Defect Triangulation via Demixing Algorithms Based on Dictionaries with Different Morphological Complexity

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Key words: Sparse coding, Anomaly detection, Structural diagnostics, Laser vibrometer.

Abstract
This work investigates the capabilities of methods based on morphological data demixing in application to anomaly detection and triangulation in solid media with significant structural complexity. The operating principle of morphological demixing is the decomposition of the spatiotemporal response of a structural system into two distinct contributions that are antithetical and complementary in terms of their morphology. While the bulk of the dynamic response can be generally represented in terms of smooth functions, the effect of nominally rare anomalies is captured through the projection of the data on a dictionary of sparse basis functions. The resulting sparse representation de facto distills the part of the response carrying the signature of the defects, thus promoting their agile triangulation. It has been shown that the triangulation of rare and weak anomalies can be enhanced through the adoption of dictionaries whose atoms feature, in addition to the above mentioned sparsity requirements, other morphological attributes that are germane to the scattered fields observed in the neighborhood of localized defects. This additional capability introduced through morphologically germane dictionaries results in a more robust discrimination between the signatures of true defects and the spurious response features due to other (often benign) structural elements. Our objective here is to explore the use of a dictionary constructed specifically to represent the anomalous response, whose atoms have morphological characteristics that mirror the structure produced in the response by a point excitation. The conclusions of our analysis are supported by numerical and laser-acquired experimental data.

1. Introduction

The advent of Scanning Laser Doppler Vibrometry (SLDV) has made available the reconstruction of high resolution wavefields, and this capability has been shown to have great potential in the field of structural diagnostics. SLDV systems allow making a series of velocity measurements (potentially on a very dense grid) on the exterior of a vibrating object, which can be aggregated in the form of a collection of snapshots of the wavefield at different time instants. This type of sensing provides spatially rich data which can be analyzed frame by frame via image processing (IP) and computer vision (CV) techniques. The additional information available from full wavefield reconstructions (as opposed to the parsimonious data acquired from sparse sensor arrays) has led to the development of diagnostics techniques that can operate with minimal requirements in terms of knowledge of the material properties, and virtually without information about the performance
of the structure in its pristine state (baseline data). Some contemporary approaches used to identify and visualize structural anomalies from laser-acquired wavefield data include: methods based on space-time DFT \[1\], wavenumber-space filters \[2\], Laplace filters \[3\], and approaches based on saliency analysis \[4\].

A variety of diagnostic methods based on compressive sensing (CS) have recently populated the SHM literature \[5\]. These approaches typically seek a sparse approximation of the response data through its projection onto an over-complete basis; an example of this paradigm is Matching Pursuit Decomposition (MPD), which has been shown to be a powerful diagnostics tool \[6\]. In describing compromised media, one can leverage obvious structural differences between damaged and undamaged behavior, which often correspond to differences (albeit possibly minute) in the morphological structure of the corresponding wavefields \[7\, 8\].

Another class of structural diagnostic techniques that have recently begun to emerge exploit the spatial implications of the dichotomous nature of the wavefield response. These methods effectively separate, or demix, the wavefield data into two coexisting components: one representing the bulk response in the pristine portion of the system, the other capturing the behavior of the wavefield in the neighborhood of the defect. The underlying idea is to represent both components in terms of multiple representative bases, or dictionaries, where the dictionaries are either specified a priori \[9\] or learned from the data itself \[10\]. In this paper, we describe a methodology based on demixing of the response data performed according to its morphological characteristics, with the objective of inferring the presence and location of defects. The demixing procedure uses intelligently prescribed dictionaries which are endowed with a befitting spatial structure that allows them to emulate the morphology of the response component they are to describe. Implementation is achieved via a straightforward extension of the basis pursuit denoising (BPDN) problem \[11\], where the spatial structure of the dictionaries allows for a formulation which is highly computationally efficient.

2. A Demixing Technique for Defect Detection

Upon inspection of the dynamic response of a medium with embedded defects, e.g., an aluminum sheet with a partial hole excited with a tone burst (shown in the top image of Fig. 1), one can observe the existence of both diffuse and localized features in the wavefield. As the defect acts as a scatterer for the impinging wave, the localized features are predominantly concentrated around the defect, whereas the bulk of the pristine portion of the domain displays the conventional spatial characteristics observed in wavefields established in undamaged media. The upshot of this observation is that we can identify the portion of the wavefield associated with the defect by recognizing the regions with compact (or low support) spatial structure. Thus, one way to identify the presence of the defect is to separate, or demix, the full wavefield data into two complementary components classified according to their morphological structure.

Let \( X \in \mathbb{R}^{N \times T} \) denote the dynamic response of a structure, stored as a matrix where the columns are arrays encoding the velocity or displacement measurements at the \( N \) nodes of the system at \( T \) different time instants. Our objective is to decompose \( X \) into the sum of two distinct contributions: \( X_1 \in \mathbb{R}^{N \times T} \), which captures any morphologically compact features, as they would
Figure 1: Illustrative overview of the demixing process. The original wavefield is processed by the demixing algorithm according to Eq. (3), which effectively separates the response into the component associated with the defect and the component which approximates the response in the undamaged portion of the medium.

be observed in the neighborhood(s) of material defects, and $X_2 \in \mathbb{R}^{N \times T}$, which captures all large and spatially diffuse features, which make up the bulk of the response in the undamaged portion of the medium.

$$X \approx X_1 \underset{SPATIALLY\ LOCALIZED}{\downarrow} + X_2 \underset{SPATIALLY\ SMOOTH}{\downarrow}.$$  \hspace{1cm} (1)

To enforce the demixed representation shown in (1), we construct a *dictionary*, or representative basis for each component, each endowed with befitting morphological characteristics appropriately selected to describe its respective class. For each contribution, we consider a matrix factorization of the kind:

$$X_1 \approx D_1 A_1 \quad \text{and} \quad X_2 \approx D_2 A_2.$$  \hspace{1cm} (2)

Here, $D_1 \in \mathbb{R}^{N \times K_1}$ and $D_2 \in \mathbb{R}^{N \times K_2}$ are *a-priori* defined dictionaries, (in our case we set $K_1$ and $K_2$ to $N$ for simplicity) and $A_1 \in \mathbb{R}^{N \times T}$ and $A_2 \in \mathbb{R}^{N \times T}$ are arrays of temporal coefficients whose columns indicate which columns of the corresponding dictionary are utilized in the representation at each time instant. The individual columns of the dictionaries $D_1$ and $D_2$ are referred to as *atoms*.

To solve for the unknown coefficient of influence arrays $A_1$ and $A_2$ and obtain the aforemen-
tioned factorization, we invoke a *basis pursuit denoising* optimization problem of the form:

\[
\minimize_{A_1 \in \mathbb{R}^{K_1 \times T}, A_2 \in \mathbb{R}^{K_2 \times T}} \frac{1}{2} ||X - D_1 A_1 - D_2 A_2||_F^2 + \tau_1 ||A_1||_1 + \tau_2 ||A_2||_1
\]  

(3)

where \( \tau_1, \tau_2 > 0 \) are user-specified regularization parameters. In (3), the first term in the objective function is essentially a standard squared error term (the subscript \( F \) denotes the Frobenius norm, which is the matrix analog of the Euclidean norm for vectors), which controls the “goodness of fit” to the data and ensures that the overall approximation in (1) is sufficiently accurate. The second and third terms are parsimony terms: the presence of the \( \ell_1 \) norm tends to produce solutions in which \( A_1 \) and \( A_2 \) are sparse, which allows the columns of \( X_1 \) and \( X_2 \) to be individually well-approximated using relatively few components of their respect dictionaries. Enforcing reconstruction using only a small number of atoms from each dictionary is important as this promotes an efficient representation of each component via a dictionary whose atoms directly model its morphological structure instead of relying on a brute force reconstruction that employs a myriad of ill-suited atoms. As an extreme example, we could argue that a large set of orthogonal functions can be used to accurately decompose a signal of arbitrary complexity, but such choice would be suboptimal from an anomaly detection standpoint as the reconstruction would not possess appealing localization attributes.

3. Using Morphologically Germane Dictionaries

Hitherto, our approach has been deliberately kept general without any explicit mention of the specific structure of the dictionaries \( D_1 \) and \( D_2 \); we now discuss the underlying logic behind the specific dictionary selection used to achieve the desired detection capabilities. To approximate the smooth component of the response associated with the wavefield in the undamaged portion of the domain, we choose \( D_2 \) to be the Discrete Cosine Transfer (DCT) matrix, whose atoms correspond to digitized cosine functions of different frequencies. By this choice, \( D_2 \) acquires the appealing property that the atoms form an orthogonal basis, which in turn significantly reduces computation time and complexity. More relevant to the task of diagnostics is the selection of the dictionary associated with the response in the neighborhood a defect. The key idea is to select functions that closely mimic the morphological structure of the wavefield in close proximity to a defect. Without compromising the agnostic attributes of the method, we invoke the intuitive assumption that the defect will indeed act as a scatterer and produce a signature in the response whose structure will locally resemble that of the applied excitation. Accordingly, we choose our dictionary \( D_1 \) to be a collection of digitized radial functions, each centered at different points in the domain, and displaying a spatial morphology which mirrors that of the original excitation.

In our case, the excitation is given by a tone burst \( E(t) \) which has the general expression

\[
E(t) = \sin(\omega_c t) H(t), \quad \text{where} \quad H(t) = \frac{1}{2} \left( 1 - \cos \left( \frac{2\pi t}{T - 1} \right) \right)
\]  

(4)

where \( \omega_c \) is the carrier frequency of the signal, \( H(t) \) is the Hann window, and \( T \) is the number of time instants the window is to include. Fig.(2) illustrates the early stages of the response...
produced by a 5 cycle tone burst; for completeness, the excitation $E(t)$ is visually shown in the inlay figure. Fig. (2b) shows a similarly generated wavefield scattering against a defect. Our choice for the atoms used to describe $D_1$, shown in the inlay of Fig. (2b), is inspired by the ring structure observed around the scatterer. Despite the obvious resemblance, upon close inspection, one may notice that the radial decay in our model atom is much more abrupt than that observed in the wavefield upon interaction with a defect. This apparent limitation, however, can be an advantageous attribute for two reasons: 1) The atoms have a sparser structure, which is computationally efficient, 2) If the medium is highly heterogeneous, the precise ringed structure observed in the scattered field away from the immediate proximity of a defect may be lost; therefore, using spatially localized atoms for $D_1$ allows us to still capture the behavior of the scattered field in media where significant wavefield distortion may occur.

As matrix $D_1$ in Eq.(3) is non-orthogonal, a straightforward closed-form solution is unavailable, and we must therefore resort to iterative procedures to solve the convex minimization problem. To this end, we consider an alternating approach where the variable coefficient matrices $A_1$ and $A_2$ are individually held fixed while one step in the descent iteration is performed, and this alternation is repeated until convergence is reached. The particular gradient method we use here is the well known FISTA algorithm [12], where we exploit both the sparsity and the block-Toeplitz structure of the $D_1$ dictionary to achieve convergence efficiently.

4. Results from Numerical Simulations and Laser-Acquired Experimental Data

We examine the efficacy of the method by testing it against two challenging defect detection problems. Before presenting the results, it is appropriate to take a brief digression to discuss a post-processing step that is embedded in the method to enhance the demixing procedure and to automate the defect triangulation process. This procedure consists of constructing a spatial map, which we refer to as a superatom (in analogy to our previous work [13]), which depicts a nonlinear aggregation (across time) of the frames of $X_1$ and is meant to highlight the most persistence features across the sparse component set. The superatom features a partitioning of the domain into rectangular re-
Figure 3: Challenging defect detection problem using simulated data. a) Schematic of a thin aluminum plate with stiffening ribs, random distributed perturbation of the Young’s modulus and embedded defect; a snapshot of the wavefield is superimposed onto the structure. b) Four random frames of the anomalous response, out of which only two correctly pinpoint the defect. c) After the energy in each spatial partition is aggregated, a ranking algorithm is used to determine the most likely candidate partition(s). d) Super atom intelligently aggregating the partition data and correctly triangulating the location of the defect.

regions, where the selection of the partition(s) which is deemed the most likely to contain defects is based on a ranking algorithm that takes into consideration three parameters: 1) the maximum pixel magnitude in each partition 2) the number of non-zero pixels in each partition 3) the number of non-zero pixels in all adjacent partitions. In essence, the triangulation procedure identifies the partitions of the superatom that have many non-zero elements of high magnitude while being isolated from other partitions that also contain non-zero pixels.

The first case we consider probes the method’s ability to localize an elusive defect whose signature is buried in noise due to interference from reflections caused by a benign structural element, as well as widespread material perturbations. To this end, we model a thin Aluminum plate excited by an actuator located in the middle of the bottom edge of the domain and introduce in the plate two sources of structural and material heterogeneity: 1) we increase by 50% the thickness and Young’s modulus within four thin bands (two horizontal and two vertical), as shown in Fig. 3(a), to model the effect of four stiffening ribs; 2) we modulate by 25% the Young’s modulus of a random subset of elements (30% volume fraction) of the FE mesh. A localized anomaly is introduced in the domain by reducing (by a single order of magnitude) the Young’s modulus of the material inside a small region, to simulate a soft inclusion (a variation of thickness, simulating a partial hole, was also tried with equivalent results), as schematically shown in Fig. 3(a), where the propagating wavefield (showing pronounced distortion due to heterogeneity) is superimposed to the schematic. To obtain the data to feed to our demixing algorithm, we record and vectorize the nodal displacements and we arrange them as columns of the response data matrix X. Fig. 3(b) shows four random frames of the anomalous component X₁; after the superatom is constructed,
a ranking algorithm assigns a value to each partition, whose value is associated with how likely each partition is to contain a defect (as shown in Fig. 3(c)). Finally, a superatom is constructed, as shown in Fig. 3(d), which correctly pinpoints the defect.

Finally, we test the method against experimentally acquired data. We consider an Aluminum plate (dimensions $61 \times 61$ cm and thickness 2.54 mm) containing a defect but also featuring benign structural heterogeneity in the form of a stiffening rib glued to the rear surface between the defect and the excitation source. The defect is a cylindrical 6-mm diameter partial hole which spans roughly half the plate thickness. We excite the plate with a piezoelectric transducer which generates a 5-cycle burst with carrier frequency $f_c = 200$ kHz. The wavefield is reconstructed from surface velocity data using the Polytec PSV-400-3D Scanning Laser Doppler Vibrometer (SLDV). Fig. 4(a) shows the heavy interference in the wavefield which is predominantly the interactions between the incident wavefield and multiple reflections from the rib and the boundaries. The performance of our algorithm is highlighted in Fig. 4(b), where it can be seen that the superatom allows successful triangulation of the anomaly despite the pronounced level of wavefield distortion.

We duly note the similarity of this configuration to that in our previous work in [14], as our intent is precisely to illustrate the improvements in performance under the new developments.

5. Conclusion

In this work we have presented a virtually model- and material-agnostic defect detection methodology for solid media that exploits the spatially dense scans made available via SLDV systems. The method exploits the contrasting wavefield morphology between undamaged regions and areas that are in close proximity to a defect. In particular, it capitalizes on the fact that, when defects are impinged upon by a wavefront, the structure of the scattering event closely mimics that of a locally applied excitation. As a result, demixing the full response splits the data into two compo-
nants: one associated with the wavefield in the undamaged portion of the medium and the other associated with the locally anomalous behavior. We have shown the method to be effective against both synthetic and experimental data even in the presence of significant wavefield distortion.


