A Baseline-Free Damage Detection Method Based on Node Displacement in Mode Shape

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Abstract
Most widely studied SHM methods need baseline data at undamaged stage and a high number of sensors attached to the structure, which make these methods hard to be applied in aircraft structures for online monitoring. The idea of this paper is to determine a practical damage indicator based on the mechanical information in structural design. In this paper, a damage index based on node displacement method from the area of mechanical vibration is proposed. In the mode shape, the displacement at the node is always zero according to definition. When damage appears on the structure or the structural stiffness is changed, the displacement at the original node will become nonzero. The advantages of this method are: (1) baseline-free, (2) only small number of sensors is required. Parameter studies of damage location are analyzed by FEM. An experiment conducted on a beam together with a transfer function based damage index demonstrates the advantage of this method. In addition, the influence of noise, damage sensitivity, and the influence of sensor location are discussed.

1. INTRODUCTION

Structural health monitoring (SHM) has become one of the most important topics for aircraft structural engineers, as for its benefits in enhancing safety, lightweight and reducing life-cycle cost [1]. There are many emerging new SHM technologies: Guided wave, acoustic emission, vibration, strain, and sensor rupture [2]. In addition, sensor technologies are mostly studied in PZT, fiber optic, MEMS, nanotube, etc. Among them, vibration based SHM technology has been demonstrated to be one of the most reliable method [3].

Several vibration based damage detection methods have been proposed. Natural frequency based method uses the change in the frequency to detect damage, which is one of the first to use vibration information for damage detection [4]. Kim’s research shows that mode shape based method is more sensitive to local damage than frequency based method [5]. Hamey [6] used mode shape curvature, which takes second derivatives of the mode shape, to detect delamination in a composite beam, as this method is more effective to small damage. Similar to mode shape curvature, some researchers used fractional modal strain energy as a damage indicator [7].

Recently, Xu et al. [8] proposed a damage detection method based on longitudinal vibration shapes to locate two edges of a damaged section. Ciambella and Vestroni [9] introduced a filtering procedure for modal curvatures to locate damage more effectively.
Some of the vibration based methods have demonstrated their effectiveness in SHM or nondestructive testing (NDT). However, there are three main problems for these methods: damage location, the number of sensors, and the baseline. Methods based on frequency, which is a global parameter, are less sensitive to local damages, especially for concentrated damages [9]. For mode shape based methods (mode shape deflection, mode shape curvature, and strain energy), the accuracy is highly dependent on the number of sensors deployed on the structure [10]. In addition, these methods rely on the baseline of the undamaged structure. Although some researchers proposed a baseline-free method which uses the smoothed damaged mode shape as the health mode shape [11], the accuracy of damage detection is restricted by the measurement of the mode shape and the smoothing method.

Some researchers also found that the nodes in mode shape at its corresponding natural frequency can be affected by damage. Wolff and Richardson’s research [12] found that the position of node will change when damage appears on the structure. Dilena and Morassi [13] investigated this method on a thin beam in bending vibration. However, the method they proposed requires multiple impulse tests on the structure to determine the movement of the node which makes the method hard to be applied for online application. On the other hand, Yeung and Smith [14] measured the frequency response on the node point to access the damage information, using FEM information of a bridge. More studies should be conducted on the sensitivity and quantification of this node based method.

In the present paper, a baseline-free damage location method based on the node displacement in mode shape is proposed. A transfer function based damage index is proposed to quantify the noise effect and the damage sensitivity of the method. This paper first gives a parameter study on a double clamped beam by FEM. Then the damage location procedure is proposed. After that, an experiment conducted on a double clamped steel beam is used to validate this method. A transfer function based damage index is proposed to normalize the damage detection result. At last, the sensitivity of the proposed method is analyzed.

2. GENERAL IDEA

In the mode shape, the displacement at the node at the corresponding natural frequency is always zero. If damage appears, the displacement at node will become nonzero. Based on the idea, this section demonstrates a beam simulated by ABAQUS (commercial finite element software ABAQUS 6.13) to verify this idea and illustrates a damage location procedure.

2.1 FEM analysis

A beam is chosen with the dimension of 1 m × 0.01 m × 0.01 m, both sides are clamped, as illustrated in Figure 1. The material properties are: ρ = 7850 kg/m³, E = 200 GPa, v = 0.26. The beam is modelled with 3 × 3 × 250 C3D20R element. The damage is simulated as stiffness loss in the elements. The damage severity is a, thus the Young’s Modulus of the damage element is \( E' = (1-a)E \). Three different locations of damage with the same damage severity \( a = 0.1 \) are taken into consideration sequentially. The beam is separated into 50 sections for defining the location of damage. These three different locations are 0.250m (13th section), 0.375m (19th section), and 0.5m (25th section), from the left end, as illustrated in Figure 1.

The mode shapes of mode 2 and mode 3 are taken into consideration. Mode 2 has one node in the middle, while mode 3 has two nodes (Length = 0.36 m and 0.64 m). For each damage location case, the mode shape difference is calculated by taking the absolute subtract of the damaged mode shape and the health mode shape, as shown in Figure 2. As the node
displacement of health mode shape is always zero, the mode shape difference at nodes can be recognized as the node displacement of the damaged beam. From Figure 2, the node displacement is no longer zero when damage appears. In addition, the node displacement will have a different response when the damage locates at different places. In particular, when the damage is located at the place where the amplitude of the mode shape is big, the node displacement is significant.

2.2 Damage location procedure

After deploying 3 accelerometers on the nodes of mode 2 and mode 3, it is possible to determine the damage location in 5 sub-areas (A1 to A5) as illustrated in Figure 3. Based on the response of node displacement from Figure 2, an algorithm to determine the damage location can be proposed as illustrated in Figure 4. The node displacements at the nodes are \( N_1, N_2, \) and \( N_3 \), as shown in Figure 3. In the FE results, if node displacement is less than \( 1 \times 10^{-3} \), it is considered as 0. It should be mentioned about the algorithm, that under the condition that \( N_2 = 0, N_1 \neq 0, \) and \( N_3 \neq 0 \), if \( N_1 > N_3 \), the damage is located in A1, as showed in Figure 2. In contrast, if \( N_1 > N_3 \), the damage is located in A5. But under the circumstance that \( N_2 \neq 0, N_1 = 0, \) and \( N_3 = 0 \), damage in A2 or A4 cannot be distinguished. Based on the algorithm, the location of the damage can be determined among the 5 sub-areas.
3. EXPERIMENT STUDIES

3.1 Experiment settings

A steel beam with the dimension of 985 mm × 45 mm × 10 mm is chosen. The beam is double clamped on a steel workbench as shown in Figure 5. The effective length of the beam between two clamps is 800 mm. The material properties are: \( \rho = 7850 \text{ kg/m}^3 \), \( E = 200 \text{ GPa} \), \( \nu = 0.26 \). Three accelerometers (M352C15, PCB Piezotronics, Inc), which are Acc1, Acc2, and Acc3, are chosen in this experiment. The gain factor of the charge amplifier is 100, and the sample frequency of data acquisition devices is 5000 Hz. In the host computer, DASYLab is adopted to control the data acquisition devices, read and store the data from accelerometers.
The measurement system is illustrated in Figure 6 (a). A magnet block which weights 40 g is used to simulate damage, considering as a system change (change in mass) in the structure. A tap hammer as shown in Figure 6 (b) is used for the impact testing.

The accelerometers should be deployed at the exact location of nodes, so a sensor location calibration should be carried out before test. First, the accelerometers are located at the approximate nodes location based on the FEM information similar as in Figure 3. Then, the location calibration is applied by adjusting the position of the accelerometers based on the impact test to minimize the amplitude of certain mode in frequency domain.

### 3.2 Experiment results

Fast Fourier transform (FFT) is adopted to transform the time domain signal into frequency domain. The frequency responses of three accelerometers are shown in Figure 7. Health (Y axis is on a logarithmic scale). The amplitude of frequency responses at 2nd and 3rd natural frequency can be recognized as the node displacements. All three accelerometers have response at 1st mode (72 Hz). In the 2nd mode (189 Hz), Acc2 has a small amplitude ($2 \times 10^{-4} \text{ m/s}^2$), as Acc2 locates at the node of 2nd mode. In the 3rd mode (377 Hz), the amplitudes of Acc1 ($6 \times 10^{-4} \text{ m/s}^2$) and Acc3 ($5 \times 10^{-4} \text{ m/s}^2$) are very small comparing to Acc2, as Acc1 and Acc3 locate at the nodes of 3rd mode. The health stage of the structure can be determined by the small value of three accelerometers.
Damages (D1 to D5) are located at 5 sub-areas (A1 to A5, refer to Figure 3) and the frequency responses are acquired sequentially, as shown in Figure 7 - D1 to D5. In Figure 7 - D1, as damage is located at A1, the amplitudes of Acc2 at 2nd mode and Acc1/Acc3 at 3rd mode are changed significantly. In addition, the amplitude of Acc1 is higher than Acc3 which demonstrate that the damage is located on the left side. Similar procedure can be applied with the instruction of damage location algorithm as shown in Figure 4.

### 3.3 Damage index (DI)

The amplitude at natural frequency of mode 2 and mode 3 can be recognized as the node displacement which can act as a baseline-free DI. However, as the external impact is not the same each time, the node displacement is not the same even if the structure is not changed. Thus, this paper proposes a transfer function based DI which is independent to the external load and only related to the structural characteristics.

The Fourier transform of external impact load is defined as $F_0(\omega_k)$. The Fourier transform of Acc1 and Acc2 are defined as $U_1(\omega_k)$ and $U_2(\omega_k)$. (Considering symmetry, only Acc1 and Acc2 are taken into consideration.) $U_1(\omega_k)$ and $U_2(\omega_k)$ are related to $F_0(\omega_k)$.

For each impact we have the frequency response function:

\[
H_{1,0}(\omega_k) = \frac{U_1(\omega_k)}{F_0(\omega_k)} \quad (1)
\]

\[
H_{2,0}(\omega_k) = \frac{U_2(\omega_k)}{F_0(\omega_k)} \quad (2)
\]

where, $\omega_k$ represent the frequency at $k^{th}$ mode.

Hence, by dividing Eq. (1) to Eq. (2), we define a frequency response ratio $S_{1,2}(\omega_k)$ which is independent to the external load and only related to the structural characteristics.

\[
S_{1,2}(\omega_k) = \frac{H_{1,0}(\omega_k)}{H_{2,0}(\omega_k)} = \frac{U_1(\omega_k)F_0(\omega_k)}{U_2(\omega_k)F_0(\omega_k)} = \frac{U_1(\omega_k)}{U_2(\omega_k)} \quad (3)
\]

Similarly, $S_{2,1}(\omega_k)$ can be acquired. $S_{1,2}(\omega_3)$ and $S_{2,1}(\omega_2)$ can be recognized as normalized DIs of the response of Acc1 and Acc2 respectively.

Considering the variations in the frequency response and the natural frequency is changed when the structure is damaged, an improved DI which measures the range (maximum value - minimum value) around the natural frequency is proposed. Based on the considerations, improved DIs ($S_1$, $S_2$, and $S_3$) are proposed as following:

\[
S_1 = \frac{\max[U_1(\tilde{\omega}_1)] - \min[U_1(\tilde{\omega}_1)]}{\max[U_2(\tilde{\omega}_1)]} \quad (4)
\]

\[
S_2 = \frac{\max[U_2(\tilde{\omega}_2)] - \min[U_2(\tilde{\omega}_2)]}{\max[U_1(\tilde{\omega}_2)]} \quad (5)
\]

\[
S_3 = \frac{\max[U_3(\tilde{\omega}_3)] - \min[U_3(\tilde{\omega}_3)]}{\max[U_2(\tilde{\omega}_3)]} \quad (6)
\]

where, $\tilde{\omega}_2 - \Delta \omega \leq \tilde{\omega}_2 \leq \tilde{\omega}_2 + \Delta \omega$ , $\tilde{\omega}_3 - \Delta \omega \leq \tilde{\omega}_3 \leq \tilde{\omega}_3 + \Delta \omega$ , $\tilde{\omega}_k$ represent the natural frequency of the health structure, the frequency range $\Delta \omega$ is set to be 10 Hz in this case.
The original data which are directly acquired from Figure 7 are listed in Table 1. Based on Eq. (4), (5), and (6), the DIs are illustrated in Table 2. After setting a proper threshold, the damage location can be determined by the algorithm as shown in Figure 4.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Mode 2 ($\omega_2$)</th>
<th>Mode 3 ($\omega_3$)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Acc1 ($U_1$)</td>
<td>Acc2 ($U_2$)</td>
<td>Acc3 ($U_3$)</td>
</tr>
<tr>
<td>Health</td>
<td>27.3</td>
<td>0.2</td>
<td>27.6</td>
</tr>
<tr>
<td>D1</td>
<td>24.0</td>
<td>0.7</td>
<td>25.2</td>
</tr>
<tr>
<td>D2</td>
<td>28.3</td>
<td>0.9</td>
<td>30.1</td>
</tr>
<tr>
<td>D3</td>
<td>39.2</td>
<td>0.2</td>
<td>39.6</td>
</tr>
<tr>
<td>D4</td>
<td>32.9</td>
<td>1.2</td>
<td>31.6</td>
</tr>
<tr>
<td>D5</td>
<td>34.1</td>
<td>1.1</td>
<td>33.2</td>
</tr>
</tbody>
</table>

Table 1: Amplitude and frequencies of three accelerators at different stages

<table>
<thead>
<tr>
<th>Stage</th>
<th>DI ($\times 10^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_1$</td>
</tr>
<tr>
<td>Health</td>
<td>0.82</td>
</tr>
<tr>
<td>D1</td>
<td>5.10</td>
</tr>
<tr>
<td>D2</td>
<td>0.67</td>
</tr>
<tr>
<td>D3</td>
<td>3.38</td>
</tr>
<tr>
<td>D4</td>
<td>0.82</td>
</tr>
<tr>
<td>D5</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Table 2: DI under different damage location

4. SENSITIVITY ANALYSIS

The influence of noise should be taken into consideration when setting the threshold. In addition, damage sensitivity of this method is studied. As this method is highly dependent on the sensor location, the influence of sensor location is also discussed.

4.1 Influence of noise

The hammer impact test is repeated for 5 times for a health stage and a damage stage respectively. In this case, the damage (D1) is simulated by a 40 g magnet block located at A1 area (refer to Figure 6).

<table>
<thead>
<tr>
<th>Times</th>
<th>DI (Health, $\times 10^{-2}$)</th>
<th>DI (Damage, $\times 10^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>1</td>
<td>0.89</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>0.79</td>
<td>0.66</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>0.82</td>
<td>0.78</td>
</tr>
<tr>
<td>5</td>
<td>0.82</td>
<td>0.77</td>
</tr>
<tr>
<td>Average</td>
<td>0.86</td>
<td>0.74</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.07</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Table 3: DI under different experiment times
As shown in Table 3, DIs at health stage are near $0.8 \times 10^{-2}$ and the standard deviations are near $0.07 \times 10^{-2}$, this can be considered as the influence of noise. In addition, the average DI for this damage case is near $4 \times 10^{-2}$, which is much higher than the health stage. In this case, a threshold of 0.01 is proper for damage detection.

### 4.2 Damage sensitivity and threshold setting

In this part, eight different severities of damages are arranged in A1 area (refer to Figure 3). The information of the damages is listed in Table 4. Several small magnet blocks are adopted to simulate damage with small weight. The weight of the tested beam is 2826 g. The weight ratio is calculated by dividing damage weight with the beam weight. For each damage severity, the hammer impact test is repeated for 5 times.

The DI results are illustrated in Figure 8. As the noise induced DI is near $0.8 \times 10^{-2}$, the threshold is set to be 0.01 with a damage of 10 g (weight ratio: 0.35%). In addition, the DI increased with the damage weight. Thus, it has the potential to quantify the damage.

<table>
<thead>
<tr>
<th>Damage No.</th>
<th>h</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>d4</th>
<th>d5</th>
<th>d6</th>
<th>d7</th>
<th>d8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage weight (g)</td>
<td>/</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>40</td>
</tr>
<tr>
<td>Weight ratio (%)</td>
<td>/</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.28</td>
<td>0.35</td>
<td>0.49</td>
<td>0.64</td>
<td>1.42</td>
</tr>
<tr>
<td>$S_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average ($\mu_1$)</td>
<td>0.74</td>
<td>0.70</td>
<td>0.73</td>
<td>0.75</td>
<td>0.98</td>
<td>1.17</td>
<td>1.22</td>
<td>1.67</td>
<td>3.43</td>
</tr>
<tr>
<td>Standard deviation ($\sigma_1$)</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.10</td>
<td>0.08</td>
<td>0.09</td>
<td>0.06</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>$S_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average ($\mu_2$)</td>
<td>0.86</td>
<td>1.34</td>
<td>1.41</td>
<td>1.51</td>
<td>1.72</td>
<td>1.92</td>
<td>2.24</td>
<td>2.54</td>
<td>4.35</td>
</tr>
<tr>
<td>Standard deviation ($\sigma_2$)</td>
<td>0.07</td>
<td>0.08</td>
<td>0.05</td>
<td>0.07</td>
<td>0.03</td>
<td>0.07</td>
<td>0.08</td>
<td>0.06</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 4: Damage information

![Figure 8: DI with damage weight](image)

### 4.3 Influence of sensor location

A quantified study is conducted in this part to measure the effect of sensor location on the DI value on the no damage condition. Accelerometer 2 is chosen to move five distances (1 mm, 2 mm, 3 mm, 4 mm, and 5 mm). The same hammer impact test procedure is applied.

Figure 9 shows the influence of sensor location on DI. DI increase dramatically with the sensor distance, even a 1 mm error in sensor location will lead to a similar DI as a 10 g magnet block.
5. CONCLUSIONS

In this paper, a baseline-free damage detection method based on the node displacement in mode shape is proposed. A transfer function based DI is proposed and adopted to normalize the result. An experiment conducted on a steel beam demonstrates that after setting the DI threshold of 0.01, the method can determine the damage location among 5-sub areas. The DI caused by noise is about 0.008. The minimum damage can be located by this method is 10g (weight ratio: 0.35%; DI: 0.01). As DI is increased with the damage, this method can be adopted to quantified damage. In addition, even a 1 mm error in sensor location will cause a DI similar to a 10 g block, so the sensor location calibration before the test is very important. This method has the potential to be applied to a more complex structure, as every structure should have a node in its mode shape. For more complex structures, laser vibrometer can be applied to find the nodes.

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