An Assessment of the Effect of Random Noise in the Frequency Measurements of a Vibration-Based SHM System with Neural Network Damage Predictor

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Abstract
The implementation of Structural Health Monitoring (SHM) systems in modern industrial designs is a challenging task that comprises multidisciplinary aspects of engineering, from numerical simulation to sensor installation, data acquisition and post-processing. In the framework of vibration-based SHM, this work describes and prospects the use of Artificial Neural Networks (ANNs) with supervised learning procedure for embedded damage prediction in isolated structural components. Parametric Detailed Finite Element Models (DFEMs) are developed to retrieve the modal fingerprints caused by damages in different locations and severities. The effect of random deviations introduced in the sensor frequency measurements to simulate environmental in-service conditions, as well as the use of random noise in the training set of the network to alleviate prediction errors, are statistically analysed and discussed.

1 INTRODUCTION
In recent years, the interest of aerospace, automotive and civil industries in the development of fully smart structures and systems has encouraged the research in the field of Structural Health Monitoring (SHM). Remarkable advances have been achieved in the study of techniques for structural diagnosis and prognosis [1]. In particular, vibration-based SHM is based on the principle that embedded damage yields a change in the dynamic response of a structure. The analysis of this dynamic shift by means tailored methods [2] may allow a valuable prediction of the damage pattern and its location within the structure. In this basis, Vibration Based Damage Identification (VBDI) methods [3,4,5] and automated Operational Modal Analysis (OMA) systems for continuous identification of modal parameters [6] are being deployed and tested in civil engineering architectures.

Far from a straightforward procedure, damage detection requires tuned numerical simulations, mainly finite element (FE) models to extract modal data [7], and tools able to retrieve the damage parameters from measured data –potentially menaced by environmental noise [8]–, this is, able to solve the so-called inverse problem of SHM.

This work revisits the application of Artificial Neural Networks (ANNs) for damage recognition from labelled modal data. Emphasis is placed on the capability of ANNs to tackle
the non-uniqueness nature of the solution of the inverse problem. The investigation focuses on the assessment of the impact on the damage predictions of random noise introduced in the frequency measurements that feed the Neural Network. Numerical experimentation with a statistical treatment of the ANN predictions is reported and discussed.

2 NUMERICAL INVERSE PROBLEM WITH UNCERTAIN MODAL DATA

In vibration-based SHM, the parameters \( \theta_d \) that define a structural damage are inferred from the measurement of the shift of quantities \( D \) that characterize the dynamic response of the monitored structure:

\[
\Delta D = D^{(d)} - D^{(0)} = H K \theta^{(d)} + C \theta^{(d)} + M \theta^{(0)}
\]

(d) and (0) being, respectively, a damaged and a reference intact status, and \( K, C \) and \( M \) denoting the stiffness, damping and mass matrices of the system, which are perturbed with the embedded damage \( \theta_d \) and a set of environmental variables \( \theta^e \) that usually possess stochastic nature. This study is limited to modal analysis, in which the shift \( \Delta D \) is typically taken as an array of residuals [2] in frequencies \( \Delta \omega_j = \omega_j^2 - \omega_j^{2(0)} \), mode shapes \( \Delta \phi_j = \phi_j^{(d)} - \phi_j^{(0)} \), modal forces \( \Delta MFR_{j} = (K^{(0)} - \omega_j^{2(0)} M^{(0)}) \phi_j^{(d)} \), etc. In addition, methods employing mode shape derivatives (curvatures) or indexes such as the Modal Assurance Criterion (MAC) or the Modal Strain Energy (MSE) have been developed as damage indicators [3,7]. The ANN predictions hereinafter are performed employing directly the shift in eigenfrequencies: \( \Delta \omega_j = \omega_j^2 - \omega_j^{2(0)} \) is used. Once the variations in the natural frequencies are measured (\( \Delta D_m \)), the inverse problem deals with the estimation of the damage parameters \( \theta_d^* \):

\[
\theta_d^* = H^{-1}(\Delta D_m, \theta^e)
\]

It is widely known that problem (2) suffers from non-uniqueness and instability of their solutions, e.g. Figure 1 gathers the first ten iso-frequency curves in the space of the damage parameters \( \{ \theta_{A,d}, \theta_{B,d} \} \) defined in section 3. The roughly identified intersection point in Figure 1a provides an estimation \( \theta_d^* \) for measurements \( \Delta D_m \) coincident with the nominal shifts. A small random deviation of \( \pm 0.2\% \) in the expected \( \Delta D_m \) (Figure 1b) yields a diagram with multiple potential solutions (clusters of curves intersections). Besides, the curves corresponding to the two lowest frequencies (main drivers of the estimation) do not intersect. Thus, the solution to (2) becomes inaccessible.

The inverse problem can be tackled in practice via machine learning procedures. The concept of ANN embraces a collection of information processing systems able to build numerical approximations by means of a learning process from a given set of labelled data. These models use schemes of interconnected neurons acting as nodes whose weights are adapted during the learning phase, usually consisting in a minimsisation procedure of certain cost function. In spite of recent investigations have boosted the development of a powerful deep learning technology, this work focuses on the statistical analysis of the predictions provided by simple single layer perceptrons, and the challenges found when measurements are affected by unforeseeable environmental noise. Given the set of M training data \( L^T = \{ \Delta \omega_{1,m}, \Delta \omega_{2,m}, \ldots, \Delta \omega_{M,m} \} \) associated to multiple locations and severities of damage \( \theta_d \), the ANN prediction \( \theta_d^* \) is built as a weighted sum of the contributions of H neurons of the
hidden layer of the network:

$$\Theta_{d,k}^* \left( \Delta \omega_{m}, p^* \right) = \sum_{i=1}^{H} v_{ki}^* S \left( \sum_{j=1}^{n} w_{ij}^* \Delta \omega_{j,m}^2 + u_{ki}^* \right)$$

(3)

where \( \left( \Delta \omega_{m}^2 \right)^T = \{ \Delta \omega_{1,m}^2, \Delta \omega_{2,m}^2, \ldots, \Delta \omega_{n,m}^2 \} \) are the n measured shifts of eigenfrequencies of the damaged structure, \( S \) is an activation function and \( p^* \) is the array of intrinsic parameters of the ANN, \( p^T = \{ u_{ki}, v_{ki}, w_{ji} \mid i = 1, \ldots, n; j = 1, \ldots, n; k = 1, \ldots, K = \text{len}(\Theta_d) \} \), to be determined through a minimization process of the quadratic norm \( J(p) \):

$$J(p) = \frac{1}{2} \sum_{\omega_i \in \Delta \omega_i^*} \left( \Theta_d \left( \Delta \omega_{i,m}^2, p \right) - \theta_{d,i}^* \right)^T \left( \Theta_d \left( \Delta \omega_{i,m}^2, p \right) - \theta_{d,i}^* \right)$$

(4)

this is, \( p^* = \arg \min_p J(p) \). For this purpose, multivariable optimization codes such as the Levenberg-Marquardt algorithm (LMA) or the standard and the limited-memory Broyden-Fletcher-Goldfarb-Shanno (BFGS & L-BFGS) algorithms have proven excellent performances.

![Figure 1: First ten iso-frequency curves for cracked beam problem, (a) measurements coincident with nominal shifts; (b) random deviation of ±0.2% in the measured shifts.](image)

Since the cost of the construction of the dynamic fingerprint \( L^T \) from tested damaged specimens can be prohibitive, numerical models properly correlated [3] are commonly used. These FEMs provide nominal eigenfrequencies that, in general, are not be retrieved in actual in-service structural life due to the intrinsic uncertainty of measurement conditions. A statistical characterization of the measurement noise is absolutely valuable for the confidence in the predictions. The baseline for this work is the selection of normal distributions [8] to characterize the measured frequency shifts. Under this hypothesis, the ANN prediction in (3) reads \( \Theta_{d,k}^* \left( N \left( \Delta \omega_{m}, \sigma_i^2 \right), p^* \right) \), \( N(\mu, \sigma^2) \) being a normal distribution of mean \( \mu \) and standard deviation \( \sigma \), and \( \sigma \), the expected [9] in-service standard deviations in the measurements of the modal frequencies. In this work, the repeated single-layer-perceptron ANN predictions of the same damage parameters with different perturbing noise in the measurements are
transformed into a normal statistical distribution. Furthermore, an extended training set composed by the nominal shifts and by \( S_E \) additional random normally-distributed shifts is proposed to feed the ANN:

\[
EL_t^T = \left\{ \Delta \omega_{1,t}^2, \Delta \omega_{2,t}^2, \ldots, \Delta \omega_{M,t}^2, \ldots, N(\Delta \omega_{1,t}^2, \sigma_t^2), N(\Delta \omega_{2,t}^2, \sigma_t^2), \ldots, N(\Delta \omega_{M,t}^2, \sigma_t^2), \ldots \right\}
\]

(5)

where \( \sigma_t \) is the standard deviation of the Gaussian noise for training de ANN. The workflow logic of the process, including the FE generation and correlation phase, and the introduction of uncertainty on the ANN predictions is illustrated in Figure 2.

At this point, several questions arise: is a better predicting performance retrieved from an ANN if it is trained with an extended set \( EL_t^T \)? i.e., can the training noise alleviate the detrimental effect on damage estimations of the in-service noise? How is the prediction affected if the ANN is trained with a level of noise different from the actual noise is the in-service measurements? Assuming the optimum selection of the training noise, how does the ANN behave when it is fed with a localised anomaly in the measurements attributable to a breakdown in the sensor device? Would an underestimation –or a false negative– of the damage be possible in such circumstances? The paper presents two numerical experiments aimed at answering these questions. The study is performed with a FEM-single-layer-perceptron ANN algorithm developed for detecting cracks at isolated component scale. The extrapolation of the results to the assembly scale is matter of further research.

3 NUMERICAL EXPERIMENTS

The results included in this section have been obtained from the undamped bending vibration modes of a cracked cantilever beam. The beam was discretised in the 3D space with parametric DFEMs built by Python scripting for Abaqus solver. The mesh parameter \( h_{max} \) was determined by convergence analysis in intact beam conditions versus the solution available in [10]. Two non-dimensional parameters have been employed to define the damage in the beam, \( \theta_{d,A} = d_c / H \) (ratio of the crack length to the characteristic dimension of the cross-section of the beam) and \( \theta_{d,B} = x_c / L \), (ratio of the crack position to the length of the beam). The validation of the numerical eigenvalues was performed by comparison with the results published by Friswell and Penny [11]. MatLab Neural Network Toolbox technology has been used to compute \( \Theta_{d,k}^* \). Regularised shift magnitudes have been considered to balance their weight in the algorithm. The distributions of the ANN predictions have been normalised with built-in functions of the MatLab statistical module. A representative range of standard deviations of 0.01% – 1% for the in-service measurements has been considered [11,12,13].

3.1 Noise introduction in training data

A total of 237 experiments were run in order to build the normal distribution of each prediction \( \Theta_{d,k}^* \). ANNs of \( H = 50 \) neurons were trained with sets \( L_t^T \) initially sized at \( M = 275 \) data. Extended sets \( EL_t^T \) with \( S_E = 20, 50 \) and 100 were tested. The errors \( \chi_i^i = (\mu_i - \Theta_{d,i}) / \Theta_{d,i} \), where \( \mu_i = (\Theta_{d,i}^*)_{mean} \), \( i = A, B \), are presented in Figure 3 for multiple
Figure 2: Workflow for the assessment of noise effect on ANN predictions in vibration-based SHM.
combinations \( \{\theta_{d,A}, \theta_{d,B}\} \) and different noises \( \sigma_i \) and \( \sigma_i \). The regions of highest accuracy in both damage parameters correspond to large crack sizes positioned close to the free end of the cantilever beam. In such regions, errors in the order of \( \chi^A = 2-3\% \) (Figure 3a) and \( \chi^B = 1-2\% \) (Figure 3b) can be retrieved from the ANN with the proper selection of \( \sigma_i \). A remarkable loss of accuracy is found for small cracks and for damages near the clamped end of the beam, configurations with a less pronounced modal fingerprint. Figures 3b-d reveal a severe penalisation in \( \chi^B \) in local regions. More complex ANNs of deeper learning might solve these issues, adapted to the particular nature of each analysed structural configuration.

The effect of dissimilar noises \( \sigma_i \) and \( \sigma_i \) is clearly identified from the numerical tests. A first assessment can be stated from the comparison of Figures 3a-b (\( \sigma_i = 0.1\% \), \( \sigma_i = 0.1\% \)) with Figures 3c-d (\( \sigma_i = 1.0\% \), \( \sigma_i = 0.1\% \)). Training with insufficient noise ANNs of SHM systems whose acquisition sensors may be subjected to severe stochastic in-service conditions yields, according to this study, useless and, of course, incorrect damage predictions. The confirmation of this observation is given in Figure 4, which provides the global average errors \( \chi^i_{\text{avg}} \) for all the damage couples \( \{\theta_{d,A}, \theta_{d,B}\} \) along the beam. It is noticed the different orders of magnitude of \( \chi^A_{\text{avg}} \) and \( \chi^B_{\text{avg}} \), explained by the significant contribution of the local regions of poor performance in \( \chi^B \) to the average value \( \chi^B_{\text{avg}} \). The global errors are plotted for different pairs \( \sigma_i \) and \( \sigma_i \), optimum performance being found for the diagonal \( \sigma_i = \sigma_i \). This is, the effect of undesirable uncertainty in the measurements can be partially mitigated by training the ANNs with such uncertainty. Under-training \( \sigma_i < \sigma_i \) leads to a remarkable loss of the capability of the ANN to estimate the damage parameters. On the other hand, over-training \( \sigma_i > \sigma_i \) does not improve essentially the accuracy of the predictions from the global standpoint of the structure. On the contrary, over-training might have detrimental effects, but of inferior severity than the ones of under-training. It has been checked that the ANN predictions are decreasingly sensitive to the number of additional sets \( S_E \) used, provided \( S_E \) is sufficiently high. Results presented correspond to \( S_E = 50 \).

### 3.2 Anomaly in sensor data

Anomalous in-service measurements can be associated to exceptional events in the operation of the structure (out of the statistically foreseeable conditions), or directly to a breakdown in the sensor system installation. Thus, uncertainty analysis should not only be aimed at the alleviation of the Gaussian deviations in the measurements, but also at the study of the response of the ANN predictor when a completely random anomaly shunts expected measurements distributions aside. The numerical experiment presented in this section employs a normal distribution of measurements \( \mathcal{N}(\Delta \bar{\omega}_m^2, \sigma_s^2) \) as input of an ANN trained with optimal noise, \( \sigma_i = \sigma_i \) according to the results exposed in previous section. In particular, \( \sigma_i = \sigma_i = 0.1\% \) are selected. The reference measurements \( \Delta \bar{\omega}_m^2 \) are replaced by an array of measurements \( \Delta \bar{\omega}_m^2 \) with anomalous components. The study is limited to one single anomaly in the mode \( j \), \( \Delta \bar{W}_{j,m}^2 \), which is introduced as a completely random, no normalised value in the range defined by the minimum and the maximum frequencies:

\[
\left(\Delta \bar{\omega}_m^2\right)^T = \left\{\Delta \omega^2_{m,1}, \ldots, \Delta \omega^2_{j-1,m}, \Delta \bar{W}_{j,m}^2, \Delta \omega^2_{j+1,m}, \ldots, \Delta \omega^2_{m,n}\right\}
\]
Figure 3: ANN prediction errors $\chi^A$ and $\chi^B$ for different training and in-service noises.

Figure 4: Global ANN prediction errors $\chi^A_{\text{avg}}$ and $\chi^B_{\text{avg}}$. Deviations $\sigma_t$ and $\sigma_s$ in %.
The “one anomalous sensor” scenario described above may not be directly translatable to the case of vibration-based SHM, since individual channels of the sensor system do not measure individual eigenfrequencies. The analysis is performed as an illustrative study case of networks in which a localised anomaly in sensor data may yield unrecognisable false negative predictions, endangering the structural integrity. Figure 5 gathers the predictions and the actual damage parameters of 237 experiments in 275 damage points $\theta_d$, for three different anomalous frequency shifts in $\omega_1$ (a-b), $\omega_5$ (c-d) and $\omega_9$ (e-f). The experiment shows that the anomaly in the lowest frequency (to a great extent, the lowest frequencies drive the predictions in vibration-based SHM) causes the ANN to predict a approximately constant value $\Theta_{d,A} \approx 0.3$, while the estimation $\Theta_{d,B} < 0$ (negative in all the points $\theta_d$) lacks sense and would result in an immediate warning of the incorrect operation of the system. Anomaly in higher frequencies can yield underestimations of the crack size for logical positions $\Theta_{d,B}$. This is observed in the last prediction tests of diagrams of $\omega_5$ (c-d) and, specially, of $\omega_9$ (e-f). In Figure 5f, an acceptable correlation of $\Theta_{d,B}$ and $\theta_{d,B}$ is found for cracks ranging from medium to large sizes. The experiment suggests that the single-layer-perceptron ANN algorithm is prone to provide roughly detectable underestimations of the damage in the structure when it is fed with anomalous high frequencies. As the weight of the lower frequencies in the prediction is higher, the easiness of detection of random anomalies introduced in the first vibration modes should be also higher.

4 CONCLUSIONS

The presented FEM-ANN integration focuses on the performance of a single-layer-perceptron used to locate embedded damage in a cantilever beam. The system predicts the size and position of a crack from its modal fingerprint, i.e. from the measured shifts in the eigenfrequencies produced on the damaged structure.

The study assesses how random noise introduced in the sensor frequency measurements to simulate environmental in-service conditions affects the predictions. A statistical approach based on the Gaussian normalisation of the in-service noise is employed. Experiments reveal a severe detrimental effect of environmental noise with moderate standard deviations if the ANN is not properly trained. The optimum operation point found is the training with the expected in-service noise. Both under- and over-training return higher global averaged errors.

Anomalous random elements present in the set of in-service measurements can cause remarkable but unrecognisable underestimations of the embedded damage if the frequencies of high vibration modes are affected. The numerical tests show that anomalies in the leading eigenfrequencies should be discovered early from the inspection of the returned predictions. Caution must be taken before extrapolating these results to other structural scales. The evaluation of the effect of noise on deeper networks with or without alternative machine learning procedures (e.g. unsupervised training) is matter of interest for further research.

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Figure 5: ANN predictions with and without anomaly in frequency measurements, compared to nominal damage parameters.
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