General Selection Criterion for Time Lag Parameter in Applications of Stochastic Subspace Identification for Civil Structures

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Abstract
A recent work by the authors revealed that the lower limit for setting the time lag parameter in the stochastic subspace identification (SSI) analysis for stay cables can be decided with the ratio of the fundamental period to the sampling time increment. Inspired by this important criterion, an alternative stabilization diagram was proposed to exhibit the results with varying values of the time lag parameter for more conveniently distinguishing stable modal parameters of cable. Based on the alternative stabilization diagram, this study aims to investigate the measurements from different types of civil structures for extending the applicability of such a criterion to ensure stable identification results. This criterion is first validated for its applications with single measurements from stay cables and bridge decks. As for multiple measurements, it is found that the predicted threshold works well for the cases of stay cables, but makes an evident overestimation for the cases of bridge decks. This discrepancy is further explained by the fact that the deck vibrations are induced by multiple excitations coming from the passing traffic. The cable vibration signals covering the sensor locations close to both the deck and pylon ends of a cable-stayed bridge provide convincing evidences to testify this important discovery.

1. INTRODUCTION

For the applications in civil structures often subjected to narrowly banded excitations, the selection of time lag parameter in the stochastic subspace identification (SSI) analysis may have a strong influence on the determined modal parameters. The great uncertainty of actual excitations in these cases makes it particularly difficult to obtain a clear criterion for choosing the time lag parameter. Until very recently, a work by the authors [1-2] revealed that the lower limit for setting the time lag parameter can be decided with the ratio of the fundamental period of a stay cable to the sampling time increment for a valid modal identification using the conventional stabilization diagram. Inspired by this important criterion, a new methodology based on the covariance-driven SSI was established by proposing an alternative stabilization diagram to exhibit the results with varying values of the time lag parameter for more conveniently distinguishing stable modal parameters of cable. A hierarchical sifting process with three stages was also developed to automatically extract reliable modal parameters from the alternative stabilization diagram [1-2]. Equipped with this improved SSI algorithm, a related study [3] further investigated a benchmark problem for the mode identifiability of a cable-stayed bridge by analyzing several sets of known and blind bridge deck measurements.

Based on the alternative stabilization diagram, this study aims to investigate the measurements from different types of civil structures for extending the applicability of the newly discovered criterion regarding the choice of the time lag parameter to ensure stable identification results. This
criterion is first validated for its applications with single measurements from stay cables, bridge decks, and buildings. As for multiple measurements, it is found that the predicted threshold works well for the cases of stay cables and buildings, but makes an evident overestimation for the case of bridge decks. This discrepancy is further explained by the fact that the deck vibrations are induced by multiple excitations independently coming from the passing traffic. The cable vibration signals covering the sensor locations close to both the deck and pylon ends of a cable-stayed bridge provide convincing evidences to testify this important discovery.

2. CRITERION FOR TIME LAG PARAMETER AND ALTERNATIVE STABILIZATION DIAGRAM

The derivation of covariance-driven SSI typically starts from the state space description of a linear system with \( n \) degrees of freedom. If output measurements are conducted to obtain the \( 1 \times 1 \) output vector \( y(k) \) at the time instant \( t_k \) and \( x(k) \) represents the corresponding \( 2n \times 1 \) state vector, the whole set of state and output equations including the effect of noises can then be discretized with the sampling time increment \( \Delta t \) and expressed as:

\[
x(k+1) = Ax(k) + w(k) \\
y(k) = Cx(k) + v(k)
\]  

(1)

where \( A \) is the \( 2n \times 2n \) discretized system matrix and \( C \) signifies the \( 1 \times 2n \) output allocation matrix. In addition, \( w(k) \) and \( v(k) \) indicate the \( 1 \times 2n \) process noise vector and the \( 1 \times 1 \) measurement noise vector at the time instant \( k\Delta t \), respectively. These two types of noises are commonly assumed in the SSI analysis as stationary zero-mean white noises for the convenience of derivation.

Considering the output vectors measured at \( N \) consecutive time instants, they can be systematically organized into a Hankel matrix with the selection of a time lag parameter \( i \):

\[
Y_{02i-1} = \begin{bmatrix}
y(0) & y(1) & \cdots & y(j-1) \\
y(1) & y(2) & \cdots & y(j) \\
\vdots & \vdots & \ddots & \vdots \\
y(i-1) & y(i) & \cdots & y(i+j-2) \\
y(i) & y(i+1) & \cdots & y(i+j-1) \\
y(i+1) & y(i+2) & \cdots & y(i+j) \\
\vdots & \vdots & \ddots & \vdots \\
y(2i-1) & y(2i) & \cdots & y(2i+j-2)
\end{bmatrix}_{2i \times j}
\]  

(2)

where \( Y_p \) and \( Y_f \) are both \( i \times j \) with \( j = N - 2i + 1 \). Post-multiplication of \( Y_f \) by \( Y_p^T \) leads to the approximation of the so-called Toeplitz matrix. Singular value decomposition can then be conducted on \( Y_f Y_p^T \) to obtain

\[
Y_f Y_p^T = U S V^T = \begin{bmatrix} U_1 \end{bmatrix}_{i \times 2n} \begin{bmatrix} U_2 \end{bmatrix}_{i \times n_i} \begin{bmatrix} S_1 \end{bmatrix}_{2n \times 2n} \begin{bmatrix} 0 \end{bmatrix}_{2n \times n_i} \approx \begin{bmatrix} V_1 \end{bmatrix}_{i \times il} \begin{bmatrix} 0 \end{bmatrix}_{n_i \times il} \approx U_1 S_1 V_1^T
\]

(3)

where \( n_i = i - 2n \), \( U \) and \( V \) are orthogonal matrices, and \( S \) is a quasi-diagonal matrix with positive diagonal elements arranged in a decreasing order. By further taking
it has been shown [4] that the discretized system matrix $A$ in the state space can be determined by

$$A = O_i^\oplus(1:1(i-1),)O_i(1+1:i,;)
$$

where the symbol $\oplus$ denotes the pseudo inverse operation. Further with the solved eigenvalues and eigenvectors of $A$, the modal frequencies $\omega_k$’s, the damping ratios $\xi_k$’s, and the mode shape vectors at the output measurement locations $\varphi_k$’s can be directly calculated according to the theory of linear systems.

From the above review, it is obvious that the time lag parameter $i$ in Eq. (2) and the system order parameter $n$ in Eq. (3) have to be prescribed in conducting the SSI analysis. Theoretically, the time lag parameter specifies the construction of a Hankel matrix from the measured signals and the subsequent modal parameters identified in the SSI analysis should be insensitive to its value if the excitation is close to the white-noise assumption. It is not this case, however, when the excitation is narrowly banded. As for the system order parameter $n$, it indicates the selection of the largest $2n$ singular values in Eq. (3) to compose the two matrices in Eq. (4) and consequently decides the size of the discretized system matrix $A$ to finally provide $n$ sets of modal parameters. A value of $n$ sufficiently larger than the number of physical modes within the interested frequency range is usually chosen to assure the incorporation of more contributing modes.

The determination of modal parameters with the SSI analysis conventionally requires constructing the stabilization diagram to observe the stability of the identified results with the increasing value of $n$ under a designated value of $i$. Nevertheless, the performance of stabilization diagrams for the applications in civil structures can be significantly affected by the selection of time lag parameter. An effort was made in a recent research by the authors [1-2] to resolve this discrepancy in identifying the modal parameters of a stay cable. Considering the fact that all the modal frequencies of a stay cable are with values approximately in integer multiples of its fundamental frequency $f_1$, it was first pointed out that the ambient vibration signal of cable would be nearly a periodic function with a quasi-period close to the period $T_i = 1/f_i$ of the first cable mode. Therefore, all the possible independent patterns of the covariance matrix for the output vector are uniformly distributed in $Y_fY_p^T$ for the SSI analysis if

$$i \geq i_c = \frac{T_i}{\Delta t} = \frac{s}{f_i}
$$

where $s=1/\Delta t$ represents the sampling rate of measurement and $i_c$ means the critical threshold for the time lag parameter to ensure stable identification results. Even for the measurements on the other structures with irregularly distributed modal frequencies, Eq. (6) is also expected to be practically applicable because the unbiased distribution of all the possible independent patterns of the covariance matrix may still be roughly attained.

Inspired by the criterion of Eq. (6), the authors [1-2] further proposed an alternative stabilization diagram which exhibits the gradually increased time lag parameter $i$ along the ordinate and the corresponding modal frequencies along the abscissa under an assigned value of the system order $n$. By first examining the Fourier amplitude spectra (FAS) of cable
signals, it was suggested to choose the system parameter $n$ that is approximately twice of the observable peaks and determine the fundamental period of cable for setting the lower limit $i_{\text{min}} = i_1$ with Eq. (6). In addition, the upper limit of time lag parameter $i_{\text{max}}$ is decided by the need in the subsequent processes to extract reliable modal parameters. The alternative stabilization diagram has also been validated to hold the advantage in less interference from the superfluous modes for the cases of stay cables [2].

3. INVESTIGATED STRUCTURES AND MEASUREMENTS

For extensively evaluating the applicability of the critical threshold for the time lag parameter shown in Eq. (6), two cable-stayed bridges are selected in this research for investigation and will be subsequently introduced. The high resolution velocimeters VSE-15D made by Tokyo Sokushin were employed to conduct the ambient vibration measurements on these structures. A portable ambient vibration monitoring system SPC-51 with a maximum sampling frequency of 1000 Hz, also from Tokyo Sokushin, was connected with the VSE-15D sensors to collect the signals. In all the measurements, the duration was set at 300 sec with a sampling rate of $s = 200 \text{ Hz}$.

As illustrated in Figure 1, Chi-Lu Bridge locates in central Taiwan to connect the two towns Chi-Chi and Lu-Ku. It is a single-pylon symmetric cable-stayed bridge built by prestressed concrete box girders, comprising two 120 m main spans. The single-plane cable system, as shown in Figure 1, consists of 17 pairs of stay cables on each side of the pylon and is arranged in the semi-fan shape along the centerline of the girder. The ambient vibration measurements were first conducted on two cables of Chi-Lu Bridge: R33 and R17, representing the longest and medium cable, respectively. Cable R33 is 126.4 m long with an inclination angle of 26° and Cable R17 is 76.4 m long with an inclination angle of 31°. Four velocimeters were installed on each cable to record the synchronized vibration signals in the in-plane vertical direction. All these sensor locations were selected to be close to the bridge deck. Their distances to the front end of the bottom rubber constraint are also indicated in Figure 1 to be 1.5 m, 3.5 m, 5.5 m, and 7.5 m, respectively. Moreover, the ambient vibration signals at different locations of the bridge deck are also taken in this study. Five velocimeters were deployed along the central line of deck to simultaneously collect the velocity responses in the gravity direction. Denoted by D1 to D5, these five sensors were arranged at distances

![Figure 1: Chi-Lu Bridge and its sensor locations.](image-url)
of 30 m, 60 m, 150 m, 180 m, and 210 m from the Chi-Chi end.

As depicted in Figure 2, Shin-Tong Bridge is a three-span (75m+175.6m+75m) cable-stayed bridge connecting Miau-Li and Kong-Kuang located in northern Taiwan. It is a double-pylon symmetric bridge made by steel box girders and its cable system consists of 34 stay cables, 9 on the outer side and 8 on the inner side of each pylon. Installation of sensors close to the pylon end of a stay cable is usually difficult in practical implementation. It is fortunate that this research group had an opportunity in 2014 to conduct a measurement project on Shin-Tong Bridge for evaluating a novel method used in accurately estimating the cable force [5]. The ambient vibration measurements were taken on Cables C16 and C13, both on the outer side of the pylon close to the Miau-Li end. Cable C16 is 86.8 m long with an inclination angle of 31° and Cable C13 is 63.1 m long with an inclination angle of 36°. A total of four sensors were adopted to record the synchronized vibration signals of each cable in the in-plane vertical direction. Three velocimeters were mounted close to the deck end. On the other hand, the forth velocimeter near the pylon end needs to be installed with the help of a hired crane. For Cable C16, the distances of sensors to the front end of the bottom rubber constraint are 4.0 m, 5.5 m, 7.0 m, and 72.6 m, respectively. For Cable C13, the corresponding distances are 3.5 m, 5.0 m, 6.3 m, and 52.0 m, respectively.

4. CHOICE OF TIME LAG PARAMETER FOR DIFFERENT MEASUREMENTS

In addition to more effectively reduce the interference from the deceptive modes in SSI analysis, the alternative stabilization diagram recently proposed [1-2] also provides an excellent tool to investigate the critical threshold for the time lag parameter. Even though the formula of Eq. (6) was found to be adequately accurate in the case of cable measurements [2] to ensure stable modal frequencies, it seems to overestimate the lower limit for an appropriate choice of time lag parameter in the case of bridge deck measurements [3]. For conveniently clarifying the applicability of Eq. (6) in various types of structures, the current study starts in this section by examining the alternative stabilization diagrams for the cases simply with a single measurement. The same inspection on similar cases with multiple measurements will be followed for further discussions.

4.1 Single measurement

In this section, a single measurement either from the stay cable of Chi-Lu Bridge or the deck of Chi-Lu Bridge is considered for performing the SSI analysis. It is apparent from Eq. (3) that $i_0 - 2n = n_i \geq 0$ or $i \geq 2n/l = i_0$. In other words, the lowest possible value of the time lag parameter $i_0$ is controlled by the choice of the system order parameter $n$. For the
cases of a single measurement \((l = 1)\), \(i_0 = 2n\). To clearly observe the varying trend of identified modal frequencies, the lower limit of time lag parameter is consistently set at \(i_{\text{min}} = i_0\) in the subsequent alternative stabilization diagrams.

Taking the ambient vibration measurement at 7.5 m from the front end of the bottom rubber constraint for Cable R33 as the first example, the system order is fixed at \(n = 20\) because there are around 11 peaks observable in the interested frequency range from 0 to 10 Hz of the FAS for this measurement. The SSI analysis is then performed with the time lag parameter increasing from \(i_{\text{min}} = i_0 = 40\) to \(i_{\text{max}} = 320\) for establishing the alternative stabilization diagram shown in Figure 3(a) where the identified frequencies values are symbolized by black cross signs together with the corresponding FAS displayed in the background. Since the fundamental frequency of Cable R33 is \(f_1 \approx 0.9\) Hz, the critical threshold can be determined to be \(i_c = 200/0.9 \approx 220\) according to Eq. (6) and is also illustrated in Figure 3(a) with a horizontal dashed line in red. Figure 3(a) obviously confirms that all the identified cable frequencies are stable until the value of time lag parameter gets close to the criterion of \(i \geq i_c = 220\). Moreover, the alternative stabilization diagram corresponding to the measurement at 7.5 m from the bottom rubber constraint for Cable R17 is further presented in Figure 3(b). In this case, the system order parameter is also set as \(n = 20\) based on the observation of approximately 8 peaks in the targeted frequency range from 0 to 15 Hz of the FAS. With the fundamental frequency of \(f_1 \approx 1.63\) Hz for Cable R17, its critical threshold \(i_c = 200/1.63 \approx 120\) can then be obtained and the SSI analysis is conducted from \(i_{\text{min}} = i_0 = 40\) to \(i_{\text{max}} = 220\) for constructing the alternative stabilization diagram. Without any surprise, the frequency associated with each cable mode reaches its stable values in Figure 3(b) when the level of \(i = i_c = 120\) is reached.

The SSI analysis is then conducted with the single deck measurement D2 of Chi-Lu Bridge. Since there are 4 clear peaks (0.60 Hz, 0.98 Hz, 1.85 Hz, and 3.15 Hz) recognizable in the interested frequency range from 0 to 5 Hz of its FAS, the system order is chosen as
n = 10 and the time lag parameter is increased from $i_{\text{min}} = i_0 = 20$ to $i_{\text{max}} = 440$ in this case. In addition, the critical threshold can be obtained by $i_c = 200/0.6 \approx 330$ with the fundamental frequency $f_1 \approx 0.6$ Hz. Based on the corresponding stabilization diagram illustrated in Figure 4(a), it is of no doubt that all the deck frequencies can be steadily identified when the criterion of $i \geq i_c = 330$ is approached.

### 4.2 Multiple measurements

As illustrated in Figure 1, four sensors were installed on Cable R33 of Chi-Lu Bridge at different locations near the deck end to measure the ambient vibration signals. With the system order fixed at $n = 20$ and the time lag parameter increasing from $i_{\text{min}} = 40$ to $i_{\text{max}} = 320$, these four simultaneous measurements ($l = 4$) are adopted to perform the SSI analysis for constructing the alternative stabilization diagram portrayed in Figure 5(a). Comparing Figure 5(a) with the corresponding stabilization diagram in Figure 3(a) for a single measurement, it is evident that the critical threshold for the time lag parameter is close to the same predicted value of $i_c = 220$ in both cases. Additionally, the alternative stabilization diagram for four simultaneous measurements taken on Cable 17 with the system order parameter set at $n = 20$ is also plotted in Figure 5(b) from $i_{\text{min}} = 40$ to $i_{\text{max}} = 220$. Likewise, the critical threshold around $i_c = 120$ demonstrated in Figure 5(b) for multiple measurements of Cable R17 is similar to that shown in Figure 3(b) for a single measurement.

As also displayed in Figure 1, five velocimeters (D1 to D5) were distributed along the deck of Chi-Lu Bridge to conduct the ambient vibration measurements. With the same parameter selections ($n = 10$ and $i$ from $i_{\text{min}} = 20$ to $i_{\text{max}} = 440$) as those for the case of single measurement in the SSI analysis, the alternative stabilization diagram corresponding to these five simultaneous measurements ($l = 5$) is shown in Figure 4(b). Focusing on examining the four dominant deck frequencies demonstrated in Figure 4(a), Figure 4(b) discloses that the critical threshold for the time lag parameter is undoubtedly decreased from
\( i_c = 330 \) in the case of single measurement to a much lower value at around 120 in the case of multiple measurements. Therefore, a thinner dashed line in red is adopted in Figure 4(b) to indicate that \( i_c = 330 \) determined by Eq. (6) is definitely an overestimation in this case. Other than the four clear frequencies identified in Figure 4(a) for the single deck measurement D2, it is also noteworthy that two more steady frequencies are found in Figure 4(b) for the case with five deck measurements. This phenomenon is not unusual in the practical applications of SSI analysis.

5. EFFECT OF INDEPENDENT EXCITATIONS

Other than the cases of stay cables [1-2], it has been demonstrated in the previous section that Eq. (6) is also valid for the application with a single measurement from the bridge deck. To explain the exceptional case in which Eq. (6) obviously overestimate the critical threshold for the analysis with multiple measurements of the bridge deck, the key factor may also reside in the characteristics of excitations. For the two types of signals investigated in Section 4, their independent sources of excitation are not the same. The cable measurements of Chi-Lu Bridge are all taken near the deck end and there is only one major excitation source from the motion of bridge deck in this case. On the other hand, the deck measurements of Chi-Lu Bridge are influenced by multiple excitations independently coming from the passing vehicles in the neighborhood of each sensor location. It is consequently suspected that numerous independent excitations in the case with multiple measurements of the bridge deck create an input more widely distributed in the frequency domain. Therefore, the critical threshold for the time lag would be significantly lowered in this case to approach the situation that the identified results are insensitive to \( i \) under the white noise excitation.

To solidify the above conjecture, the ambient vibration measurements on Cables C16 and C13 of Shin-Tong Bridge would provide a perfect testimony. In addition to the installation of three velocimeters near the deck end of cable, the other velocimeter was mounted on a location close to the pylon end. Since the excitation for the pylon is considerably different from that for the deck, inclusion of the measurement near the pylon end in the SSI analysis is
likely to widen the contributed frequency range of excitation. For comparison, the alternative stabilization diagram associated with the three measurements \( l = 3 \) near the deck end of Cable C16 is shown in Figure 6(a) and the other corresponding to the addition of one more measurement close to its pylon end \( l = 4 \) is illustrated in Figure 6(b). With 8 clear peaks observed in the interested frequency range from 0 to 10 Hz of its FAS, the system order is fixed at \( n = 20 \) and the time lag parameter is increased from \( i_{\min} = i_0 = 10 \) to \( i_{\max} = 265 \) in this case. Moreover, the critical threshold can be predicted by \( i_c = 200/1.2 \approx 165 \) considering that the fundamental frequency of this cable is \( f_1 \approx 1.2 \text{ Hz} \). It is obvious that the critical threshold for the time lag parameter follows the estimated value of \( i_c = 165 \) pretty well in Figure 6(a). This trend is similar to that for multiple cable measurements of Chi-Lu Bridge.

Figure 6: Alternative stabilization diagrams for multiple measurements of Cable C16.

Figure 7: Alternative stabilization diagrams for multiple measurements of Cable C13.
discussed in the previous section. Nevertheless, the corresponding threshold is substantially lowered to a value at approximately 80 in Figure 6(b). Such a phenomenon due to the inclusion of one more measurement near the pylon end resembles the observations in Figure 4(b) for multiple deck measurements of Chi-Lu Bridge. The same comparison is also made for the measurements on Cable C13 with the alternative stabilization diagrams displayed in Figures 7(a) and 7(b) for the case of three measurements near the deck end (l = 3) and that adding one more measurement close to its pylon end (l = 4), respectively. For this analysis, the system order is chosen as n = 20 and the time lag parameter is increased from \( i_{\text{min}} = 40 \) to \( i_{\text{max}} = 265 \). With the fundamental frequency of \( f_1 \approx 1.61 \text{Hz} \) for Cable C13, its critical threshold \( i_c = 200/1.61 \approx 125 \) can then be determined from Eq. (6). As expected, the critical threshold for the time lag parameter is noticeably decreased from around \( i_c = 125 \) in Figure 7 (a) to a much lower value at about 60 in Figure 7(b). With all the above powerful evidences, it is not difficult to deduce that the identified results from the SSI analysis for the signals caused by more independent excitations would reach stable values under a much lower threshold of the time lag parameter than that estimated by Eq. (6).

6. CONCLUSIONS

Based on the alternative stabilization diagram by varying the time lag parameter in the SSI analysis, this study investigates the measurements from different types of structures to extend the applicability of a recently noticed criterion to ensure stable identification results. Such a criterion is first validated for its applications with single measurements from stay cables and bridge decks. Regarding multiple measurements, it is found that the predicted threshold works well for the cases of stay cables, but makes an evident overestimation for the case of bridge decks. This discrepancy is further explained by the fact that the deck vibrations are induced by multiple excitations independently coming from the passing traffic. The cable vibration signals covering the sensor locations close to both the deck and pylon ends of Shin-Tong Bridge provide convincing evidences to testify this important discovery.

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