Active Structural Health Monitoring of Reinforced Concrete Structures using Piezoelectric Smart Aggregates

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Abstract
Research and development of active monitoring systems for reinforced concrete structures should lead to improved structural safety and reliability. Implementation of active structural health monitoring systems with capability of damage detection and structural diagnosis represent one of the main challenges in this field of research. In this paper we propose a numerical modeling of damage detection process in a concrete beam with piezoelectric smart aggregates, which may be used both as actuators and sensors. The modeling procedure involves modeling of piezoelectric smart aggregates – developed using implicit finite element method and modeling of the wave propagation – developed using explicit finite element method. Based on the numerically generated actuation waves and sensors signals, a one-dimensional damage index based on energy variations of the output sensor signals is formed using the wavelet signal decomposition and the principle of root-mean-square deviation. The paper presents the original numerical models with parametric analysis of the damage index variation problem depending on the size, position and orientation of the cracks.

Keywords: embedded piezoelectric sensors, condition monitoring, damage detection, wave propagation

1. INTRODUCTION
Structural Health Monitoring (SHM) is an interdisciplinary engineering field that deals with innovative methods of monitoring structural safety, integrity and performance without affecting the structure itself or particularly impairing its operation [1]. Non-destructive methods (NDM) of the damage detection together with the active structural health monitoring (ASHM) methods determine the future in the condition monitoring of civil engineering buildings. This paper presents a non-destructive method for damage detection of reinforced concrete (RC) structures with piezoelectric smart aggregates (PZT SA) embedded into the structure itself. The application of the NDM based on wave propagation has been researched over a few decades, whereas the use of PZT SA for the purpose of ASHM of reinforced concrete structures has been still developing.

Numerical simulations based on the implementation of finite element (FE) methods can significantly contribute to the development of new NDM for the damage detection based on wave propagation. The computer power has been increased tenfold every five years, and with growing power, expectations for more accurate predictive analysis have also risen. Besides the advantage of the computer simulations widely used in industry and science to get first insight in the numerical analysis before the experimental investigation has been performed, another very important task should be tackled – to evaluate the reliability of the results.
obtained by virtue of simulations, before the first experimental investigation and the production of first prototypes have been done. Thus the use of the computer simulations can reduce the development expenses to a large extent and also contribute to development of the new methods of ASHM, but on the other hand the errors that may occur by incautious analyses of the results may lead to significant financial losses.

In the recent years, the use of piezoelectric elements for active monitoring of reinforced concrete structures has been expanding and gaining important place among the contemporary methods whose practical implementation is expected in the future. PZT SA are used e.g. for determination of early strength of concrete in situ, for determination of the impact force in the event of vehicle collision with the RC structure or for the active damage detection and monitoring of structural status [2,3]. Detection and localization of errors and damage occurring during gluing of carbon strips on the reinforced concrete beams is presented in the paper [4]. Also, the method for detection of irregularities in reinforcement bonding with concrete with the aid of inserted piezoelectric sensors in the RC structure was presented in [5]. Detection of damage caused by dynamic action in RC frame structures of bridge beams, RC walls and piles was experimentally analyzed and presented in [6-9].

In this paper the original analysis of one-dimensional damage index is presented, depending on the variation of wave propagation energy with the change of the damage position, size and orientation. Besides, the efficiency of the numerical models for damage detection by PZT SA is also presented.

2. MODELLING PZT SA ACTUATOR

Piezoelectric smart aggregate is studied based on the three-dimensional finite element model. Smart aggregate model consist of two parts: a small concrete block (30x30x10mm) and a piezoelectric patch (12.7x12.7x0.25mm) embedded within the concrete block (as shown in Fig. 1).

FE modeling of piezoelectric materials is based on constitutive equations for coupled electro-mechanical behavior. In this paper modeling is performed using the FE software ABAQUS. The constitutive equations of the coupled electro-mechanical behavior for a linear piezoelectric medium can be presented in the form:

\[
\sigma_{ij} = -D^{el}_{ijkl} \varepsilon_{kl} - e_{mj}^{el} E_m
\]

\[
q_i = e_{ij}^{po} \varepsilon_{jk} + D^{po(e)}_{ij} E_j
\]

with following notation: \(\sigma_{ij}, \varepsilon_{ij}\) represent the mechanical stress and strain tensors respectively, \(D^{el}_{ijkl}\) is the elastic stiffness matrix defined at zero electrical potential gradient, \(e_{mj}^{el}\) piezoelectric stress coefficient matrix; \(E_j\) electrical potential gradient vector, \(q_i\) the electric displacement and \(D^{po(e)}_{ij}\) material’s dielectric properties strain matrix.

Contact between the PZT patch and the surrounded concrete was defined by the surface based Tie Constraints. Boundary conditions on the concrete block are defined to external edges by constrained all degrees of freedom. Quasi-static analysis was performed by applying a constant electrical voltage (from 0V to 100V with 10V step) and deformation of the concrete block was monitored at a predetermined point (Fig. 1). Since there is a linear relationship between the electric charge and displacement of PZT SA, deformation of the concrete block from quasi-static analysis can be used as an input parameter for the modeling of wave propagation.
Starting point for modeling of the wave propagation using the Explicit Finite Element Method (EFEM) is the second Newton law written in matrix form [10]:

\[ F = M \cdot A \] (3)

with:

\[ F_i^m = -\int_{\Omega} \left( \sigma_{ij} \Phi_{M,j} \right) d\Omega + \int_{\Gamma_i} f_i \Phi_M d\Omega + \int_{\Gamma_t} g_i \Phi_M d\Omega \] (4)

\[ M_{MN} = \int_{\Omega} \left( \rho \Phi_M \Phi_N \right) d\Omega \] (5)

\[ A^N_i = \ddot{u}^N_i (t) \] (6)

Detailed explanation of the terms in equations (3)-(6) can be found in [10]. The system defined by equation (3) represents a system of differential equations of second order in time, which in general case can be linear or nonlinear. In order to solve this system, the explicit scheme based on the method of central differences for the approximation of acceleration, velocity and displacement has been used. Assuming that the time range \((0, T)\) is steadily divided into \(N\) even sub-domains \([t_n, t_{n+1}]\), where \(0 = t_0 < t_1 < ... < t_N = T\), \(t_{n+1} - t_n = \Delta t = T / N\), displacement, velocity and acceleration differentiated through time and approximated by the method of central differences, can be expressed in the vector form:

\[
\begin{align*}
\partial \mu_{n+1/2}^h &= \left( u_{n+1}^h - u_n^h \right) / \Delta t = \dot{u}_{n+1/2}^h \\
\partial \mu_n^h &= \left( \partial \mu_{n+1/2}^h - \partial \mu_{n-1/2}^h \right) / \Delta t = \left( u_{n+1}^h - 2u_n^h + u_{n-1}^h \right) / \Delta t^2 \\
\dot{u}_n^h &= \left( \dot{u}_{n+1/2}^h - \dot{u}_{n-1/2}^h \right) / \Delta t \\
\end{align*}
\] (7)

\[
\begin{align*}
\partial \mu_n^{h+1} &= \partial \mu_{n+1}^h + \partial \mu_n^{h} \Delta t \\
u_{n+1}^h &= u_n^h + \partial \mu_{n+1/2}^h \Delta t \\
\end{align*}
\] (8)

For the approximation of acceleration by the method of central differences, which is presented by equations (7) and (8), the equation (3) has been defined in the form:
\[ \partial_t u^h_n = M^{-1} F_n \]  

Equation (9) is stable if the time step \( \Delta t \) is less than or equal to the critical time step \( \Delta t_{\text{crit}} \), which for the non-damping systems has been given depending on the biggest frequency in the smallest finite element.

\[ \Delta t \leq \Delta t_{\text{crit}} = \frac{2}{\omega_{\text{max}}} \]  

For the modeling of the wave propagation, with the assumption that small deformations can occur, it has been adopted that the critical time step is the time of propagating the waves through the smallest finite element:

\[ \Delta t \leq \Delta t_{\text{crit}} = \frac{\Delta L}{c_L} \]  

with: \( \Delta L \) – the smallest characteristic dimension of the finite element, \( \Delta t \) – the time step and \( \Delta t_{\text{crit}} \) – the critical time step.

By the analysis of equation (11) it is easy to come to a conclusion that for the smaller finite elements the smaller time step is required. Regarding that in each time step the calculation of characteristic values has been done, with the appliance of smaller finite elements, the analysis becomes more and more demanding. Furthermore, since it is usual to use at least ten finite elements per wavelength, for modeling of the ultrasound wave propagation even relatively small models are very demanding from the computational point of view.

The diagonal mass matrix is one of the most important characteristics of the explicit finite element method that makes this method extremely efficient and practical. When the diagonal mass matrix is used, the step in which the acceleration is counted by the second Newton law, equation (9), it comes to a simple division without the need of calculating the inverse matrix. This fact lowers the computation time of models and makes EFEM very efficient for modeling the wave propagation.

4. ROOT MEAN SQUARE DEVIATION (RMSD) ONE-DIMENSIONAL DAMAGE INDEX

The output signal \( S \) of the sensor can be decomposed into \( 2^n \) signals denoted as \( \{X_1, X_2, \ldots, X_{2^n}\} \). Each of the signals \( X_j \) can be represented in the following way:

\[ X_j = [x_{j,1}, x_{j,2}, \ldots, x_{j,m}] \]  

where \( m \) – represents the number of measured data of the time signal, and \( n \) – represents the level of the wavelet signal decomposition (\( n=3 \) having been assumed in the paper). Energy of decomposed signals can be represented by the following expression:

\[ E_{i,j} = \|X_j\|^2 = x_{j,1}^2 + x_{j,2}^2 + \ldots + x_{j,m}^2 \]  

where \( i \) – is the time index and \( j \) – the frequency range, \( j = 1,2,\ldots,2^n \). By calculating energy of all decomposed signals, an energy vector can be calculated for an RC element in undamaged state \( E_h \) as well as for an element in the damaged state \( E_o \):
\[ E_h = \left[ E_{h,1}, E_{h,2}, \ldots, E_{h,n} \right], \quad E_o = \left[ E_{o,1}, E_{o,2}, \ldots, E_{o,n} \right]. \]  

Damage index (DI) based in the output signal energy variation is formed as a root-mean-square deviation:

\[
DI = \sqrt{\frac{\sum_{j=1}^{2n} (E_{o,j} - E_{h,j})^2}{\sum_{j=1}^{2n} E_{h,j}^2}}
\]  

Damage index can have values from 0 for undamaged structures to 1 for a totally damaged structure where the wave could not reach the sensor. Also, from the equation (15) it can be concluded that the higher the difference between the output signal energy of a damaged and undamaged RC structure, the higher the damage index. Based on this fact, it is possible to monitor the damage of a RC structure through time, monitoring the variation of the damage index.

5. NUMERICAL MODELS

An RC element having dimensions (0.6×0.2×0.2m) with the reinforcement 4Φ20 and two PZT SA represents the analyzed model (Fig. 2). Parametric analysis of the damage index variation was performed, depending on the position (L_p), size (H_p) and orientation (α_p) of the crack in the mentioned RC element. A total of 80 models were analyzed, and their characteristics and designations are presented in Table 1 and Table 2. The position of the cracks, for models with vertical crack, was varied from the value 0.15m to 0.45m with the increment of 0.05m. The lengths of the cracks 0.05, 0.08, 0.11 and 0.14m were analyzed as well as the crack angles from 60° to 120°, with 10° counterclockwise increments.

![Figure 2: Geometric characteristics of RC elements.](image)

Concrete and reinforcement were modeled as linear-elastic materials with following material characteristics: Young’s elasticity modulus of concrete is $30\times 10^9$ Pa and of steel $210\times 10^9$ Pa; Poisson’s ratio of concrete is 0.2 and of steel 0.3, while the density of concrete is 2400kg/m$^3$ and of steel 7800kg/m$^3$.

Displacements obtained from the model of PZT SA were used as an input parameter for modeling of the wave propagation, performed in ABAQUS/EXPLICIT software package. Function of the displacement variation used in the analysis is 3.5-cycle Hanning windowed tone burst signal with duration of $T_{sig}=3.5\times 10^{-3}$ s and central frequency $f_{cen}=100$ kHz.
Table 1: Markings and characteristics of models with vertical cracks.

The contact of the concrete and the reinforcement was defined with the aid of *Tie Constraint* surface contact available in ABAQUS/EXPLICIT analysis with the potential for rotational degrees of freedom. The crack was modeled as an opening in the model, having the thickness of a finite element with a length and orientation defined for each individual model separately. The total duration of the simulation was $T_{sim}=0.001s$, and the adopted time increment was $\Delta t=2e^{-7}s$, which satisfies the critical time increment. The applied finite elements were C3D8R eight-node prismatic finite elements with reduced integration and Hourglass control.

6. RESULTS AND DISCUSSIONS

Figures 3 and 4 represent the wave propagation through RC elements with vertical and inclined crack damage. The elements were cut through the medium vertical plane in the direction of RC element in order to better display propagation of the waves inside the RC elements. In Fig. 3b, 3c, 3d it can be seen that a part of the waves reflects from the cracks and returns to the actuator, weakening the propagating wave and reducing the energy of the output signal. The other part of the wave passes near the cracks, propagating through the RC element and reaching the PZT SA sensor. The models with crack lengths of 0.11 and 0.14 m are characterized by the delay of the direct incoming wave in the sensor, which is not the case with the models having cracks of 0.05 and 0.08 m. A similar interpretation of the results applies also to Fig. 4.
### Table 2: Markings and characteristics of models with inclined cracks.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>MODEL NAME</th>
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<tr>
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<td>$L_p$ [m]</td>
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<tr>
<td>$H_p$ [m]</td>
<td>0 0.05 0.08 0.11 0.14</td>
<td>$H_p$ [m]</td>
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<td>M 11-2</td>
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<tr>
<td>M 11-3</td>
<td>M 12-3</td>
</tr>
<tr>
<td>M 11-4</td>
<td>M 12-4</td>
</tr>
<tr>
<td>M 11-5</td>
<td>M 12-5</td>
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<tr>
<td>$L_p$ [m]</td>
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<tr>
<td>$H_p$ [m]</td>
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<tr>
<td>$\alpha_p$ [°]</td>
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<tr>
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<tr>
<td>$H_p$ [m]</td>
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<tr>
<td>$\alpha_p$ [°]</td>
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<th>MODEL 15</th>
<th>MODEL 16 – HEALTHY BEAM</th>
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<tr>
<td>M 15-5</td>
<td>M 16-5</td>
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<tr>
<td>$L_p$ [m]</td>
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</tr>
<tr>
<td>$H_p$ [m]</td>
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</tr>
<tr>
<td>$\alpha_p$ [°]</td>
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Figures 5 and 6 display the values of the damage index for analyzed models depending on the position, length and orientation of the crack. The values are presented on the geometry of numerical models in order to more easily monitor the change of the damage index for the models with vertical (Fig. 5) and with inclined cracks (Fig. 6).

Fig. 5 shows that the change of the damage index value is not drastically depending on the position of the crack and that the values most frequently differ up to several percents. Also, the shape of the damage index variation is convex for the models with crack length of 0.05 m while in case of the models having crack lengths of 0.11 and 0.14 m the shape is concave. The model with the crack length of 0.08 m has approximately same values of the damage index, with the mild convex trend. In case of the model with slant cracks (Fig. 6) the variation of the damage index depending on the gradient of the cracks does not exceed 5% except in case of the models M 9-2 and M 15-2. Also the models with vertical cracks do not exceed the mentioned percentage except the models M 7-2 and M 6-2. A small percentage of DI variation depending on the position and orientation of the crack leads to the conclusion that DI is in direct relation with the size of the crack, position of the PZT SA actuator-sensor as well as geometry of RC beam, and does not depend much on the position and orientation of the damage.
Figure 3: Wave propagation in Model 1 with vertical crack caused by PZT PA actuator at different time instants: 

- a) $t = 2.0172 \times e^{-5}$ s, 
- b) $t = 4.0172 \times e^{-5}$ s, 
- c) $t = 6.0172 \times e^{-5}$ s, 
- d) $t = 1.0017 \times e^{-4}$ s.

Figure 4: Wave propagation in Model 10 with inclined crack caused by PZT PA actuator at different time instants: 

- a) $t = 2.0172 \times e^{-5}$ s, 
- b) $t = 4.0172 \times e^{-5}$ s, 
- c) $t = 8.0172 \times e^{-5}$ s, 
- d) $t = 1.4011 \times e^{-4}$ s.
7. CONCLUSIONS

Modern methods of active structural health monitoring systems as well as the nondestructive damage detection certainly represent the future in the field of condition monitoring of civil engineering buildings, whereas it has been expected to develop and apply them in practical work to a large extent in the future. Numerical methods and computer simulations have an important role in the development of these methods. This paper represents the original numerical process of modeling the damage detection by means of PZT SA and wave propagation using the implicit and explicit finite element method. During the process of modeling of PZT SA actuator, the electro-mechanic characteristics of PZT materials have been taken into consideration, as well as the interaction of PZT patch and the surrounding concrete. The EFEM as the direct integration method which uses the method of the central differences and diagonal mass matrix has been very efficiently used for modeling wave propagation. The extensive parameter analysis of the change of the damage index with the change of the position, length and orientation of cracks has been carried out. On the basis of
the obtained results from parametric analysis, it can be concluded that the change of the damage index in relation to the position and orientation of the crack does not exceed the value of several percents in most of the models. Furthermore, for the vertical cracks certain regularity could be observed regarding the influence of the crack length on the DI. Thus, for the vertical cracks with length smaller than one half of the beam’s cross section height it has been noticed that the DI change goes from the vaguely convex to completely horizontal, while for the crack length greater than the mentioned value, the change of damage index is concave, as shown in Fig. 5. Dependence of the DI on the orientation and size of inclined cracks was also investigated and the results are documented in Fig. 6.

REFERENCES