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Spectral Finite Element Method in Condition Monitoring and Damage Detection

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Abstract

It is well known that the dynamic behaviour of engineering structures may carry very important and crucial information that can be further used for the assessment of their condition as well as detection of any damage induced. The current interest in monitoring techniques based on the propagation of guided elastic waves requires that numerical techniques used for modelling the phenomena associated must shift into the realm of high frequency dynamics in contrast to traditional low frequency dynamic modal analysis. However, high frequency dynamics may present serious difficulties, which may have their origin in the numerical approach used. In that light the aim of the current work is to present certain results obtained by the authors related to two different spectral finite element approaches (frequency-domain and time-domain) for condition monitoring and damage detection of various engineering structures including simple 1-D and 2-D structures as well as complex 3-D structures. Also an effective numerical technique based on the application of an absorbing layer with increasing damping (ALID) for propagation of guided elastic waves in infinite or semi-infinite engineering structures of complex geometries is presented. Its concept has been not only presented by the authors, but certain relations between the layer properties and the characteristics of propagating elastic waves have been given that can help to maximise the layer performance in terms of its damping capability. Beside the assessment of the effectiveness of the numerical techniques used by the authors various numerical issues and problems that can be encountered during numerical simulations and that can strongly influence their accuracy or even determine their existence are discussed and presented.

Key words: Guided waves (Lamb waves), Condition monitoring, Vibration analysis, Modelling and simulation, Spectral Finite Element Method, Numerical analysis.

1 INTRODUCTION

Condition monitoring of engineering structures is very closely related to damage detection despite the fact that the former aims rather at the observation of structural behaviour in order to monitor certain parameters that can be closely correlated with the proper operation of these structures, these being: vibration, temperature, stress, deformation, and many more [1]. In that context damage detection makes one step more and seeks for any abnormalities in these parameters trying to associate them with the presence or development of damage in various forms. Both approaches are very strongly backed up by various experimental and numerical techniques trying to validate their findings as well as to increase their effectiveness and accuracy [2].

A special branch of the techniques that can be used for condition monitoring and damage detection in engineering structures are various techniques based on the propagation of guided elastic waves and their subsequent interaction with damage [3]. The development of very precise Laser Scanning Doppler Vibrometry (LSDV) in 1-D and 3-D in past decades together with the development of appropriate numerical techniques represented by the Spectral Finite Element Method (SFEM) resulted in a very rapid increase in this particular field of research, which can be illustrated by the results of numerical calculations presented in Fig. 1.

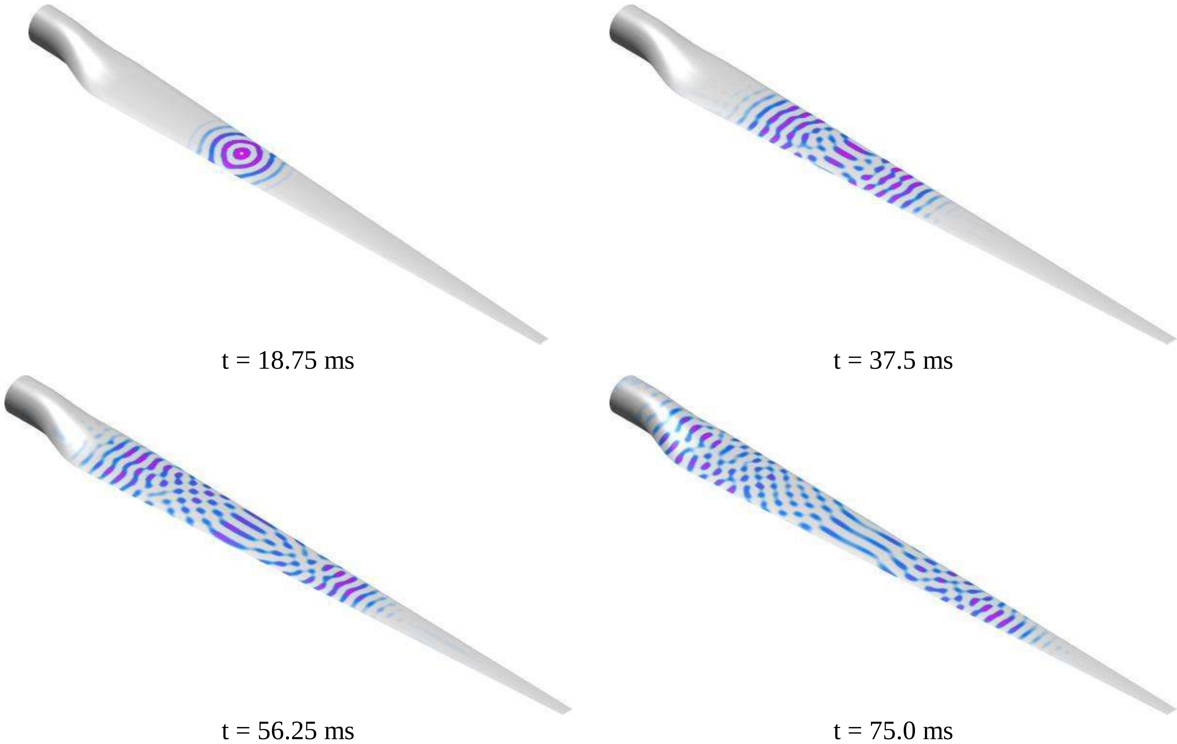

Figure 1: Calculated patterns of propagating guided elastic waves in a composite wind turbine blade with a fatigue crack (crack located near the blade root) according to a six-mode theory of shells [3] – results of numerical simulation obtained by TD-SFEM for a 5 sine pulse excitation signal of 15 kHz modulated by the Hann window.

2 SPECTRAL FINITE ELEMENT METHOD

SFEM is a numerical technique that is particularly well suited to study and investigate high frequency dynamics of engineering structures, especially the wave propagation phenomena. It is based on orthogonal shape functions that can take form of orthogonal Lobatto or Chebyshev polynomials, in the case of the Time-domain Spectral Finite Element Method (TD-SFEM) [4], or trigonometric functions representing analytical solutions (real or complex), in the case of the Frequency-domain Spectral Finite Element Method (FD-SFEM) [5], also known in the literature as the Fast Fourier Transform (FFT) based SFEM. Thanks to that the convergence rate of TD-SFEM is exponential with the degree of approximation polynomials, while for FD-SFEM with the number of FFT points.

Since SFEM is based on the classical Finite Element Method (FEM), when it comes to the discretisation of the domain under investigation, various damage modelling techniques known and used in FEM can be easily adopted [6, 7]. However, as every numerical technique SFEM in both time-domain and frequency-domain formulations, is not free of its drawbacks, as it is summarised and presented in Tab. 1.

FD-SFEM	TD-SFEM
• Structural elements can be modelled by one spectral finite element	• Structural elements require uniform meshing by many spectral finite elements
• Very low computational cost / fast computations	• Relatively high computational cost / relatively slow computations
• Suitable for 1-D problems of simple geometries	• Suitable for 1-D, 2-D and 3-D problems of complex geometries
• Simple modelling of structural damage	• Simple modelling of structural damage
• Suitable for lower order theories of structural elements / higher order theories difficult to implement	• Suitable for all available theories of structural elements / higher order theories easy to implement
• Requires special elements to model and analyse structures of finite dimensions	• Natural modelling and analysis of structures of finite dimensions
• Natural modelling of infinite or semi-infinite structures thanks to through-off elements	• Requires special elements to model and analyse structures of infinite or semi-infinite dimensions
• Prone to problems related with FFT properties	• Prone to problems related with inertia matrix integration quadrature
• Extension to 2-D or 3-D problems very difficult	• Time integration schemes may be unstable
	• Reduced quality of results in the regime of high frequencies due to underestimation of inertia properties by GLL quadrature
	• Reduced quality of results in the regime of high frequencies due to periodic properties of numerical models

Table 1: Frequency-domain Spectral Finite Element Method vs. Time-domain Spectral Finite Element Method.

2.1 Frequency-domain Spectral Finite Element Method

FD-SFEM appears as a fast computational method thanks to the application of FFT and very accurate in the case of simple 1-D structures of simple geometries or material properties. In most cases only one spectral finite element is required to model entire structural element, which greatly helps to reduce the computational cost. Due to the properties of FFT itself FD-SFEM requires a special class of spectral finite elements that are called through-off elements and that help to avoid signal leakage from surrounding time windows, associated with FFT, and to propagate signal energy outside the element [7]. These special elements must be derived analytically, therefore the application of FD-SFEM is usually limited to simple and lower order theories of structural elements, such as elementary single-mode or two-mode Mindlin-Herrmann rod theories, as well as classical single-mode or two-mode Timoshenko beam theories. The application of FD-SFEM to analyse wave propagation phenomena in 2-D structural elements presents a serious difficulty due to complicated analytical calculations that are required at every stage of the element definition and therefore FD-SFEM in 2-D is not widely reported in the literature. However, it should be said that the special through-off elements used by FD-SFEM in a natural way help to model infinite or semi-infinite structures.

2.2 Time-domain Spectral Finite Element Method

TD-SFEM, when compared to its FD-SFEM counterpart, appears as a substantially slower numerical technique, since structural elements are meshed in a traditional manner, typical for FEM, with a spatial density resulting from the characteristics of propagating waves [8]. However, TD-SFEM easily deals not only with 1-D or 2-D but also 3-D structures of complex geometries. TD-SFEM requires no through-off elements, since signal leakage problems do not exist there, but modelling of infinite or semi-infinite structures causes problems, especially that the analysis of wave propagation requires the same mesh density within the whole structure under investigation. As a solution come various numerical techniques of perfectly match layers (PML) [8, 9], but they require to combine the frequency and time domain formulations and therefore are not always suitable. An alternative can be absorbing layers with increasing damping (ALID) [10], but in this case a certain problem can arise how to characterise the layer properties in the most general manner. Finally, a serious issue may represent a numerical model itself. Since TD-SFEM employs spectral finite elements to build up a model of the structure under investigation, in exactly the same manner as FEM, the model must carry some information about the properties of the elements used. One of such properties is the degree of approximation polynomial, which is directly correlated with the level of discontinuity of the stress fields on the spectral finite element boundaries. These discontinuities, having periodic or near periodic nature, may significantly influence the model behaviour in the regime of high frequency dynamics, which is the realm of wave propagation phenomena [11]. For this reason it is very important to ensure that the numerical models employed are not affected by their periodic properties, as they can significantly influence results accuracy introducing additional dispersion or even determine result existence.

At this place it should be mentioned that the use of Lobatto and Chebyshev polynomials have also a certain influence of the quality of numerical results. Since Lobatto polynomials are orthogonal with no weight [12] their application results in the diagonal distribution of inertia properties within the characteristic inertia matrix of spectral finite elements in the case of 1-D or 2-D. In order to achieve that a special integration rule is used, known as the Gauss-Lobatto-Legendre (GLL) quadrature, which underestimates the inertial properties due to its accuracy [13]. Moreover, the application of GLL quadrature in the case of 3-D spectral finite elements ignores the existence of off-diagonal coupling terms, which additionally underestimates the distribution of inertia properties. The effect of this underestimation manifest strongly just in the regime of high frequency dynamics. Chebyshev polynomials are orthogonal with a certain weight [13] and as a consequence of that the distribution of inertia properties within the characteristic inertia matrix must contain off-diagonal elements in 1-D, 2-D and 3-D. Additionally, the integration rule used in that case is exact based on the Gauss-Legendre quadrature [3].

3 NUMERICAL STUDIES

It should be noted that an increasing demand for higher computation accuracy results in larger numerical models that are expected to provide more reliable data of computer simulations, which next can be used for testing various condition monitoring or damage detection algorithms. Unfortunately, such an approach not always leads to a desired direction, since all numerical models are inherently inaccurate due the fact that they are based on certain more-or-less clear modelling assumptions. In that context as a very important issue appears the order of the theories [14-16] of structural elements employed by such numerical models. This reveals as an extremely important factor in the case of both SFEM formulations, since the method operates in a very wide range of frequencies and high frequency regimes, which are required for the observation of the phenomena associated with the propagation of guided elastic waves is structural elements.

2.2 TD-SFEM for propagation of guided elastic waves in 1-D structures

Historically, as the first come numerical models used to investigate dynamic behaviour and wave propagation in the case of various 1-D structures built up of rods, shafts and beams, that were formulated based on elementary or classical single-mode theories [14].

However, simple single-mode 1-D theories are of limited use, since they are characterised by very low accuracy in the regime of high frequencies, although they can be successfully employed to study low frequency dynamic behaviour. Moreover, it should be strongly emphasised that the propagation of guided elastic waves is not only multi-mode, but also dispersive, in its nature. This means that with an increase in the frequency an increasing number of wave propagation modes takes part in the wave motion as well as the shapes of propagating wave packets become gradually distorted over the distance travelled. The existence of these modes result from a coupled interaction between structural boundaries and two primary waves present in 3-D unbounded media, these being the longitudinal and shear waves [3].

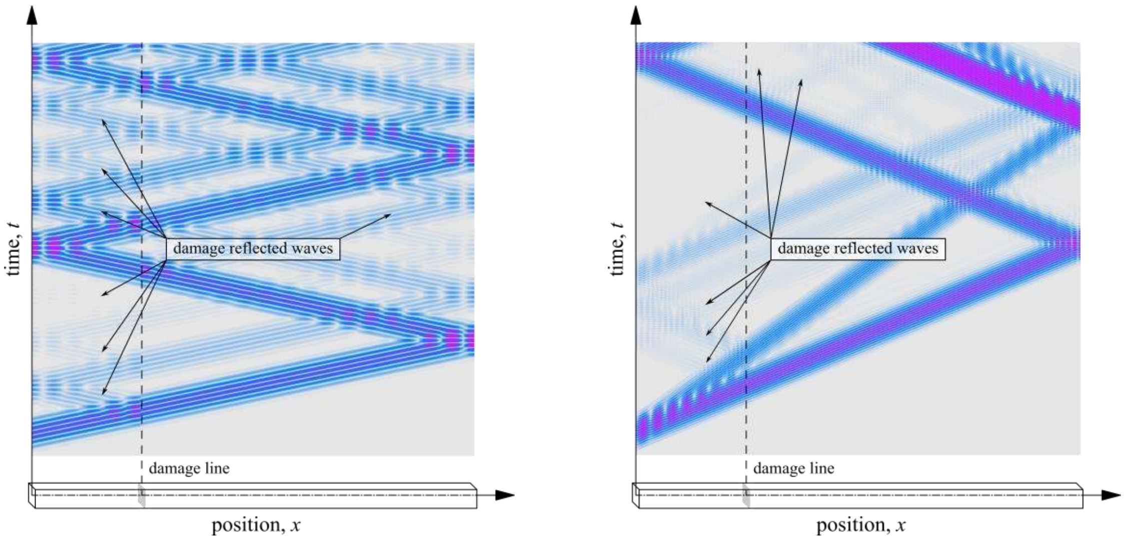

Figure 2: Calculated patterns of propagating guided elastic waves in an aluminium bar with damage – results of numerical simulation obtained by TD-SFEM according to: single-mode Love theory for longitudinal symmetric waves in rods [14], for a 5 sine pulse excitation signal of 15 kHz modulated by the Hann window (left), four-mode theory for flexural antisymmetric waves in beams [15], for a 10 sine pulse excitation signal of 100 kHz modulated by the Hann window (right).

The multi-mode nature of elastic wave propagating within structural elements requires that all possible types of wave motion, within a given range of frequencies excited, always take part in that motion and that the modes can mutually exchange one into another. This means that any type of structural discontinuity (boundary or damage) results in the generation of all possible wave propagation modes, however, their amplitudes may be different. It is interesting to note that mode conversion at structural discontinuities may be used a sensitive damage indicator in condition monitoring or damage detection in order to differentiate transverse and longitudinal cracks or to estimate their orientation as well as in the case of structural elements made out of laminated composite materials to differentiate cracks from delaminations [3].

The issues discussed above are illustrated by Fig. 2. It can be seen that in the case of the longitudinal symmetric waves propagating within the bar (left) the employed Love theory of rod behaviour allows for a single wave propagation mode. Its interaction with damage results in the generation of reflected and transmitted waves. Their amplitudes and the moments they reach certain measurement points may function as indicators of the damage position and intensity [3]. Unfortunately, this image becomes somehow blurred when a greater number of wave propagation modes takes place in wave motion because of interchanging symmetric longitudinal to antisymmetric flexural modes of wave motion and in response to a change in the range of frequencies excited (right).

2.3 TD-SFEM propagation of guided elastic waves in 2-D and 3-D structures

Mode conversion resulting from the presence of various types of structural discontinuity (boundary or damage) becomes much more prominent in the case of the analysis of 2-D structures. However, in the case of 3-D structures an additional issue can arise from the presence of mode coupling due to due to geometry (local curvature).

Apart from fast signal attenuation, which is contrary to 1-D structures, an increased number of wave propagation modes travelling within 2-D and 3-D structures under investigation, strongly influences the effectiveness, robustness and accuracy of any algorithms intended to be applied for condition monitoring or damage detection purposes and which are based on the results of numerical simulation by TD-SFEM [3], as seen in Fig. 3.

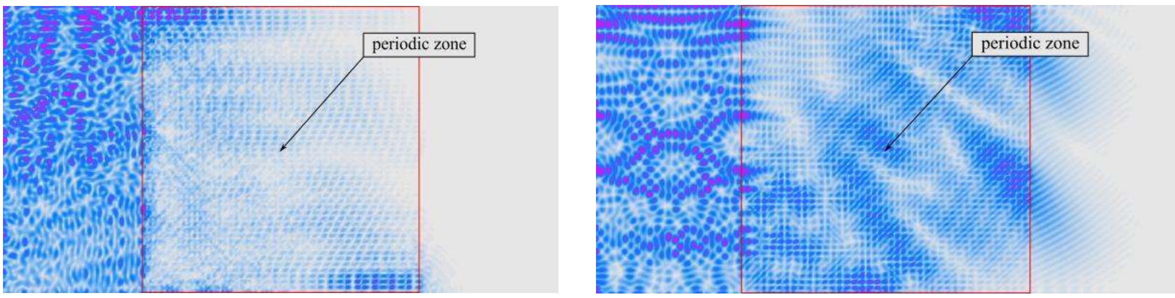

Figure 3: Calculated patterns of propagating guided elastic waves in a semi-periodic aluminium panel – results of numerical simulation obtained by TD-SFEM according to: two-mode elementary theory for longitudinal symmetric waves in membranes [3], for a 8 sine pulse excitation signal of 154 kHz modulated by the Hann window (left), three-mode Mindlin-Reissner theory for flexural antisymmetric waves in plates [3], for a 8 sine pulse excitation signal of 75 kHz modulated by the Hann window (right).

When the effects arising from attenuation, mode conversion and coupling become significant to a degree that practically prevents direct use of the information carried out by propagating elastic waves, an alternative approach must be sought in order to extract damage related features from wave signals obtained either numerically by TD-SFEM or experimentally by LSDV. An interesting solution may present methods based on the calculation of the energy carried out by propagating guided elastic waves [17]. Out of many energy indicators as a very effective one appears the root mean square (RMS) due to its simple and fast numerical calculation.

Damage detection techniques utilising RMS calculation, realised either based on computer simulation results by TD-SFEM or experimental measurements by the use of LSDV, prove to be immune to the problems described above and related to mode conversion and coupling.

An appropriate weighting technique additionally helps to compensate for signal attenuation [17]. This is well illustrated by the results of numerical simulations presented in Fig. 4. It can be seen from Fig. 4 that an appropriate weighting applied to the calculated RMS patters, based on the former propagation patters of guided elastic waves, help to sharpen the image obtained, clearly indicating all damage related features despite the above mentioned issues arising from the attenuation, mode conversion and coupling.

Figure 4: Calculated weighted RMS patterns of propagating guided elastic waves in: a composite wind turbine blade with a crack (left), an aluminium wing section with damage (right), shell structure according to a six-mode theory of shells [16] – results of numerical simulation obtained by TD-SFEM for a 8 sine pulse excitation signal of 75 kHz modulated by the Hann window.

3 CERTAIN NUMERICAL ISSUES

Solving practical engineering problems related to condition monitoring or damage detection by the use of such numerical tools as FD-SFEM or TD-SFEM requires a deep knowledge about the problem nature as well as the tool characteristics. As mentioned before in the majority of engineering problems the application of TD-SFEM seems to have better grounds due to the method versatility and robustness, especially in the case of 2-D and 3-D problems, when compared to FD-SFEM.

Investigation of high frequency dynamics or guided elastic wave propagation problems in engineering structures by TD-SFEM requires an appropriate numerical representation of the structure under consideration in order to ensure that all details of wave propagation phenomena are appropriately captured. This enforces spatial discretisation on the same or higher order that the characteristic dimensions of damage related features. When the geometrical dimensions of the structure are much larger than the characteristic dimensions of damage related features then the resulting numerical models tend to be impractically big.

3.1 Finite vs. infinite structures

FD-SFEM is a natural way is well suited to deal with semi-infinite or infinite engineering structures [5, 7]. In the case of TD-SFEM a special type of non-reflecting boundary must be used in order to mimic appropriately the dynamic behaviour of the structures at large distances from excitation points. This can be achieved by the application of ALID [10], which is in fact a layer of spectral finite elements characterised by increasing damping properties, which are well-tuned to the characteristics of propagating elastic waves.

This can be well seen in Fig. 5, which presented the application of ALID for a problem of the propagation of guided elastic waves in a semi-infinite U-shape aluminium shell structure. It can be seen that when the properties of ALID are well choses the amplitudes of the signal reflected form ALID boundaries can be neglected, which corresponds to the behaviour of boundaries at infinity. A technical problem may present the selection of ALID characteristics, however, this problem is discussed in details in [10]. It can be summarised that ALID lengths stays at least equal to the length of the longest propagating elastic waves, while the values of the layer scaling parameters α and β are selected as equal to 6 and 1, respectively. However, for better quality results of numerical simulations it is recommended to use ALID layers twice this length with the values of the scaling parameters α and β equal to 7 and 3, respectively [12].

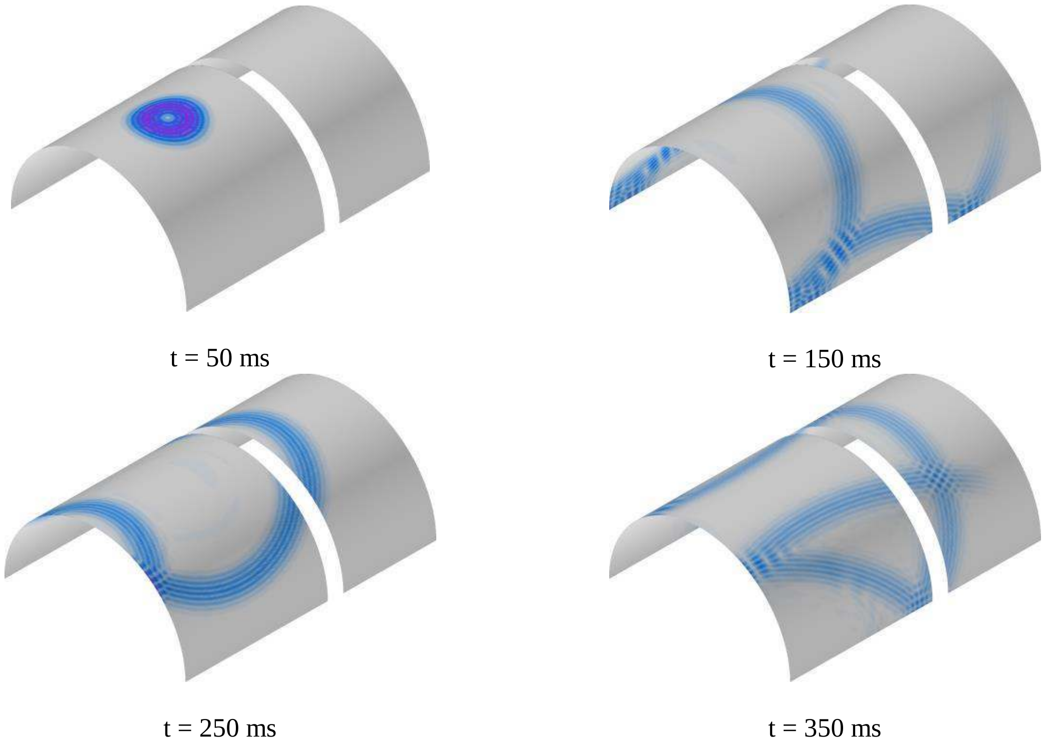

Figure 5: Calculated patterns of propagating guided elastic waves in a semi-infinite U-shape aluminium shell structure, with the use of ALID, according to a six-mode theory of shells [16] – results of numerical simulation obtained by TD-SFEM for a 8 sine pulse excitation signal of 75 kHz modulated by the Hann window.

3.2 Periodic properties of numerical models used

It is well known that numerical solutions obtained by the use of FEM are continuous over the whole domain under consideration, however, they are not smooth. This is due to the fact that the continuity of solution derivatives, represented by stress/strain fields within finite elements is not required as a condition that guaranties the convergence of FEM [3]. The same can be said about TD-SFEM. The discontinuity of stress/strain fields results in certain periodic properties of numerical models, which properties can manifest themselves at certain frequency regimes. On the other hand may have a great influence on the quality, accuracy or even existence of numerical solutions obtained by TD-SFEM.

The investigation of propagation of guided elastic waves in structural elements by TD-SFEM requires very fine and uniform spatial discretisation, which additionally increases the intensity of model periodic properties. This is kind of behaviour is omnipresent, but its strongest manifestation can be observed in 1-D case, as presented in Fig. 6.

In 1-D case the frequency spectrum of the numerical model obtained by TD-SFEM is effectively divided into a number of discontinuous regions/spectra, which number is equal to the degree of approximation polynomial p. This regions are separated from each other by certain frequency band-gaps, within which there are no natural vibrations – no elastic waves can propagate within the frequencies specified by the gaps. In 2-D and 3-D case the observed behaviour is more complicated and has an additional directional nature, which indicates or forbids certain directions of wave propagation, is a similar sense as presented in Fig. 6.

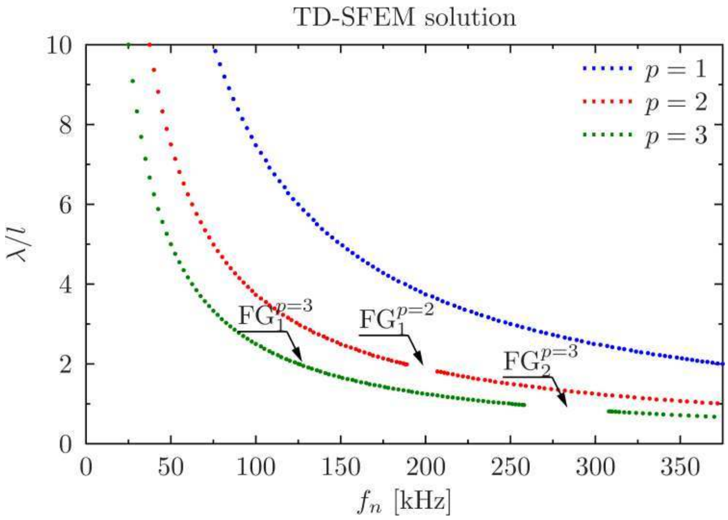

Figure 6: Frequency band gaps of a 1-D periodic medium, calculated numerically by the TD-FEM, for various degrees of approximation polynomials p according to the elementary theory of rods [14], based on Chebyshev node distribution.

As a consequence of that results of numerical computations by FEM or TD-SFEM can be considered as polluted by the above mentioned model periodic properties. Therefore the existence of frequency band-gaps or their directional properties should be closely examined in the case of analysis of structural dynamics at high frequency regimes, such as is the analysis of the propagation of guided elastic waves in order to ensure the highest quality of numerical results obtained. This is of special importance in the case of condition monitoring or damage detection.

9 CONCLUSIONS

In the current paper certain aspects of the application of SFEM in condition monitoring and damage detection have been discussed. It has been shown by the authors that the spectral approach based on the classical FEM formulation appears as an effective and robust numerical tool to investigate problems related to the propagation of guided elastic waves in structural elements or high frequency dynamic responses. Both time-domain and frequency domain SFEMs have been compared and their strength and weaknesses have been presented.

Also certain results of numerical simulations by the use of TD-SFEM have been presented illustrating the versatility of the method, especially in the context of damage detection in shell-like structures, based on the propagation of guided elastic waves. The paper also discusses well-known issues and problems that arise from the multi-mode nature of propagating guided elastic waves as well as the mode conversion and coupling or signal attenuation. In order to overcome the problems resulting from these wave characteristics an effective signal processing technique utilising the signal energy rather than its amplitude has been proposed. Also a numerical technique known as ALID for dealing with semi-infinite or infinite engineering structures has been shown and discussed. Additionally the authors have pointed out on certain numerical features of TD-SFEM resulting from its application in the regime of high frequencies, which may arise from the periodic nature of numerical discrete models used. It has been demonstrated that in some cases the intensity of this periodic behaviour may be prominent enough to dominate structural behaviour either polluting and falsifying calculated structural responses.

10 ACKNOWLEDGEMENTS

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