Structural damage detection using the estimated vibration response

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Abstract
The damage index named Auto Correlation Function at Maximum Point Value Vector (AMV), which has been proposed by the authors previously just needs the structural vibration response (displacement, velocity or acceleration) before and after damage to locate the damage. One can get the best detection results for the displacement response, while there may exist some false-positives when the acceleration response is used. But in reality it is much easier to get the acceleration signals, so how to convert it into displacement and velocity signals is of great importance. A special estimation procedure to convert the different types of the response signals is utilized in this paper. The obtained acceleration response is first transformed into the frequency domain, in which it is converted to the displacement and velocity response, respectively. The estimated displacement and velocity signals are found to have only very small deviations from the real original ones, such that they can be used for the AMV-based damage detection method. Several cases of the stiffness reduction localization of a 12-story shear frame structure show that usage of the estimated displacement and velocity response gives better detection results compared to the original acceleration response. Furthermore, there are no more false alarms, which makes the AMV-based damage detection method suitable for more applications.

Keywords: Auto correlation function, Damage detection, Estimated vibration response, White noise

1. INTRODUCTION
During the functional age of many in service structures, e.g. skyscrapers, bridges, stadiums and aircrafts, some local damages will occur that reduce the safety, reliability and durability, some may even cause catastrophic failures that result in economic and human life loss. How to detect these damages as soon as they appear in the structures has received considerable attention of the researchers in the last several decades.
In the literature, several approaches that use the auto/cross correlation functions of the vibration responses for damage detection are reported [1-9]. Based on the concept of Cross Correlation Function Amplitude Vector (CorV) [5], Inner Product Vector (IPV) [6] and Natural Excitation Technique (NExT) [10], Zhang and Schmidt [8, 9] proposed a damage index named Auto Correlation Function at Maximum Point Value Vector (AMV). The stiffness reduction detection of a 12-story frame structure showed that the AMV-based damage detection method can locate the damage effectively, even in the presence of noise and has a better detectability compared to the other correlation-function-based damage detection methods. [9] shows that one can get better detection results if the displacement response is used. The usage priority of different response types for the AMV-based damage detection is displacement response first, velocity response second and acceleration response last. In a laboratory test rig, one of the modern potentiometers or LVDT transducers could be
used to measure the absolute displacement directly, as static reference points are available. But on a moving vehicle this is not possible. It is also not simple to measure the velocity directly in real life. On the other hand, accelerometers are robust, simple to use and readily available cheaper transducers. So, once the acceleration response is obtained and if it cannot be used for damage detection directly, how to convert it into displacement and velocity responses that can be used for damage detection is very interesting. This will be discussed in this paper.

2. DAMAGE INDEX

The auto correlation function is defined as the cross correlation function of a signal with itself. When time lag \( T = 0 \), the value of the auto correlation function is also its maximum value, then AMV [8, 9] from different measurement points of the structure is defined as

\[
R_{i}^{AMV} = [R_{i,1}(0), R_{i,2}(0), ..., R_{i,n}(0)]^T
\]

where \( R_{i,j}(0) \) is the auto correlation function of the response from measurement point \( i \) when the time lag \( T = 0 \). From the definition of the auto correlation function, there is a constant \( X_p^2/2 \) related to the amplitude of the excitation [8, 9]. In order to eliminate the influence, the vector \( R_{i}^{AMV} \) is normalized by its root mean square value, expressed as

\[
\overline{R}_{i}^{AMV} = \frac{R_{i}^{AMV}}{rms(R_{i}^{AMV})} = [\overline{R}_{i,1}, \overline{R}_{i,2}, ..., \overline{R}_{i,n}]^T
\]

where the root mean square value \( rms(R_{i}^{AMV}) = \sqrt{\frac{1}{n} \sum_{j=1}^{n} R_{i,j}^2} \).

Thus, the damage index for the AMV-based damage detection method is given as

\[
D_{i}^{AMV} = [D_{i,1}, D_{i,2}, ..., D_{i,n}]^T
\]

where \( D_{i} \) is the relative change of the \( i \)th element in the vector \( \overline{R}_{i}^{AMV} \) with

\[
D_{i} = \frac{\overline{R}_{i,d} - \overline{R}_{i,u}}{\overline{R}_{i,u}}
\]

where the superscript \( d \) denotes the structure is damaged and \( u \) denotes the structure is undamaged.

In order to make the damage location more clearly, the AMV-based damage location index is defined as

\[
D'_{AMV,i+0.5} = D_{AMV,i+1} - D_{AMV,i}
\]

where the local maxima of \( D' \) corresponding to the abrupt change in the damage index \( D \), which is considered due to the damage that occurred in the structure. Then the damage is located between the measurement points \( i \) and \( i+1 \), if \( D' \) has a local maximum value in the measurement point \( i+0.5 \).

3. DAMAGE DETECTION

3.1 Simulation model

A 12-story shear frame structure (Fig.1 in [8]) is chosen as the numerical simulation model to study the effectiveness of the estimated response in detecting the damage in this paper, which
can also be considered as a 12-DOF discrete system, as shown in Fig.2. The mass $m_i$ of each floor is 1 kg and the stiffness coefficient $k_i$ of each floor is 20,000 N/m. Rayleigh damping $C = \alpha M + \beta K$ is adopted. The excitation force is applied on the 12th floor of the structure.

![Figure 1: 12-DOF discrete system](image)

The stiffness coefficients of the floor 2, 5, 8 and 11 are reduced by 5% representing the damage, and the damage cases are named as D2, D5, D8 and D11, respectively. The excitation force has a sample frequency of 1024 Hz with a duration of 16 s and magnitude of 1 N. Responses from the intact structure and different damage cases are calculated using the Wilson-\theta method. The damage index can be easily obtained when the auto correlation function of the response is calculated using their inner products.

### 3.2 Estimation procedure

If there is an acceleration signal $\ddot{x}(t)$, we may in principle integrate it to obtain velocity and in turn integrate the velocity to find the displacement. It is well known that this has to be carried out very carefully as there are several pitfalls that may cause a lot of errors. Another way to obtain the estimated velocity and displacement signal is to convert the acceleration signal in the frequency domain.

If $\dot{X}(f)$ is the infinite Fourier transform of the acceleration signal

$$\ddot{x}(t) = \int_{-\infty}^{\infty} \dot{X}(f) e^{2\pi i ft} df$$

(6)

the corresponding Fourier transform of the velocity signal $\dot{x}(t)$ is $\dot{X}(f)$, such that

$$\dot{x}(t) = \int_{-\infty}^{\infty} \dot{X}(f) e^{2\pi i ft} df$$

(7)

and the corresponding Fourier transform of the displacement signal $x(t)$ is $X(f)$, i.e.,

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{2\pi i ft} df$$

(8)

By definition, the acceleration is the rate of change of velocity

$$\ddot{x}(t) = \frac{d}{dt}\{\dot{x}(t)\}$$

(9)

Substituting the relevant Fourier form of $\dot{x}(t)$ in Eq.(7) into (9) yields
\[
\ddot{x}(t) = \frac{d}{dt}\left[\int_{-\infty}^{+\infty} \check{X}(f)e^{2\pi if \phi} df\right] \\
= \int_{-\infty}^{+\infty} \check{X}(f) \frac{d}{dt}[e^{2\pi if \phi}] df \\
= \int_{-\infty}^{+\infty} 2\pi if \cdot \check{X}(f)e^{2\pi if \phi} df
\]

Comparing this with the Fourier representation of \(\ddot{x}(t)\) in Eq.(6) yields
\[
\check{X}(f) = 2\pi if \cdot \check{X}(f)
\]

Thus,
\[
\dot{X}(f) = \check{X}(f)/(2\pi f) = \check{X}(f)/i\omega
\]

Similarly,
\[
X(f) = \check{X}(f)/(-\omega^2)
\]

So, the velocity and displacement signals can be converted from the acceleration signal. The estimation schedule is given in Fig.2 and can be expressed as follows:

1. Get the acceleration response is \(\ddot{x}(t)\);
2. Use the fast Fourier transform (FFT), transform the time domain acceleration response \(\ddot{x}(t)\) into the frequency domain \(\check{X}(f)\);
3. Use the Eqs.(12) and (13), get the Fourier transform \(\check{X}(f)\) of velocity response \(\dot{x}(t)\) and the Fourier transform \(X(f)\) of the displacement response \(x(t)\). The frequency range is set at \([fs/1000, fs]\), where \(fs\) is the sample frequency (sample rate);
4. Use the inverse fast Fourier transform (IFFT), transform the frequency domain velocity \(\check{X}(f)\) into the time domain velocity response \(\dot{x}(t)\) and transform the frequency domain displacement \(X(f)\) into time domain displacement response \(x(t)\).

From Eqs.(12) and (13), as \(f\) becomes zero then \(\check{X}(f)\) and \(X(f)\) derived from \(\check{X}(f)\) become indeterminate. So, in reality there is a low frequency limit, which is often referred to as ‘1/f noise’. In most dynamics situations, we are interested in much higher frequencies but in case of whole body dynamics, then direct integration is needed. Fortunately, it is at low
frequencies where direct integration is least error prone. When we are dealing with digital systems, ‘low frequency’ is low relative to the sample rate, which is set at $fs/1000$.

3.3 Damage detection results

Reference [9] shows that the displacement- and velocity-based AMV have a very good detectability at various excitation frequency ranges, but when the excitation ranges covers all
the natural frequencies of the structure, i.e. under white noise excitation, the acceleration-based AMV will have some false alarms. So in this section, the acceleration response obtained from the structure under white noise excitation is converted into displacement and velocity responses to check the effectivity of the proposed AMV. Fig.3 shows the real velocity response and the estimated velocity response from the real acceleration response in the measurement point 1 for the intact structure during the time [7.5, 8.5] using Eq.(12). Fig.4 shows the real displacement response and the estimated displacement response from the real acceleration response in the measurement point 1 for the intact structure during the time [7.5, 8.5] using Eq.(13). From these two figures, it is obvious that the estimated responses are almost the same as the real ones, they show only very small deviations. So the estimated response can be used for AMV-based damage detection procedure.

Fig.5 displays the AMV-based damage location index calculated from the real and estimated response signals. The green solid line is the damage location index calculated from the real acceleration response, of which the legend is ‘acc-r’, the pink dotted line is the damage location index calculated from the estimated velocity response from the real acceleration response, of which the legend is ‘acc2vel’, and the red dash-dotted line is the damage location index calculated from the estimated displacement response from the real acceleration response, of which the legend is ‘acc2dis’. Note that the pink dotted lines and the red dash-dotted lines show peak values of the damage location index at the measurement points 1.5, 4.5, 7.5 and 10.5 for each graph in Fig.5, respectively. So the damage is located in the 2nd, 5th, 8th and 11th floor, respectively. This coincides well with the damage simulated in Section 3.1. Thus, using the estimated response signal, the damage locations can be precisely detected. Besides, comparing to the green solid lines in each graph in Fig.5, i.e. the damage location
index from the real response, there are no other peak values. As a result, there is no false alarm anymore, which improves the results in the AMV-based damage detection method.

6. CONCLUSIONS

In this paper, a special procedure is utilized to estimate the displacement and velocity response from the real acceleration response in the frequency domain. The estimated response is then used for damage localization in the AMV-based damage detection method. Several different cases in a 12-story frame structure show that the AMV-based damage detection method can locate the damage very well and has no false positives compared to the previous research. Thus, the AMV-based damage detection method can be used in more applications.

REFERENCES