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A novel approach towards fatigue damage prognostics of composite materials utilizing SHM data and stochastic degradation modeling

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Abstract

A prognostic framework is proposed in order to estimate the remaining useful life of composite materials under fatigue loading based on acoustic emission data and a sophisticated Non Homogenous Hidden Semi Markov Model. Bayesian neural networks are also utilized as an alternative machine learning technique for the non-linear regression task. A comparison between the two algorithms operation, input, output and performance highlights their ability to tackle the prognostic task.

Keywords: SHM, remaining useful life, machine learning techniques, composite materials, acoustic emission

1. INTRODUCTION

The fatigue performance of composite materials is a field of much research effort the last decades. Especially in the aerospace industry where the damage tolerance philosophy is prevalent analytical and numerical tools have been utilized to design taking into account fatigue. Lately, and after increasing the Technology Readiness Level of several structural health monitoring (SHM) after damage has been diagnosed, located and characterized. In prognostics, probably the most important quantity to estimate is the remaining useful life (RUL) of a specific component or sub-structure properly sensorized and monitored on a permanent basis.

Composite materials undergoing fatigue loading are reported to fail in a rather stochastic fashion even under constant (i.e. deterministic) amplitude fatigue. This is attributed to their complex multi-phase nature, the plethora of inherent defects (fiber misalignment, voids, resin rich, resin poor areas) that cannot be absolutely controlled during the usual manufacturing processes and thus the stochasticity of their macroscopic mechanical properties. The result is that coupons from the same material batch, being manufactured under the very same process, tested under the same machine in alike conditions can fail in totally different number of cycles. Stochastic modeling and more specifically Markov chains have been utilized as early as the 1980s (see Bogdanoff and Kozin [1] and later Rowatt and Spanos [2], Pappas et al. [3]) but the Markovian assumption regarding degradation i.e. future degradation states are independent of the past states, is not generally valid in engineering practice [4]. Recently, Chiachio et al. in [4] re-visited the idea of Markov chain modeling of fatigue damage in composites under a purely Bayesian framework utilizing stiffness reduction histories from 16 glass-fiber coupons. However, these approaches utilize mostly stiffness degradation data as damage indicator assuming the Markovian property for the degradation property and utilizing simple Markov chains. Secondly, they do not exploit at all SHM data. Very recently [5-6] Liu et al. [5] and Chiachio et al. [6] realized remaining fatigue life estimations of a composite component under fatigue utilizing SHM measurements and damage propagation physical models. In [5] real-time acousto-ultrasonic measurements are correlated with stiffness degradation and integrated in a Bayesian inference framework which provides with RUL estimates. Physics-based degradation models suffer from the fact that there is no widely accepted theory for the progressive failure of composites and the aforementioned works involve the modeling of specific damage mechanisms such as delamination and matrix cracking without accounting for their interactions. The proposed framework of condition-based reliability assessment is driven by the increased usage of composite materials in high-end applications in industries such as the aerospace, automotive and wind energy among others and the need to enhance our understanding of the damage process and the health assessment of a subcomponent or a structure during service life has become more pressing than ever.

2. MACHINE LEARNING ALGORITHMS

2.1 Non-Homogeneous Hidden Semi Markov Models (NHHSMM)

The damage process in composite materials can be considered as a stochastic hidden process which manifests itself only indirectly through for instance SHM or NDT data. Multi-state degradation modeling is an appropriate framework to model damage in composites which gradually accumulates and increases during service loading. To this direction the most interesting and mathematically rich stochastic models are Hidden Semi Markov Models. These models were extended in [7,8] to take into account non-homogeneity i.e. age dependence during state transitions. To fully describe a NHHSMM the definition of a series of elements is required. The number of possible discrete degradation health states (*N*), the transition diagram which defines the connectivity between the states and the allowed transitions, the transition rate's statistical function (λ), the observations i.e. the SHM damage-sensitive feature(s) $y_{1:t}$ and the number of discrete feature values (m) after the observations quantization. The number of hidden states obviously refers to the number of discrete levels of degradation. In a Maximum Likelihood Estimation (MLE) approach, Moghaddass et al. [7,8] demonstrated a procedure to maximize $\Pr(y^{(k)}|\boldsymbol{\theta})$, i.e. define the model parameters $\boldsymbol{\theta}$ which maximize the probability of the K available for training observation sequences $\boldsymbol{y^{(k)}}$.

$$L(\theta,\boldsymbol{y^{(1:K)}}) = \prod_{k=1}^{K} Pr\left(\boldsymbol{y^{(k)}}|\boldsymbol{\theta}\right) \xRightarrow{L'=log(L)} L'(\boldsymbol{\theta},\boldsymbol{y^{(1:K)}}) = \sum_{k=1}^{K} log(Pr(\boldsymbol{y^{(k)}}|\boldsymbol{\theta})) \Rightarrow$$

$$\boldsymbol{\theta}^* = arg\max_{\boldsymbol{\theta}}\left(\sum_{k=1}^{K} log\left(Pr\left(\boldsymbol{y^{(k)}}|\boldsymbol{\theta}\right)\right)\right) \quad (1)$$

Utilizing Baum's auxiliary function, the above optimization task is reduced to a set of independent equations for the re-estimation of the elements of $\boldsymbol{\Gamma}$, $\boldsymbol{B}$. The mathematical treatment leads to two re-estimation equations:

$$\omega_{1,1}^{r}(\boldsymbol{\theta_{old}},\boldsymbol{\theta}) = \sum_{k=1}^{K} Pr\left(\boldsymbol{y^{(k)}}|\boldsymbol{\theta_{old}}\right)^{-1} \times \sum_{j=1}^{N}\sum_{a=0}^{d_k}\sum_{d=1}^{d_k-a} \log(\varepsilon_a^{(k)}(\mathrm{r,j,d}|\boldsymbol{\theta}) \times \kappa_a^{(k)}\left(\mathrm{r,j,d},\boldsymbol{y^{(k)}}|\boldsymbol{\theta_{old}}\right) \quad (2)$$

where $1\leq r \leq N-1$ giving thus N-1 equations and

$$b_i(w) = \frac{\sum_{k=1}^{K}\left(Pr\left(\boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right)^{-1} \times \sum_{t=1}^{d_k} \gamma_t\left(i, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right)\delta_{O_t^{(k)},w}\right)}{\sum_{k=1}^{K}\left(Pr\left(\boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right)^{-1} \times \sum_{t=1}^{d_k} \gamma_t\left(i, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right)\right)} \quad (3)$$

where $1 \le w \le m$ and the terms $\varepsilon_a^{(k)}(\mathrm{i,j,d}|\boldsymbol{\theta})$, $\kappa_a^{(k)}\left(\mathrm{r,j,d}, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right)$ and $\gamma_t\left(i, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right)$ are introduced in order to simplify the MLE process (refer to [21-22] for details) and are defined as follows:

$$\varepsilon_a^{(k)}(\mathrm{i,j,d}|\boldsymbol{\theta}) = \Pr\left(X_n = j, t_{a+d-1}^{(k)} < T_n \le t_{a+d}^{(k)} \middle| X_{n-1} = i, t_{a-1}^{(k)} < T_{n-1} \le t_a^{(k)}, \boldsymbol{\theta}\right),$$

$$\kappa_a^{(k)}\left(\mathrm{r,j,d}, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right) =$$

$$\Pr\left(X_n = j, t_{a+d-1}^{(k)} < T_n \le t_{a+d}^{(k)}, X_{n-1} = i, t_{a-1}^{(k)} < T_{n-1} \le t_a^{(k)}, \boldsymbol{y}^{(\boldsymbol{k})} \middle| \boldsymbol{\theta}_{\boldsymbol{old}}\right),$$

$$\gamma_t\left(i, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right) = Pr\left(Q_t = i, \boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}_{\boldsymbol{old}}\right),$$

with X_n being the state of the component after the nth transition, T_n the time of the nth transition, Q_t the current hidden state and $t_i^{(k)}$ the ith observation time point of the kth observation/SHM data sequence $\boldsymbol{y}^{(\boldsymbol{k})}$.

The MLE approach begins with a random initialization of $\boldsymbol{\Gamma}$, $\boldsymbol{B}$ and via the use of the re-estimation equations (2) and (3) it aims to the iterative maximization of the $\sum_{k=1}^{K} log\left(Pr\left(\boldsymbol{y}^{(\boldsymbol{k})}\middle|\boldsymbol{\theta}\right)\right)$ value. This procedure concludes to a parameter vector $\boldsymbol{\theta}$ which describes the most probable model for a given training data set.

Regarding the prognostics, the mean Remaining Useful Life (RUL) is the quantity of interest in a condition-based monitoring framework. It can be estimated via eq. (4) as the integral of the conditional reliability function $R\left(t\middle|y_{1:t_p}, L > t_p, M\right) = Pr\left(L > t\middle|y_{1:t_p}, L > t_p, M\right)$ i.e. the probability that the composite material/component continues its operation after a time point t (less than life-time L) further than the present time t_p. This is a definition which is conditional on SHM data i.e. the observation sequence $y_{1:t_p}$. Details on the calculation of the conditional reliability function can be found in [7,8].

$$\widehat{RUL}\left(t\middle|y_{1:t_p}, L > t_p, M\right) = \int_0^\infty R\left(t + \tau\middle|y_{1:t_p}, L > t_p, M\right)d\tau \quad (4)$$

In prognostics, an estimate of the uncertainty that follows the mean RUL estimation is of utmost importance, in order to give a confidence of the predicted mean value. The calculation of confidence intervals is based on the calculation of the a% and (1-a)% lower and upper percentiles respectively. It can be easily proved [5,7] that the cumulative distribution function (CDF) for RUL can be defined at any time point utilizing the conditional reliability according to the following:

$$Pr\left(RUL_{t_p} \le t\middle|y_{1:t_p}, M\right) = 1 - R(t + t_p \mid y_{1:t_p}, M) \quad (5)$$

2.2 Bayesian Feedforward Neural Networks

An alternative approach handles the RUL estimation as a nonlinear regression task mathematically described as:

$$y(\boldsymbol{x}) = RUL(\boldsymbol{x}) = f(\boldsymbol{x}, \boldsymbol{\theta}) + \boldsymbol{e} \quad (6)$$

Where $RUL(\boldsymbol{x})$ is the output RUL estimated at the ith time instant during the coupon's

lifetime, $f(\boldsymbol{x},\boldsymbol{\theta})$ the nonlinear mathematical model trained throughout the training process which takes as input the damage feature vector **x** obtained from SHM and is characterized by a number of model parameters $\boldsymbol{\theta}$, **e** is random noise. Let us note the different use of y variable in contrast to section 2.1. y here denotes the output i.e. the RUL of a nonlinear regression scheme. The concept is to use historical data $\mathbf{D}=\{(\mathbf{x}_1,\mathbf{y}_1),\ldots,(\mathbf{x}_N,\mathbf{y}_N)\}$ to estimate $f(\boldsymbol{x},\boldsymbol{\theta})$ and consequently be able to predict the RUL(x_{new}) for any new SHM input x_{new}.
A state-of-the-art model utilized for nonlinear regression is the classical feed-forward artificial neural network (FFANN). In FFANN the mathematical function f correlating the outputs with the inputs can be expressed as $f(\boldsymbol{x},\boldsymbol{w})$ where **w** are the weight and bias parameters involved in the various layers of the network. A limitation of the conventional FFANN is that it is of deterministic nature and its predictions do not entail confidence limits. From a prognostic point of view this is not acceptable as any prediction must be accompanied by a confidence interval to make sense. This is why we resort to Bayesian FFANN (BFFANN). The Bayesian approach to neural networks learning and prediction processes is based on the Bayesian inference. This means the integration of the weights **w** and the quest of its PDF function instead of searching for a single vector of **w**. In this approach all parameters are treated as random variables. A hierarchical Bayesian modeling of the FFANN as first discussed in [9] utilizes the Bayes theorem to obtain the posterior likelihood on weights **w** as follows:

$$p(\boldsymbol{w}|\boldsymbol{D}) \sim p(\boldsymbol{D}|\boldsymbol{w})p(\boldsymbol{w}) \tag{7}$$

Where $p(\boldsymbol{D}|\boldsymbol{w})$ is the likelihood function and $p(\boldsymbol{w})$ the prior PDF on weights **w.** From the Principle of Maximum Information Entropy it follows that if we assume a zero mean Gaussian for the error term in (6), i.e. $e_i \sim N(0,1/b)$ where b is the inverse variance, then the observed output will also be Gaussian, $y_i \sim N(f(x_i,\boldsymbol{w}),1/b)$.
Consequently, the likelihood function is given by the following:

$$p(\boldsymbol{D}|\boldsymbol{w}) = \prod_{i=1}^{N} p(y_i|\boldsymbol{x},\boldsymbol{w}) = \prod_{i=1}^{N} (2\pi/b)^{-\frac{1}{2}} \exp\left[-\frac{b}{2}\,\{y_i - f(x_i,\boldsymbol{w})\}^2\right] \tag{8}$$

In a regression problem such as the prediction of a quantity e.g. the RUL of a composite structure conditional on historical training data, the target RUL y_{new} is computed by applying the rule of summation in probability theory marginalizing out the weight **w**.

$$p(y_{new}|x_{new},\boldsymbol{D},a,b) = \int p(y_{new}|x_{new},\boldsymbol{w},b)\, p(\boldsymbol{w}|\boldsymbol{D},a,b)d\boldsymbol{w} \tag{9}$$

having predetermined the hyperpriors a,b.
Eq. (9) is very important since it provides the whole PDF of the random variable y_{new} or RUL(x_{new}). Thus mean, variance etc can be subsequently calculated. The integral in (9) is generally not analytically tractable since **w** is of high dimension. Computational approaches such as the Markov Chain Monte Carlo Sampling (MCMC) render the calculation viable via the estimation of the integral by a finite sum:

$$p(y_{new}|x_{new},\boldsymbol{D}) \approx \frac{1}{m}\sum_{i=1}^{m} p(y_{new}|x_{new},w_i) \tag{10}$$

Where w_i are samples generated from the posterior PDF $p(\boldsymbol{w}|\boldsymbol{D},a,b)$. Conventional MCMC algorithms such as the Metropolis-Hastings have proved rather inefficient in the case of BFFANN where **w** can be of considerably high dimension. Hybrid Monte Carlo, a stochastic sampling algorithm which incorporates the gradient of Hamiltonian energy information to search more effectively the sample space in regions of higher likelihood. More details can be found in [10].

3. CASE STUDY

Constant amplitude fatigue tests were performed in nine open-hole coupons towards the validation of the proposed stochastic modeling for damage diagnostics and remaining useful life prognostics in composite materials. Plates from carbon/epoxy prepregs were manufactured in house via the autoclave process. The stacking sequence was a typical quasi-isotropic $[0/45/-45/90]_{2s}$ and rectangular open-hole coupons were cut in dimensions 300x30 mm^2. A central hole of 6 mm diameter was drilled subsequently to all coupons. Tensile tests indicated a mean tensile failure load of 28kN. Tension fatigue at 10 Hz loading frequency and ratio R=0.1 were performed in an Instron hydraulic universal testing machine (Figure 4a). The maximum loading level was kept constant at 90% of the ultimate tensile load in order to minimize the time needed for the tests. Table 1 summarizes the cycles to failure distribution between the tested coupons. Evidently, the scatter in the cycles to failure is quite large, an expected result of the stochasticity in the material properties and material inhomogeneities inherent in composites.

Coupon #	**Fatigue test conditions**	**Cycles to failure (x 10^3)**	**Cycles to end-of-life threshold (Figure 1) (x 10^3)**
A1	R=0.1, f=10Hz, F_{min}= 2.5 kN F_{max}=25.2 kN	286	285
A2		159	121
A3		116	116
A4		146	135
A5		109	76
A6		317	306
A7		261	224
A8		53	37
A9		204	160

Table 1. Cycles to failure of tested coupons

AE was recorded in situ via a PCI-2 A/D acquisition board system provided by Envirocoustics Ltd (Mistras Group). Pre-amplification of 40 dB and band-bass filtering of 100-1200 kHz was applied using general purpose voltage pre-amplifiers. A threshold of 50 dB was adequate to reject hydraulic noise and the choice of the timing parameters was PDT=50μsec, HDT=100μsec and HLT=300μsec. The single AE sensor is a wideband piezoelectric sensor type WD 100-900 kHz manufactured by PAC, USA. Figure 1 depicts the resulted degradation histories of all coupons in terms of windowed cumulative RA (Risetime/amplitude). This feature came out after assessing the monotonicity of several other AE features. Windows of 2.5 minutes were used to calculate the proposed feature. An end-of-life threshold is necessary for the NHHSMM as it requires the last observation to be unique i.e. non hidden. On the contrary BFFANN does not require such a threshold which can be considered an advantage. The degradation histories kept for the training of both models were formed with data up to this threshold though in order to secure comparability. Moreover, the NHHSMM requires quantized data and to this direction a simple k-means clustering scheme was used (see [11] for details).

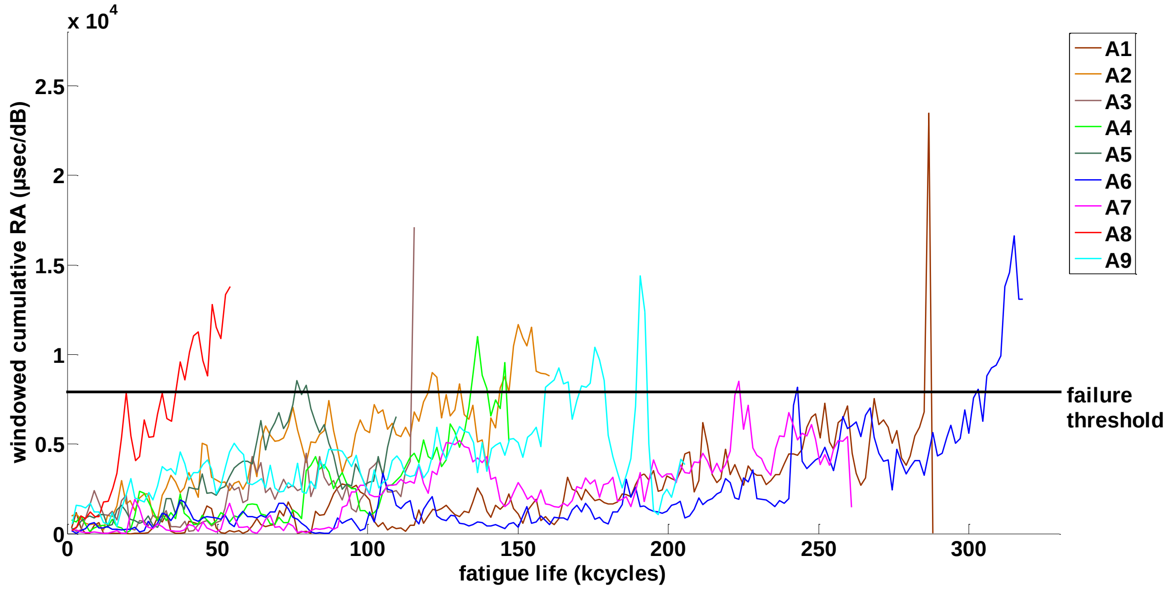

Figure 1: Degradation histories of 9 coupons in terms of windowed cumulative RA

The application of a k-fold leave-one-out cross-validation scheme in the case of the NHHSMM gives the parameters involved in $\boldsymbol{\theta}$=[$\boldsymbol{\Gamma}$,$\boldsymbol{B}$]. A similar training approach gives the distribution of the parameters **w** of the BFFANN.

The BFFANN was designed with two layers and 20 hidden neurons to model the relationship between the inputs and the output. Training of FLNN model was done within 20 epochs of scaled conjugate gradient (SCG) optimization method. An isotropic Gaussian prior for weights was assumed with an inverse variance of 0.1. Also the noise model was defined as a normal distribution with inverse variance hyperparameter equal to 0.1. Inference and prediction was done by using Gibbs sampler and 1000 steps of Hybrid Monte Carlo (HMC) algorithm [10].

The results of the application of the two machine learning algorithms in the degradation histories from coupons A2 and A9 are presented in Figures 2 and 3.

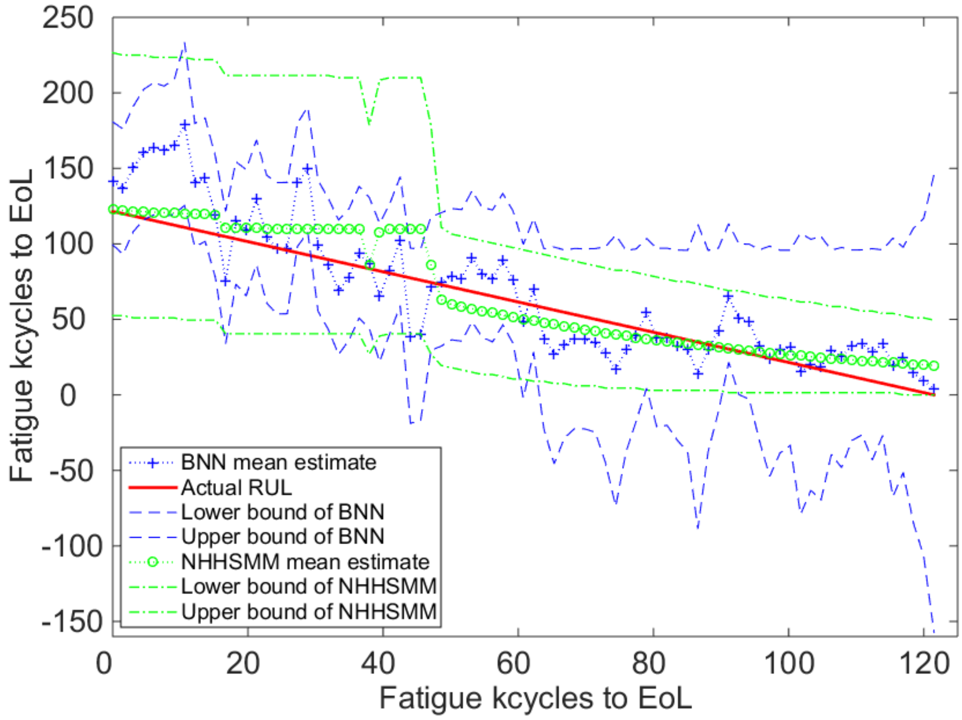

Figure 2: Prognostic results for coupon A2

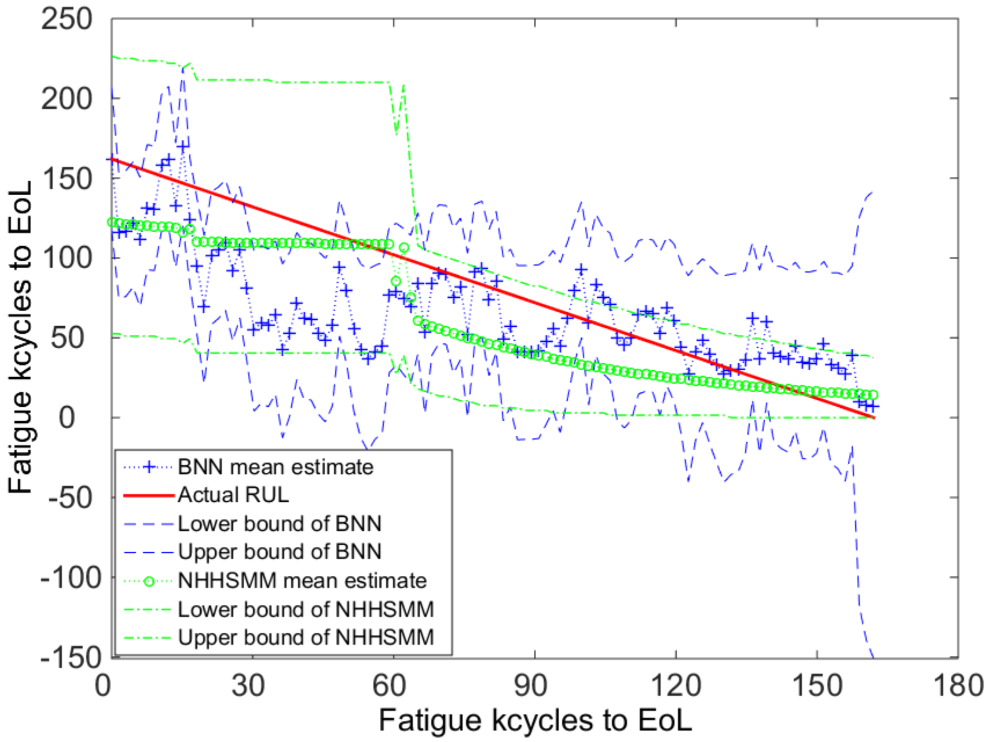

Figure 3: Prognostic results for coupon A9

4. PROGNOSTIC PERFORMANCE METRICS

For a more quantitative assessment certain prognostic performance criteria were utilized. Squared correlation coefficient (SCC), Mean Square Error (MSE), Precision, Cumulative Relative Accuracy (CRA) and Mean absolute percentage error (MAPER) were used as the basic prognostic performance criteria introduced in [11]. These criteria were measured between the predicted RUL and the actual RUL. Precision, MAPER and CRA are defined as:

$$\text{Precision}=\sqrt{\frac{\sum_{t=1}^{N}(y(t)-\hat{y}(t))^2}{N-1}}, \qquad \text{MAPER}=\frac{1}{N}\sum_{t=1}^{N}\left|\frac{100\cdot y(t)}{RUL_{actual}(t)}\right|, \qquad \text{CRA}=\frac{\sum_{t=1}^{N}RA(t)}{N}$$

where $y(t) = RUL_{actual}(t) - \widehat{RUL}(t)$ is the prediction error and $\hat{y}(t)$ the mean prediction error, RA is the relative accuracy at each point, $RA(t)=1-\left|\frac{RUL_{actual}(t) - \widehat{RUL}(t)}{RUL_{actual}(t)}\right|$, t=1:N is the temporal instants that recordings take place and N is the total number of recordings.

In tables 2 and 3 the performance metrics defined above are presented and highlighted to show which performs better in the comparison between the two.

Coupon	MSE	SCC	PRECISION	MAPER	CRA
A1	3.90×10^{9}	0.785	53.6×10^{-11}	59.1	0.41
A2	0.52×10^{9}	0.793	1.24×10^{-11}	57.5	0.42
A3	4.65×10^{9}	0.500	24.7×10^{-11}	285.5	-1.85
A4	3.40×10^{9}	0.662	7.62×10^{-11}	142.3	-0.42
A5	5.44×10^{9}	0.873	5.45×10^{-11}	219.2	-1.19
A6	5.87×10^{9}	0.656	58.4×10^{-11}	55.17	0.45
A7	1.74×10^{9}	0.65	5.86×10^{-11}	117.1	-0.17
A8	5.21×10^{9}	0.857	5.05×10^{-11}	299.0	-1.99
A9	1.19×10^{9}	0.559	6.58×10^{-11}	61.4	0.39

Table 2. Prognostic performance metrics for BFFANN results

Coupon	MSE	SCC	PRECISION	MAPER	CRA
A1	27.4×10^{9}	0.748	110×10^{-11}	99.95	5.5×10^{-4}
A2	4.95×10^{9}	0.908	10.7×10^{-11}	99.91	9.5×10^{-4}
A3	4.47×10^{9}	0.864	5.51×10^{-11}	99.86	14×10^{-4}
A4	6.1×10^{9}	0.743	16.0×10^{-11}	99.86	14×10^{-4}
A5	1.96×10^{9}	0.815	5.14×10^{-11}	99.75	25×10^{-4}
A6	31.3×10^{9}	0.689	74.7×10^{-11}	99.94	5.8×10^{-4}
A7	16.7×10^{9}	0.918	40.7×10^{-11}	99.93	6.7×10^{-4}
A8	0.48×10^{9}	0.851	0.44×10^{-11}	99.58	42×10^{-4}
A9	8.78×10^{9}	0.905	7.6×10^{-11}	99.94	6.2×10^{-4}

Table 3: Prognostic performance metrics for NHHSMM results

5. CONCLUSIONS

A novel framework was proposed that utilizes AE data for fatigue damage prognostics in composite materials. A stochastic multi-state degradation approach (NHHSMM) versus a more conventional soft computing approach (BFFANN) were implemented to this direction.

The methodology was validated in coupon level after a fatigue test campaign. The two algorithms differ significantly in their philosophy and operation. NHHSMM is a multi-state left-to-right generalized Hidden Semi Markov Process which takes into account the ageing of the asset/components it tries to model. BFFANN attempts to find the nonlinear relationship between observations i.e. SHM data and RUL. BFFANN does not depend on an end-of-life threshold which is generally desirable as opposed to NHHSMM. Concerning the RUL predictions, special metrics were utilized to assess the prognostic algorithms performance. None of the two algorithms clearly excels but the NHHSMM gives more coherent predictions with less fluctuations and with confidence intervals that close as time passes and more data come into play. The BFFANN on the other hand, despite seeming to provide with better estimates towards the end-of-life, its confidence intervals deviate increasingly, which is definitely not desirable for a robust prognostic algorithm. Both approaches require training data and apparently the more the historical data available the better. Considering their generalization, both procedures can be extended to structures of higher than coupons complexity. Moreover, any type of SHM data that can form highly monotonic features can be successfully utilized for prognostics.

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