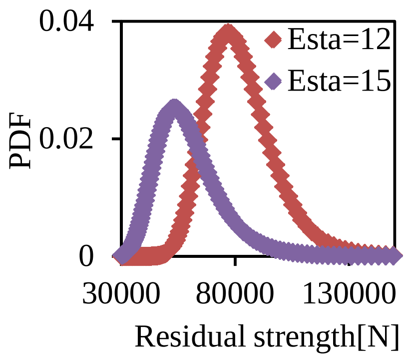


Figure 7. Estimated distribution of residual strength

3.5 Estimation of POF

It is assumed that an external force would cause buckling failure when 15 mm delamination exists. Figure 9 illustrates the results. The abscissa represents the estimated size, and the vertical axis represents PoF. Failure occurs at 15 mm or less because the external force is not constant. As shown in the figure, the PoF starts the lifting by a damage smaller than 15 mm (Area A in the figure) and is saturated with about 17 mm (Area B in the figure). From the results, it is possible to evaluate the PoF at arbitrary estimated result by using the proposed method. Figure 10 shows the plot of true damage length against estimated damage length using a diagnostic method. Area A results from the underestimation and causes accidental risk. Area B results from the overestimation and causes economic risk (i.e., unnecessary maintenances). To reduce total risk, both accidental and economic risks must be reduced; however, the effects of the two risks are not the same. Thus, reducing the probability of Area A is more important.

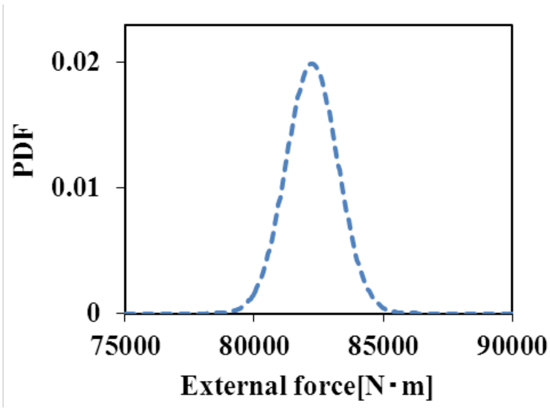


Figure 8. Assumed PDF or external force size

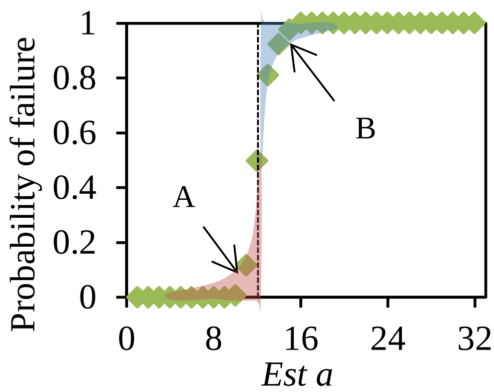


Figure 9. Estimated POF vs each estimated

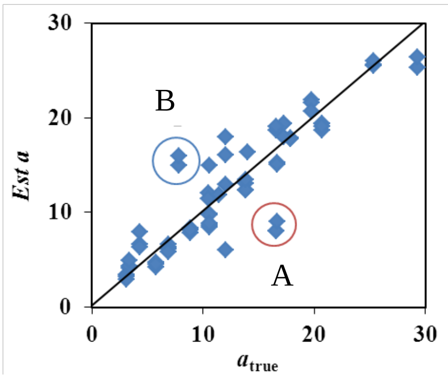


Figure 10. True damage length vs estimated damage length by the diagnostic method

4. CONCLUSION

To accurately determine the reliability of a structure based on diagnostic results, it is important to determine the probability of underestimation of the severe damage, in which causes structural failure. This paper proposes a method for determining the probability distribution of occurrence of true damage properties at arbitrary estimated results. The proposed method uses the Bayesian theorem to determine the occurrence probability of the cause event. The residual strength of the damaged structure is estimated from the distribution; PoF is determined using the limit state function method. As the result, PoF starts the lifting by a damage smaller than the critical level, and is saturated with over the critical level. In conclusion, this study confirms that the proposed method evaluates the PoF at arbitrary estimated result.

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