Damage Detection in Metallic Plates using $d_{36}$ Piezoelectric Phased Arrays

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Abstract

Recently, $d_{36}$-type piezoelectric wafers derived from lead magnesium niobate-lead titanate (PMN-PT) single crystal materials have been proposed for elastic wave generation in thin-walled structures. In contrast to traditional $d_{31}$ piezoelectric materials which generate symmetric and anti-symmetric Lamb waves, $d_{36}$ introduces these same Lamb waves plus an in-plane shear horizontal (SH) wave. Based on guided wave theory, the $SH_0$ wave mode is non-dispersive which may find many advantages uses in structural health monitoring. To detect and localize damage (e.g., cracks, corrosion) in thin walled structures, a phased array of $d_{36}$-type piezoelectric wafers is proposed. To validate the proposed phase array, extensive experiments have been conducted using an aluminum plate with four $d_{36}$ PMN-PT piezoelectric wafers surface bonded. The $d_{36}$ phased array is shown capable of accurately detecting the location of local damage in the plate. Good detection accuracy is observed indicating that this proposed methodology has the potential to provide an effective identification of damage in metallic thin walled structures.

1 INTRODUCTION

Thin walled metallic structural elements are pervasive in many structures including I-beams, pipelines, ship hulls, and aluminum aircraft, just to name a few. Many of these structures are exposed to environmental and operational conditions that can lead to damage. Structural health monitoring (SHM) systems have been proposed to monitor these structures and to use acquired data to estimate damage states in the structure; in doing so, SHM systems can provide the information needed to improve the safety and reliability of critical structures[1, 2]. While many sensing strategies have been proposed for SHM, guided wave methods have been attracting much excitement given their ability to interrogate structures at the local level. Lamb waves and shear horizontal (SH) waves are ultrasonic guided waves that exist in thin-wall structures. An attractive feature of guided waves is their ability to travel through thin walled structures over large distances with limited attenuation giving them a relatively large interrogation range[2, 3].
When damage (e.g., crack) or a defect exists in the structure, ultrasonic guided waves will reflect or scatter from it providing a signature that can be used for damage detection[4, 5]. Specifically, reflected and scattered signals received by a piezoelectric sensor can be analyzed to identify the damage type, its location and potentially its severity[6]. While Lamb waves have been extensively studied in the SHM literature, they do have some drawbacks. For example, Lamb waves are dispersive (i.e., there are multiple modes whose waves have varying velocities at different frequencies). Also, mode conversion of Lamb waves can occur at some damage sites further complicating the analysis of Lamb waveforms acquired[7, 8]. In contrast, SH0 waves are non-dispersive guided waves which can simplify the analysis of the guided wave signals acquired. The non-dispersive nature of SH0 waves have motivated the SHM community to explore its use to inspect thin walled structures including rail, plates, pipe structures, and composite structures [9-12].

In this paper, a phased array of four d36 piezoelectric actuators is designed to emit directional SH0 waves for the detection and localization of damage in metallic plate structures. First, the characteristics of SH waves are introduced. Second, the design of the d36-type piezoelectric phased array including the theory of its operation are presented. Third, numerical simulations are performed to demonstrate the focusing effect of the phase array. By programming the time delay of the excitation signal, a directional wave front emitted by the phased array can be rotated to interrogate specific areas of the structure. Finally, experimental validation of the SH phased array is conducted using a thin aluminum plate. Using the imaging method proposed herein, the location and the size of plate damage can be accurately reconstructed.

![Figure 1: Guided waves: (a) Lamb waves and SH0 waves in thin plates; (b) dispersion curves of symmetric (Sx) and antisymmetric (Ax) Lamb and SH0 waves.](image)

**2 THE CHARACTERISTIC OF SHEAR HORIZONTAL WAVE**

SH waves are waves that propagate within the plane of a thin walled structural element such as a plate (Figure 1a). In other words, the particle motions associated with SH waves are perpendicular to the wave propagating direction but parallel to the thin wall plane. The velocity of the first SH wave (SH0) is constant irrespective of frequency content (Figure 1b); the velocity is equal to the shear wave velocity for the given material. In contrast, Lamb waves are associated with particle motions that include components perpendicular to the thin wall plane. There are two types of Lamb wave modes: symmetric (S, mode) and anti-symmetric (A, mode) where the x subscript designates the mode number. Lamb waves are dispersive with group and phase velocity varying with frequency content of the signal and the geometry of the plate (i.e.,
thickness).

![Figure 2: Derivation of $d_{36}$-type piezoelectric wafer from PMN-PT bulk material.](image)

### 3 $d_{36}$-TYPE PIEZOELECTRIC PHASED ARRAYS

#### 3.1 $d_{36}$-type Piezoelectric Material

The majority of piezoelectric elements used for SHM to date have been made from lead zirconate titanate (PZT). Alternatively, piezoelectric elements can also be made from Pb(Mg$_{1/3}$Nb$_{2/3}$)O$_3$-PbTiO$_3$ (lead magnesium niobate-lead titanate or PMN-PT) single crystal materials. Figure 2 illustrates bulk PMN-PT with crystal orientation denoted; depending on how the PMN-PT material is cut, various types of thin piezoelectric elements can be created. If PMN-31%PT is created with crystal growth in the $z$ direction, poled in the [011] direction and is cut in the (011) plane, the piezoelectric element will strain axially under an electric field imposed orthogonal to the wafer. Such wafers are referred to as a $d_{31}$-type piezoelectric due to the large piezoelectric coefficient in the $d_{31}$ term (displacement in the 1 direction per unit voltage applied in the 3 direction) of the piezoelectric coefficient matrix. If the same PMN-PT is cut in the (011) plane, a piezoelectric element capable of shear strain is created. Such piezoelectric elements are termed a $d_{36}$-type piezoelectric due to the high piezoelectric coefficient in the $d_{36}$ term (in plane shear deformation per unit voltage applied in the 3 direction). A $d_{36}$-type piezoelectric is capable of producing an in-plane shear wave (SH) [13].

The value of piezoelectric coefficient of [011] poled PMN-31%PT crystals in the original coordinates is given in equation (1) which corresponds to the $d_{31}$-type piezoelectric shown in Figure 2. After a 45° rotation (which corresponds to the $d_{36}$-type piezoelectric), the matrix of piezoelectric coefficients becomes equation (2):

$$d' = \begin{bmatrix} 0 & 0 & 0 & 0 & 2736.6 & 0 \\ 0 & 0 & 0 & 183.1 & 0 & 0 \\ 771.4 & -1147 & 1454.6 & 0 & 0 & 0 \end{bmatrix} \times 10^{-12} \text{ C/N} \tag{1}$$

$$d = \begin{bmatrix} 0 & 0 & 0 & -1276.8 & 1459.8 & 0 \\ 0 & 0 & 0 & 1459.8 & -1276.8 & 0 \\ -187.8 & -187.8 & 1454.6 & 0 & 0 & -1918.4 \end{bmatrix} \times 10^{-12} \text{ C/N} \tag{2}$$
It is noted that the piezoelectric coefficient $d_{36}$ reaches its maximum 1918.4 $pC/N$, which is much higher than the values of $d_{31}$ and $d_{32}$. The piezoelectric wafer which is dominated by the $d_{36}$ term will produce two pairs of shear strains and emit SH waves in the $x$ and $y$ directions. When voltage is applied orthogonal to the piezoelectric element, the element will experience both in-plan tensile strains which emit symmetric and antisymmetric Lamb wave modes.

### 3.2 Phased array

A phased array configuration of piezoelectric elements can produce a directional wave in a structure; directionality of the wave in an SHM context provides the advantage of better localization of damage in the structure. As shown in Figure 3, the phased array consists of multiple $d_{36}$ actuators oriented in a single line. While the same excitation signal is applied, the signal is delayed between the elements to form a directional wave front that focuses on a specific point (focal point termed “P” in Figure 3a) in the structure. By controlling the time delays of the excitation signals, the directional wave beam can be steered to scan the structure in specific areas. When the direction wave front encounters damage, each component of the wave front will be backscattered by the defect. To spatially image the results of the received wave (as measured by the elements of the same phased array), all of the received signals are shifted based on the original time delay and superimposed to derive a reconstructed signal. If there is a defect at the array focal point, the superimposed signal received by backscatter will have a significant amplitude.

The distance $r_n$ from the focal point ($P$) to the $n^{th}$ wafer can be expressed as:

$$ r_n = \sqrt{(na)^2 + F^2 - 2F \cdot (na) \sin(\theta_F)} $$

$$ r_n = \frac{F}{\sin(90^\circ - \theta_F)} $$

Here, $\theta_F$ and $F$ are the angle and distance from the mid array element to P, respectively. Considering the $SH_0$ wave with velocity $c$, the pulse timings of the phased array, $\delta_n$, focused on focal point P can be represented as following relationship (for odd and even number arrays):

$$ \delta_n = \frac{F}{c} \left\{ 1 - \left[ 1 + \left( \frac{na}{F} \right)^2 - \frac{2na}{F} \sin \theta_F \right]^{1/2} \right\} + t_0, \text{ when } n = 0, \pm 1, \pm 2, \ldots \pm \left( \frac{N-1}{2} \right) $$

Figure 3: (a) Focal point of the phased array; (b) direction and amplitude of $d_{36}$ waves ($SH_0$, $S_0$, $A_0$).
\[
\delta_n = \frac{F}{c} \left[ 1 - \left( \frac{n + 0.5 \alpha}{F} \right)^2 - 2 \left( \frac{n + 0.5 \alpha}{F} \sin \theta \right) \right]^{1/2} + t_0, \text{ when } n = \pm 1, \pm 2, \ldots \pm \frac{N}{2}
\]

where the term \( t_0 \) is a constant to ensure the \( \delta_n \) does not become negative one[14]. Based on direction relative to the piezoelectric element, the amplitude of the SH \( n \) wave varies (Figure 3b). The amplitude also attenuates along a given line based on \( r_{n}^{-0.5} \). The summation of wave amplitudes (acoustic pressure, \( S_p \)) at the focal point is:

\[
S_p(t) = \sum_{n=-(N-1)/2}^{N-1/2} \frac{A_n}{\sqrt{r_n}} S_0 \left( t - \frac{r_n}{c} - \delta_n \right)
\]

If damage is present at the focal point, the ultrasonic waves are backscattered or reflected with a backscatter coefficient, \( B \). Considering the round trip of the ultrasonic waves, the signal received by each sensor of the passive phased array will be:

\[
S_s(t) = \sum_{n=-(N-1)/2}^{N-1/2} \frac{B \cdot A_n}{\sqrt{r_n + r_m}} S_0 \left( t - \frac{r_n + r_m}{c} - \delta_n \right)
\]

The assembled receive signal, \( S_R(t) \) will be enhanced and superposed \( N \) times which yields the damage location.

\[
S_R(t) = \sum_{n=-(N-1)/2}^{N-1/2} S_s \left( t - \delta_n \right)
\]

### 3.3 Focusing effect

Due to the fact that SH waves produced by \( d_{36} \)-type piezoelectric wafer are directional (see Figure 3b), different focal directions from a \( d_{36} \) array results in different focusing effects. As shown in Figure 4, when the focal direction is 10º, the main lobe of the phased array is significant in the wave field. When 40º, the main lobe is barely distinguished from the side lobe. At 70º, the amplitude of the main lobe is much higher than the side lobe; however, in this angle the focusing effect is not good because the width of the main lobe is too big.

In order to indicate the focusing effect and region of detection of a \( d_{36} \) phased array, a new parameter, \( MSR \), which is a function of angle and distance, is defined as the energy ratio of the main lobe (\( E_{Main} \)) and side lobe (\( E_{Side} \)):

\[
MSR(\theta, F) = \frac{E_{Main}(\theta, F)}{E_{Side}(\theta, F)}
\]

\( MSR \) allows the focusing effect of the whole detected region to be clearly presented as shown in Figure 5. There are two main regions which can provide accurate directional focusing of the wave. The first part is the area from about -30º to 30º relative to the orthogonal axis. Second, there is a small area near to the phased array where the array is effectively blind due to the inability of the array to form a synthesized wave front that close to the array; that is why the detected region does not start from the zero point.
In order to research and investigate the focusing and detection capabilities of the $d_{36}$-type piezoelectric phased array, numerical simulation is performed using the finite element software ANSYS. Here, solid 185 and solid 5 elements were implemented to build a numerical model of a thin plate (Figure 6). The size of the aluminum plate is $180 \times 180 \times 1 \text{mm}$ with the elasticity modulus, $E$, Poisson ratio, $\nu$, and density assumed to be 71.0 GPa, 0.33, and $2700 \text{kg/m}^3$, respectively. Four $d_{36}$-type piezoelectric wafers (with a piezoelectric coefficient matrix shown in equation (2)) with the dimension of $7 \times 7 \times 0.5 \text{mm}$ were attached on the plate surface with the spacing of 1mm. The distance from the piezoelectric patches and the edge of plate is 68mm and 75 mm in the x and y directions, respectively. The excitation signal of the piezoelectric wafers in the phased array is a tone-burst signal enclosed in a Hanning window with center frequency of 200kHz. In order to obtain accurate focusing results, the size of the element should be less than $1/20$ of the shortest wavelength among $A_0$, $S_0$, and $SH_0$ waves, and the time step should be less than $1/20$ of the central frequency $f_c$ [12]. The wavelength of the $S_0$ wave, $A_0$ wave and $SH_0$ is 27.4 mm, 6.6 mm and 15.8mm, respectively. Thus, the grid is chosen to be 0.25mm and time step is chosen as 0.1µs, both of which satisfy the aforementioned conditions.

If the aluminum plate is healthy, the focal point is 60 mm away from the central point of the phased array. The time delay of 4 piezoelectric wafers should be set as 0s, 0.336µs, 0.336µs, and 0s to achieve an angle of $0^\circ$. The focusing effect of this configuration is shown in Figure 7. $SH$ wave is in the plate plane which means it cannot be shown in the z-direction (out-of-plane). In viewing Figure 7a and 7b, it is evident that there are areas, especially along the

![Figure 6: The focusing effect of the $d_{36}$ phased array for various wave directions ($n = 4$).](image)

![Figure 5: Focusing effect of d36 phased array represented by the energy ratio of main lobe and side lobe (MSR) for $n = 4$: (a) radial view; (b) as a function of length.](image)

4  NUMERICAL SIMULATION OF D$_{36}$ PHASED ARRAY

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orthogonal axis away front array where x- and y-direction partial motion is occurring, suggesting the presence of shear waves. This is most evident in Figure 7d when all three components (x, y, and z) of partial motion are summed. Similarly, the particle motion of the y direction is parallel to the x direction (Figure 7a). Together, this indicates the SH wave is present. The focal point can be clearly distinguished in the simulation results, especially in Figure 7d. It is noted that due to the symmetry of phased array, there are 2 focal points in the plate. If the array time delays are 0s, 1.50µs, 2.77µs, 3.76 µs, respectively, then, the directional beam will focus at 30° with a 600 mm focal length. The simulation results of this case can be illustrated in Figure 8. Using the same model, a crack with the dimension of 10×1×1mm was introduced in the plate. The wave field in the y-direction of the plate corresponding to the 0° array configuration is shown in Figure 9 for various time steps in the simulation (namely, at 20µs, 25µs, 30µs, and 35µs). As we can see, there is significant reflected or backscattered ultrasonic SH0 wave from the crack, which can be used to obtain the location, direction and size of the damage.

Figure 6: Numerical model and the d36-type piezoelectric wafer phased array.

Figure 7: The focusing effect with the deflection angle of 0°: (a) x-direction; (b) y-direction; (c) z-direction; (d) total.

Figure 8: The focusing effect with the deflection angle of 30°: (a) x-direction; (b) y-direction; (c) z-direction; (d) total.
5 EXPERIMENTAL TEST

Four $d_{36}$-type piezoelectric wafers with 1mm spacing were attached to the top surface of an aluminum plate (Figure 10) in order to verify the proposed phased array method. Another passive phased array consisting of four $d_{36}$ wafers was attached at exactly the same location on the opposite side of the plate to receive backscattered waves. A National Instruments PXI Express Chassis (PXIe-1082) data acquisition system was used; the data acquisition cards used were PXIe-6361 and PXIe-6124 each supporting two analog output channels. In total, the system has 4 output channels to launch excitation signals in the four element phased array. The sampling frequency of the 4 simultaneously sampled analog inputs channels is 1 MHz per channel with 16 bits of resolution. The plate was hung using elastic rubber cords to reduce the ambient noise.

Two magnets roughly 12.7mm×25.4mm (0.5 inch × 1 inch) in area were mounted on both sides of the plate at the same location 150mm and 0° from the array (Figure 11). The magnets are intended to simulate damage by limiting the displacement of the aluminum plate at the magnet location. In order to identify the location and size of the “damage”, four 3-cycle Hanning-windowed tone burst signals at a central frequency of 40 kHz with different time delays were launched by the active phased array on one side of the plate. The direction of the ultrasonic wave beam was varied from -90° to 90° in increments of 15°. The focal length was also varied from 5mm and then increased in 2.5mm increments until 25mm. Simultaneously, the passive phased array on the opposite side of the plate was used to receive the signals of each case.

In this study, the $S_R(t)$ parameter of equation (8) is used to map damage in the plate (Figure 11b). To use this metric, the baseline signal is obtained (Figure 12a) which corresponds to no
damage (i.e., no magnets). The test is repeated with the damage condition (Figure 12b). The
time signals are lined up based on the measured excitation which is consistent between the two
cases. Once lined up, the signal is subtracted to yield the difference signal (Figure 12b). The
process is repeated for the various focal point locations to yield a map of damage (Figure 11b).
The map is accurate in identifying the magnet location and its geometry.

![Figure 11: (a) Location of mounted magnets to simulate damage; (b) damage detection results.](image)

![Figure 12: (a) Location of mounted magnets to simulate damage; (b) damage detection results.](image)

6 CONCLUSIONS

In this paper, a phased array of \( d_{36} \)-type piezoelectric wafers made from PMN-PT single
crystal materials is proposed to detect damage in metallic plate structures. This new \( d_{36} \)-type
phased array is able to emit directional SH waves in the plate and can focus those waves on a
specific point by controlling the time delay of each wafer in the array. In the numerical
simulation, an aluminum plate model was established in ANSYS to study the focusing capabilities of the phased array; the model was also used to explore waveforms with and without damage in the plate. In the experimental part, a phased array with four $d_{36}$-type piezoelectric wafers were attached on a plate to emit SH$_0$ waves scanning the plate for damage. By utilizing the received signals of all the sensors with the appropriate time delays, the location and size of the damage was accurately reconstructed.

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