Applying Extreme Value Theory for alarm and warning levels setting under variable operating conditions

Daniela TOSHKOVA[1], Nicholas LIEVEN[1], Peter MORRISH[2], Paul HUTCHINSON[2]
[1]Dept of Aerospace Engineering, University of Bristol, Queens Building, University Walk, Bristol BS8 1TR daniela.filcheva@bristol.ac.uk
[2] Beran Instruments, Hatchmoor Industrial Estate, Torrington, Devon EX38 7HP

Key words: Condition monitoring, Variable operation conditions, Automatic alarm setup

Abstract
Alarm configuration is one of the main challenges of power generation and associated industries. The configuration challenge is compounded by machines being operated under variable conditions as a change in operational condition i.e. speed or torque affects the vibration response. Thus, if the data used to determine the alarm and warning threshold levels characterises only limited range of operational conditions a false alarm may be triggered indicating onset of a fault while only the operational regimes have changed. Another possibility is the fault to be masked by change in the operational condition which leads to misdetection. Central to determining the alarm and warning threshold levels is establishing the type of the data distribution. The distributions are usually assumed to be Gaussian even though a number of possible distributions should be considered in the search of the best fit. Incorrect distribution fit may result in sub-optimal alarm configuration. In the present paper instead of considering the whole data set only maxima will be taken into account as likely to reveal an outlier. The Generalised Extreme Value distribution is proposed as a possible limit distribution for the maxima. In order to take into account the effect of the variable speed, Extreme Value Theory for non-stationary processes will be applied. The suggested approach is validated on data from an experimental gearbox.

1. INTRODUCTION

Determining alarm level for the purposes of indicating degradation or functional failure is one of the main challenges in power generation and associated industries. Usually the number of machines and their monitored parameters are numerous and diverse. Setting and re-setting alarm thresholds for every parameter for each associated machine is both time consuming and laborious.

An alarm according to standard definition [1] is "An audible and/or visible means of indicating to the operator an equipment malfunction, process deviation, or abnormal condition requiring a response." The condition is usually perceived to be abnormal when a certain threshold level is exceeded. There are two severities of alarms – warning and alarm. When the warning (or alert) threshold is exceeded the component is usually monitored more closely and trend analysis is carried out. When higher alarm threshold is reached the machine is removed from operation for maintenance. Therefore, the setting of appropriate thresholds is of paramount importance. If the levels are set too low the system may set off an alarm while the component is in a healthy state leading to so called false alarms. If the thresholds are set too high the onset of a fault may not be indicated or noticed. These types of situations are described as misdetections. Therefore, of paramount importance is to keep the number of false alarm low whilst at the same time maximising the probability of detection.

Following the above principle, Cempel [2] suggested the alarm and warning levels should be determined by applying the Neyman-Pearson theory. The thresholds are determined by establishing the distribution density or the distribution function of the condition indicator which can be estimated with the help of Chebyshev's inequality [3]. Alternatively the distributions can be approximated by Weibull or Pareto distributions.

The same approach is applied in [4, 5] for gearbox condition indicators, which are fused into health indicators. In these papers, the authors build models of the data taking into account the variation of parameters across the operational fleet and the functional conditions. Based on the models, the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) are derived. From the CDF, the warning and alarm levels for a given Probability of False Alarms (PFA) are determined. In [4] the distribution is assumed to be Gaussian. Recognising that this is not valid in all scenarios, the authors fit in [5] Rayleigh, Gaussian and Nakagami distributions to the various condition and health indicators.

Given the range of applications and potential modes of failure there are a variety of distributions that can be considered. The authors of [6] generalise the types of data which are commonly logged in condition monitoring systems, namely vibration signal measures and symmetrical process values. The authors considered four candidate distributions: Weibull, Generalised Extreme Value (GEV), Extreme Value and inverse Gaussian. They observed that GEV is the best fit for both types of data. Furthermore, the authors suggest a method for automatic alarm and warning threshold setting based on a reference value. A reference value is derived from the distribution of parameters which are calculated for data under stationary operating conditions.

In [7] a novelty index is adopted as an indicator for onset of structural damage. A continuous probability distribution for a large data model incorporating variable environmental conditions is established by using the Smirnov-Kolmogorov test [8]. The confidence interval is defined by compromising between false alarms and misdetections. The CDF is then used to derive the alarm threshold.

Thus, returning to the alarm definition, more specifically the “abnormal condition”, it is no wonder that novelty detection, outlier detection or anomaly detection are used as condition indicators for structures [9] or rotating machinery [10]. The other issue is that establishing the probability of detection is extremely challenging – if not impossible – as the data availability for various fault modes is very restricted.

In [11] the novelty characteristic is identified by monitoring the mean and the variance of a feature. The data are divided into groups. The mean and standard deviation for each group are calculated. The limits are set at mean plus k times the standard deviation. Inherent with this approach is the assumption of Gaussianity of the distribution of the feature.

In his publication [12], Roberts suggests that EVT can be used to establish a novelty threshold. Instead of looking for the best-fit distribution of the original data we are looking at the distribution of the data extremes, which are likely to represent the abnormalities. According to Fisher and Tippett Theorem [13] there are only three possibilities for the distribution of maxima or minima: Gumbel, Fréchet or Weibull. These three forms may also be regarded as special cases of the GEV distribution. In the Roberts's publication only a Gaussian parent distribution is considered, whose domain of attraction is the Gumbel distribution.

In [10] the method is applied to gas turbine engine data. As the vibration response depends on the speed, the speed range is divided into bins and the model is built using only data within the bin. The Extreme Value parameter estimation requires large datasets in order to produce robust results. A Bayesian extension to Extreme Value Theory (EVT) is suggested, which takes into consideration the uncertainty in data extremes modelling. While developing the methodology for novelty threshold determination through Bayesian EVT the authors assumed that the data exhibit a Gaussian distribution but some of data were revealed to have a multimodal distribution. In order to overcome this drawback a multimodal extension was developed in [14]. The paper also considers the multivariate case. In [15] extreme function theory is introduced, where the functions are represented by a time-series of discrete observations.

Worden et al [9] extend the application of EVT for threshold setting realising its full potential. The authors recognised that the Gaussianity assumption is too limited for ubiquitous application. They demonstrate how the tails of various parent distributions can be modelled by Extreme Value Distributions (EVD). The methodology suggested there in for establishing the novelty measure threshold can thus be used to determine the alarm levels.

In the present paper, we consider a modified method for determining alarm and warning levels for condition indicators under variable operational conditions based on EVT for non-stationary processes is proposed.

2. DETERMINING THE ALARM AND WARNING LEVELS USING EXTREME VALUE STATISTIC FOR NONSTATIONARY DATA

The alarm and warning levels can be calculated from the inverse cumulative function. However, in practice usually the distributions of the underlying data are unknown. This difficulty can be avoided – instead of looking at the data – look at their maxima as this is where the outlier is likely to be revealed. Previous evidence in the literature indicates that there are only three types of possible distributions for the maxima or tail distribution: Fréchet, Weibull and Gumbel. The results were established by Fisher and Tippett [13] and later rigorously proven by Gnedenko [16].

2.1. Classical extreme value theory

2.1.1. Extremal Types Theorem

Let $X_1, X_2, \dots, X_n$ be a sample vector of independent and identically distributed variables with distribution F, the most appropriate statistic to study the tails is

$$M_n = max\{X_1, X_2, \dots, X_n\} \quad (1)$$

The distribution of M_n can be derived by:

$$\Pr(M_n \le z) = Pr\{X_1 \le z, X_2 \le z, \dots, X_n \le z\} = Pr\{X_1 \le z\} \times Pr\{X_2 \le z\} \times \dots \times Pr\{X_n \le z\} = \{F(z)\}^n \quad (2)$$

As distribution F is unknown, approximate families of models for F^n are considered which can be estimated using the extremal data only. However, $F^n \to 0$ as $n \to \infty$. In order to avoid the degeneration of M_n to a point, the variable M_n is renormalised as follows:

$$M_n^* = \frac{M_n - b_n}{a_n} \quad (3)$$

for sequences of constants $\{a_n > 0\}$ and $\{b_n\}$.

Theorem 2.1: If there exists a sequence of constants $\{a_n > 0\}$ and $\{b_n\}$ such that

$$Pr\{(M_n - b_n)/a_n\} \to G(z) \quad \text{as } n \to \infty \quad (4)$$

where G is a non-degenerate distribution function, then G belongs to one of the following families [17]:

GUMBEL (Type I): $G(z) = \exp\left\{-\exp\left[-\left(\frac{z-\lambda}{\delta}\right)\right]\right\}, -\infty < z < \infty$ (5)

FRÉCHET (Type II): $G(z) = \begin{cases} 0 & z \le \lambda \\ \exp\left\{-\left(\frac{z-\lambda}{\delta}\right)^{-\beta}\right\} & z > \lambda \end{cases}$ (6)

WEIBULL (Type III): $G(z) = \begin{cases} \exp\left\{-\left[-\left(\frac{z-\lambda}{\delta}\right)^{\beta}\right]\right\} & z < \lambda \\ 1 & z \ge \lambda \end{cases}$ (7)

where $\lambda > 0$ is a location parameter, δ is a scale parameter and β is a shape parameter.

The classical EV distributions Gumbel, Fréchet and Weibull can be combined into a single GEV family of distributions [18]:

$$G(z) = \begin{cases} \exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)^{-1/\xi}\right]\right\} & \xi \ne 0 \\ \exp\left\{-\exp\left[-\left(\frac{z-\mu}{\sigma}\right)\right]\right\} & \xi = 0 \end{cases} \quad (8)$$

where μ is the location parameter, σ is the scale parameter and ξ is the shape parameter.

Theorem 2.1 can be reformulated for the GEV distribution as follows:

Theorem 2.2: If there exists sequence of constants $\{a_n > 0\}$ and $\{b_n\}$ such that

$$Pr\{(M_n - b_n)/a_n\} \to G(z) \quad \text{as } n \to \infty \quad (9)$$

where G is a non-degenerate distribution function, then G belongs to one of the following families:

$$G(z) = \exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)^{-1/\xi}\right]\right\} \tag{10}$$

defined on $\{z : 1 + \xi\,(z - \mu)/\sigma > 0\}$, where the parameters satisfy $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$ respectively.

2.1.2. Determination of the Thresholds

Having obtained the estimates of the parameters for the appropriate EVD, the CDF is used to establish the confidence limits. The value of the 100α percentiles of the distribution is obtained through:

GUMBEL (Type I): $z = \lambda - \delta \ln(-\ln(\alpha))$ (11)

FRÉCHET (Type II): $z = \lambda + \delta(-\ln(\alpha))^{-1/\beta}$ (12)

WEIBULL (Type III): $z = \lambda - \delta(-\ln(\alpha))^{1/\beta}$ (13)

If the CDF is expressed in terms of the generalised EVD the levels are calculated as follows:

$$z = \begin{cases} \mu - \frac{\sigma}{\xi}\left[1 - \{-\ln(\alpha)\}^{-\xi}\right] & \xi \neq 0 \\ \mu - \sigma[1 - \{-\ln(\alpha)\}] & \xi = 0 \end{cases} \tag{15}$$

2.2. Non-stationary extreme value theory

In order to establish the threshold, the data set is divided into bins where the operating and/or conditions are stable. However, this assumption is compromised by uncertainty in the data. Alternatively, a large data model is built which accommodates the variability of the conditions or environment. The disadvantage is that this approach usually leads to increasing the frequency of false alarms as change in operating conditions may be interpreted as onset of a fault. In the present paper the data will be building a model where the change in the operating conditions will be taken into account. At the same time, the further extension of the EVT to non-stationary cases will be applied in order to construct variable alarm and warning levels.

2.2.1. Covariates

It is well recognised that vibration response is a function of the operating condition [19] i.e. the response changes in line with increase/decrease of speed or torque. These types of variables are referred to as covariates [20].

In the non-stationary case the parameters of the distribution change with time or covariate. The location parameter, which needs to be estimated can be described as follows:

$$\mu(t) = \beta_0 + \beta_t VOC(t) \tag{17}$$

where β_0 and β_t are constants and $VOC(t)$ is the variable operating condition. Similarly, the non-stationarity of shape parameter can be expressed through:

$$\sigma(t) = \exp(\beta_0 + \beta_t VOC(t)) \tag{18}$$

The positivity of σ for all values of t is ensured by using the exponential function. The shape parameters are difficult to estimate precisely. However, the uncertainty in their estimation is very weakly propagated in final determination of thresholds.

2.2.2. Calculation of alarm and warning threshold levels

The procedure for determining the alarm and warning levels follows the same logic as in section 2.1.2. The only difference is that we will have several values depending on the value of the operating condition. The parameters are estimated using the method of likelihood. However, instead of estimating the location parameter μ, the coefficients β_0 and β_1 are determined. The levels are calculated based on the inverse CDF:

$$z_{oc} = \begin{cases} \mu(t) - \frac{\sigma(t)}{\xi}\left[1 - \{-\ln(\alpha)\}^{-\xi}\right] & \xi \neq 0 \\ \mu(t) - \sigma(t)[1 - \{-\ln(\alpha)\}] & \xi = 0 \end{cases} \tag{19}$$

In this manner we will be able to obtain a set of thresholds for each of the operating condition state.

3. DATA ANALYSIS

The application will be demonstrated using data set from a Gearbox Test Rig where the controlled improper backlash was seeded as a fault. The Gearbox Test Rig (Fig. 1) is described in detail in [21]. Using the Test Rig 45 tests were conducted over a wide range of speeds and torques, representing a combination of 5 running speeds, 5 torques and 3 degrees of severity of backlash. The speed was adjusted respectively at 1, 3 and 5 Hz and stabilised at ±3%. The torque was harmonically varied and the average of the torque was set at 0%, 25%, 50%, 75% and 100% of the nominal torque respectively, with the average amplitude being maintained within ±5%. The tests were conducted for 3 degrees of severity of backlash. Between 10 and 15 datasets were taken for each combination of speed, torque and backlash severity. The sampling frequency was 3200 Hz. Within this paper, only the two datasets without and maximum improper backlash will be considered. Variation in the speed the control variable (representing operating condition) will be introduced whilst the torque will be kept constant.

For the present publication the time synchronous average vibration signal is used. The signal consisted of 3076 data points which are divided into blocks of 32 data points and the maximum for the block was established. Utilising the newly formed signal, the parameters of the distribution were estimated using maximum likelihood. The parameters were estimated using *ismev* package [22], which was created for statistical modelling of extreme values. *ismev* is an R package [23], which is an open source software environment for statistical computing and graphics. Once the parameters are estimated, the thresholds for warning and alarm were determined.

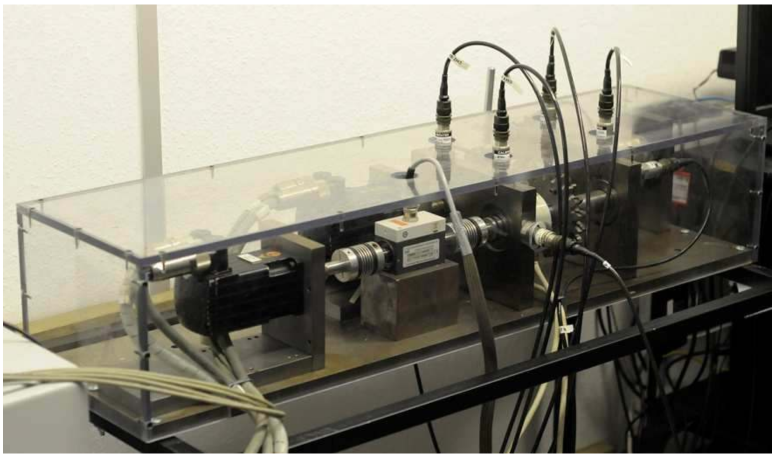

Figure 1: Instrumented gear test rig

The signal which was used for building the model is shown in Fig.2. In Fig. 3 the alarm threshold level is shown together with vibration response where there is no fault. Fig. 4 demonstrates the applicability of the suggested approach where it can be seen that the alarm is set off for the improper backlash fault.

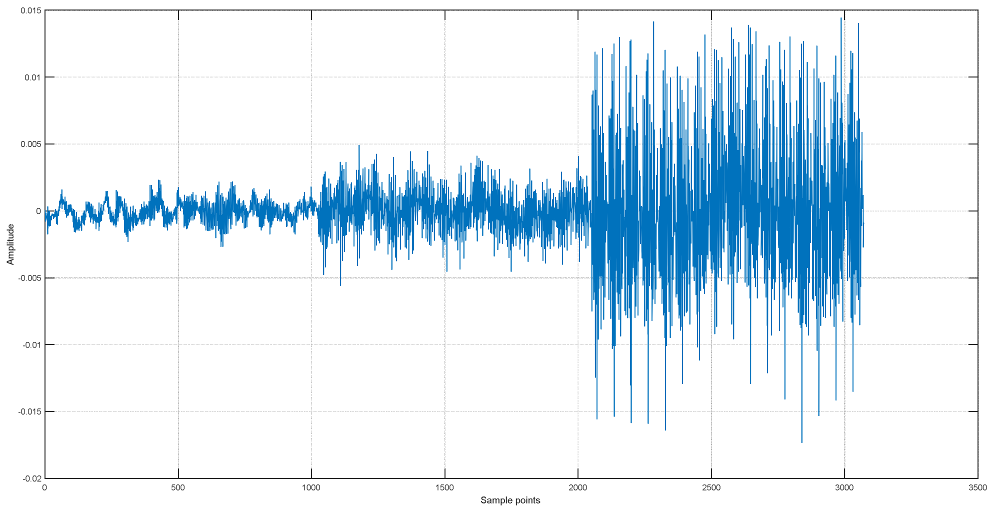

Fig.2 Dataset for establishing the parameters of the distribution

4. CONCLUSION AND DISCUSSION

The work presents potential solution of the problem of establishing warning and alarm thresholds. The non-stationary GEV theory has been applied for the calculation of the thresholds. It has demonstrated that the suggested method can be used for determining the warning and alarm thresholds under variable operating conditions. However, the uncertainty in building of a statistical model and estimating the parameters has to be taken into account. It is challenging to establish the distribution in the tails using time series even if it is time synchronous signal. Using features other than time series might produce more robust results.

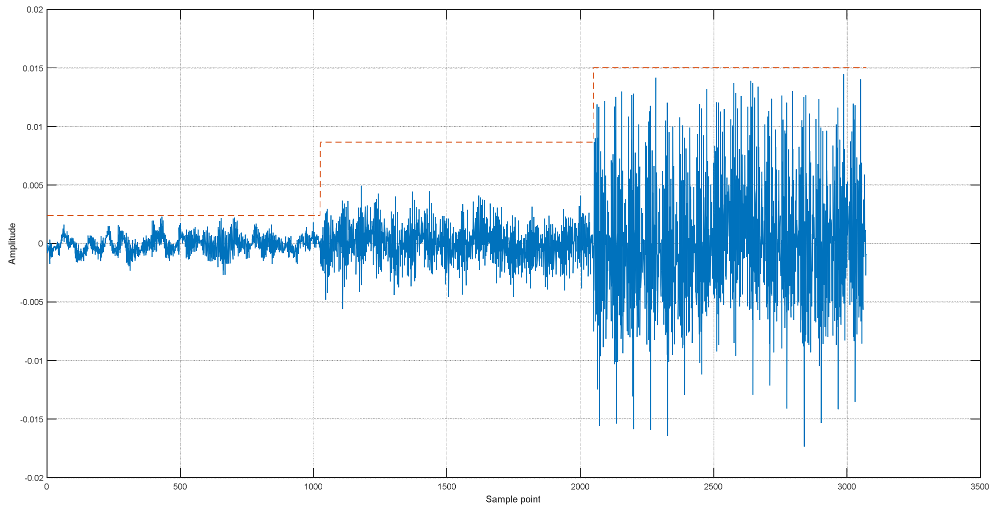

Figure 3 Signal for “healthy” state and the alarm level

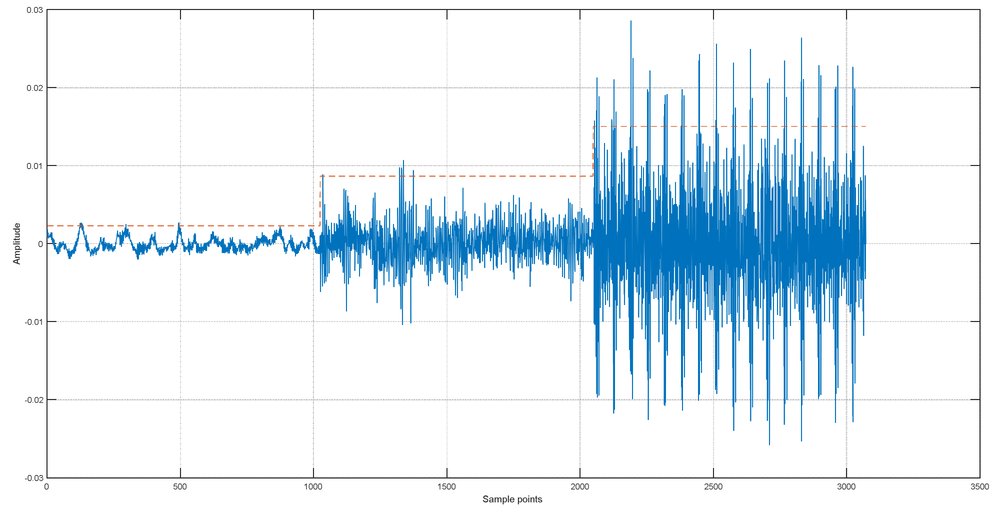

Figure. 4 Signal with improper backlash and the alarm level

Further the approach will be developed for more than one covariate. Another advancement will be related to see how the uncertainty of parameter estimates influences the number of false alarms and how this effect might be mitigated.

REFERENCES

[1] ANSI/ISA-18.2-2009 Management of Alarm Systems for the Process Industries (ISA-18.2) standard, https://www.isa.org/store/ansi/isa-182-2016,-management-of-alarm-systems-for-the-process-industries/46962105

[2] C. Cempel, Limit value in the practice of machine vibration diagnostics. *Mechanical Systems and Signal Processing*, **4(6)**, 483–493, 1990

[3] Kolmogorov. Chapter 2 “*Elements of the Theory of Functions and Functional Analysis*”, A. N. Kolmogorov, S.V. Fomin. Dover Publications ISBN 9780486406831 (1999)

[4] E. Bechhoefer, A. P. F. Bernhard. Setting HUMS Condition Indicator thresholds by modeling aircraft and torque band variance. *IEEE Aerospace Conference Proceedings*, **6**, 3590–3595, (2004)

[5] E. Bechhoefer, A. P. F. Bernhard. A generalized process for optimal threshold setting in HUMS. *IEEE Aerospace Conference Proceedings*, 1-9, 2007

[6] A. Jablonski, T. Barszcz, M. Bielecka, P. Breuhaus. Modeling of probability distribution functions for automatic threshold calculation in condition monitoring systems. *Measurement: Journal of the International Measurement Confederation*, **46(1)**, 727–738, 2013

[7] H. F. Zhou, Y. Q. Ni, J. M. Ko. Structural damage alarming using auto-associative neural network technique: Exploration of environment-tolerant capacity and setup of alarming threshold. *Mechanical Systems and Signal Processing*, **25(5)**, 1508–1526, 2011

[8] Yakir. Chapter 6 "*Extremes in Random Fields: A Theory and its Applications*", B. Yakir. John Wiley & Sons ISBN: 9781118720608 (2013)

[9] K. Worden, D. W. Allen, H. Sohn, C. R. Farrar. Damage detection in mechanical structures using extreme value statistics. *Proceedings of SPIE - The International Society for Optical Engineering*, **4693**, 289–299, 2002

[10] D. A. Clifton, L. Tarassenko, N. McGrogan, D. King, S. King, P. Anuzis. Bayesian extreme value statistics for novelty detection in gas-turbine engines. *IEEE Aerospace Conference Proceedings*, 2008

[11] M. Fugate, H. Sohn, C. R. Farrar. Vibration-Based Damage Detection Using Statistical Process Control. *Mechanical Systems and Signal Processing*, **15**, 707–721, 2001

[12] S. J. Roberts. Novelty detection using extreme value statistics. *IEE Proceedings - Vision, Image, and Signal Processing*, **146(3)**, 124, 1999

[13] R. A. Fisher, L.H.C. Tippett. Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Mathematical Proceedings of the Cambridge Philosophical Society*, **24(2)**, 180-190, 1928

[14] D.A. Clifton, S. Hugueny, L. Tarassenko. Novelty detection with multivariate extreme value statistics. *Journal of Signal Processing Systems*, ***65*(3)**, 371–389, 2011

[15] D. A. Clifton, L. Clifton, S. Hugueny, D. Wong, L. Tarassenko. An extreme function theory for novelty detection. *IEEE Journal on Selected Topics in Signal Processing*, ***7*(1)**, 28–37, 2013

[16] B. Gnedenko. Sur la distribution limite du terme maximum d'une série aléatoire. *Annals of Mathematics*, **44(2),** 423–453, 1943

[17] Farrar. Chapter 6 "*Structural Health Monitoring: A Machine Learning Perspective*", C. R. Farrar, K. Worden. John Wiley & Sons ISBN: 978-1-119-99433-6 (2012)

[18] Coles. Chapter 3 *"An Introduction to Statistical Modeling of Extreme Values"*. S. Coles. Springer ISBN 978-1-85233-459-8 (2001)

[19] Randall. "*Vibration-based Condition Monitoring: Industrial, Aerospace and Automotive Applications*", R. B. Randall. John Wiley & Sons ISBN: 978-0-470-74785-8 (2011)

[20] Everitt "*The Cambridge Dictionary of Statistics*". B. S. Everitt A. Skrondal. Cambridge University Press ISBN: 9780521766999 (2010)

[21] D. Toshkova, N. Lieven, P. Morrish, B. Titurus, W. Moore, N. Brinkworth. Gearbox Condition Monitoring under Variable Operational Conditions. *Le Cam, Vincent and Mevel, Laurent and Schoefs, Franck. EWSHM - 7th European Workshop on Structural Health Monitoring, Jul 2014, Nantes, France*, 2014

[22] ismev: An Introduction to Statistical Modeling of Extreme Values, http://www.stats.bris.ac.uk/R/web/packages/ismev/

[23] The R Project for Statistical Computing. https://www.r-project.org/