8th European Workshop On Structural Health Monitoring (EWSHM 2016), 5-8 July 2016, Spain, Bilbao
www.ndt.net/app.EWSHM2016

Output-only fatigue prediction of uncertain steel structures

Vasilis Dertimanis[1], **Eleni Chatzi**[1], **Saeed Eftekhar Azam**[2], **Costas Papadimitriou**[2]

[1] Institute of Structural Engineering, ETH Zurich, Stefano-Franscini-Platz 5, 8093 Zurich, Switzerland, {v.derti,chatzi}@ibk.baug.ethz.ch

[2] Department of Mechanical Engineering, University of Thessaly, Leoforos Athinon, Pedion Areos, 38334 Volos, Greece, {eftekhar, costasp}@uth.gr

Keywords: fatigue life, state–input–parameter estimation, dual Kalman filter, unscented Kalman filter

Abstract

A fatigue estimation framework for steel structures is proposed in this study, under realistic assumptions on the sensor network capacity and under the premise of uncertainty in the structural information available throughout the life-cycle of the monitored structure. To this purpose, in a first step, a joint input-state-parameter estimation problem is formulated, which integrates the dual Kalman filter and the unscented Kalman filter. The former aims at estimating the unknown structural excitations, while the latter solves the state and parameter estimation problem that is closely related to the estimation of stresses in critical areas. Accordingly, in a second step, a fatigue estimator is developed using material fatigue data and damage accumulation rules, which evaluates the stresses at all unmeasured hotspot locations of the structure to the fatigue damage accumulation and prognosis of the remaining fatigue life. Numerical simulations under different measurement setups and available structural properties are presented, in order to demonstrate the method's effectiveness.

1. INTRODUCTION

A number of recent advances in the development of structural health monitoring methodologies and corresponding technologies focuses on the vibration-based fatigue estimation problem [1]. Certain practical issues are however limiting the applicability of these approaches. Among other issues, we emphasize on (i) the frequent inability of deploying sensors in critical areas, especially in existing structures, (ii) the fact that not all sensors are appropriate for permanent monitoring purposes. The latter applies especially for strain gauges, which may be ideal for measuring strain fields under short-term testing campaigns, yet, they may not intended for long-term permanent measurement installations that extend up to the life-cycle of the structure. This may be attributed to their progressive degradation, even when installed by very experienced personnel. Thus, strain estimation should additionally rely on further and more robust sensor types, such as accelerometers, which are generally characterized by high accuracy class, reduced operator-induced uncertainty and, following recent developments, highly accurate low consumption solutions, such as wireless MEMS accelerometers, which permit continuous and accurate acceleration measurements when dealing with structural systems.

Returning to the first practical limitation, it is noteworthy that non-collocated response estimation at hotspot points of structure calls for model based approaches. Kalman filter (KF) is the de facto standard for stochastic state estimation, which accomplishes the estimation in two stages of prediction and update. In the prediction stage, merely a model of the system is used to predict the state of the system in a future time. In the update stage however, the so called Kalman gain is applied to partial measurements of the state at the future time in order to correct the prediction. From what precedes it can be deduced that the KF requires knowledge of the input or its probability distribution, and a model of the system. When dealing with structural dynamics, in most operational cases the input may not be known or might be non-stationary and need to be somehow estimated for obtaining accurate estimates of the state. Moreover, degradations and deteriorating phenomena may cause changes in materials properties and geometry; subsequently, the parameters of the model also might need to be updated for ensuring a reliable estimate of the state.

In dealing with the state estimation for systems with unknown input, two categories of methods could be identified: one category of methods firstly identifies the unknown input and then feed the identified input into an observer for state estimation, whereas the second category yields a simultaneous estimation of the unknown input and the state. Within the frames of the former methods, in the time domain, Nordstrom et al. [2] have proposed a deconvolution method for considering the time delay and identifying non-collocated inputs. Bernal and Ussia [3] have proposed a sequential version of the method, including stability criteria that determines the length of the time window and regularization scheme for mitigating noise effects. Lai et al. [4] have performed an extensive parametric study of the sequential deconvolution and conclude that the method could accurately identify the input in a moderately noisy environment. Kazemi Amiri and Bucher [5] have developed a new parametric impulse response matrix utilized for nodal wind load identification by response measurement. Madarshahian et al. [6] have evaluated a time reversal method with dynamic time warping matching function for human fall detection using structural vibrations, especially useful for elderly people and patients with diseases such as Alzheimer's or other forms of dementia.

A number of optimal filtering methods are developed for simultaneous estimate of the states and unknown input. Kitanidis [7] established an unbiased minimum-variance recursive filter for input and state estimation of linear systems without direct transmission; his procedure did not consider any a-priori assumption on the input. Hsieh [8] reformulated the filter proposed by Kitanidis to make its application more expedient. The latter filter is not globally optimal in the mean square error sense; to address this issue Gillijns and De Moor [9] developed a new filter for joint input and state estimation. Their filter is developed for linear time invariant systems. In recent years, Lourens et al. [10] have suggested an amendment of the method developed in [9] in order to mitigate the numerical instabilities that arise when the number of sensors exceeds the order of the model, which is typically the case for large-scale structures. The effectiveness of the proposed modification was asserted via the joint input force and acceleration estimation of a simulated steel beam, a laboratory test beam and a large-scale steel bridge. It was observed that, even though the method delivers a reasonable estimate of the accelerations, the input force estimates are affected by spurious low frequency components, which were filtered out via use of band pass filters.

It is noteworthy that in dealing with the joint state and parameter estimation, Chatzi and Fuggini [11] have suggested a technique to alleviate the issues related to the spurious low frequency components in the displacement estimates by including synthetic displacement observations into the observation vector. Within a recursive Bayesian framework, Lourens et al. [12] have for the first time applied an augmented Kalman filter (AKF) for unknown input identification in structural systems. It was concluded that the AKF is susceptible to numerical instabilities due to un-observability issues of the augmented system matrix. Naets et al. [13] performed a study of the stability of the augmented Kalman filter when applied to unknown input and state estimation and demonstrate that the exclusive use of acceleration measurements can lead to unreliable results. In order to mitigate this issue, addition of dummy displacement measurements is recommended. To address the inadequacies of the above-mentioned approaches for state estimation of the structural systems with unknown general inputs, Eftekhar Azam et al. [14] proposed a novel dual Kalman filter (DKF) for state and unknown input estimation via sparse acceleration measurements. It is demonstrated through numerical and experimental investigations that the successive structure of DKF resolves numerical issues attributed to un-observability and rank deficiency of the AKF [15].

In this article the DKF is used for dealing with the uncertainties associated with the input. Concerning the uncertainties of the model of the system, employing a traditional model updating scheme would be prohibitive due to excessive computational burden and the need for user interference [16]. Herein a recursive Bayesian filter is incorporated within the state estimation phase of the dual estimation of the input-state. In doing so, the uncertain parameters of the system are augmented into the state vector, and a Bayesian filter is employed for joint state-parameter estimation. It is noteworthy that, augmenting the parameters into the state vector, even in the case of a system represented with linear model leads to a nonlinear system. Extended Kalman filter (EKF) is often used for state estimation in case of weakly nonlinear systems. The EKF requires evaluation of the Jacobean of the state mapping at each time step. Moreover, it delivers first order of accuracy for the estimated statistics of the state. In the structural dynamics, the use of Unscented Kalman filter at the cost of higher computational cost is typically preferable to the EKF, see e.g. [17, 18]. To alleviate the computational burden of the UKF, Eftekhar Azam et al. [19] proposed a parallel implementation of it. In this study the behavior of the structure is assumed to be linear, and therefore the UKF deems suitable for the task of joint state-parameter estimation. When dealing with nonlinear structures such as systems with nonlinear hysteretic response or softening materials the use of advanced particle filters is recommended, see [20, 21].

The article is organized as follows: in Section 2 the mathematical formulation of the problem is overviewed for introducing the notations. In Section 3 the notion of input-state-parameter observer is developed based on a synergy of the dual and joint estimation concepts. In Section 4 the fatigue rule employed in this study is outlined and Section 5 is devoted to illustration of the performance of the procedure when dealing with a simulated experiment scenario. Finally, in Section 6 the results are summarized and suggestions for further research are provided.

2. PROBLEM FORMULATION

A steel structure can be modeled as a linear structural system with n DOFs in the form

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}(\boldsymbol{\theta})\mathbf{x}(t)=\mathbf{P}\mathbf{f}(t) \tag{1}$$

in which $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}(\boldsymbol{\theta}) \in \mathbb{R}^{n\times n}$ are the symmetric and positive definite mass, viscous damping and stiffness matrices, respectively, $\mathbf{x}(\mathrm{t}) \in \mathbb{R}^{n}$ is the vibration displacement vector, and $\mathbf{f}(\mathrm{t}) \in \mathbb{R}^{m}$ is the vector of excitations that act on specific DOFs of the structure, as indicated by the influence matrix $\mathbf{P} \in \mathbb{R}^{n\times m}$. It is assumed that the steel structure exhibits a certain level of uncertainty in its stiffness properties, implying that the stiffness matrix depends on the parameter vector $\boldsymbol{\theta}$, the values of which are unknown a-priori. This uncertainty may be the result of unknown/unmodelled changes due to material degradation and/or environmental influences that produce a shift to the system properties.

By defining the $[2n\times 1]$ state vector as $\boldsymbol{\xi}(t)=[\mathbf{x}^T(t)\ \ \dot{\mathbf{x}}^T(t)]^T$, a state–space representation of Eq. 1 is given by

$$\dot{\boldsymbol{\xi}}(t)=\mathbf{A}_c(\boldsymbol{\theta})\boldsymbol{\xi}(t)+\mathbf{B}_c\mathbf{f}(t) \tag{2a}$$

$$\mathbf{y}(t)=\mathbf{H}_c(\boldsymbol{\theta})\boldsymbol{\xi}(t)+\mathbf{D}_c\mathbf{f}(t) \tag{2b}$$

where the matrices of the state equation (Eq. 2a) are given by

$$\mathbf{A}_c(\boldsymbol{\theta})=\begin{bmatrix}\mathbf{O}_n & \mathbf{I}_n\\ -\mathbf{M}^{-1}\mathbf{K}(\boldsymbol{\theta}) & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}\ \text{and}\ \mathbf{B}_c=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{P}\end{bmatrix} \tag{3}$$

and the matrices of the output equation (Eq. 2b) depend on the type of structural response available. For the case of vibration acceleration measurements at specific DOFs it follows that

$$\mathbf{H}_c(\boldsymbol{\theta})=\mathbf{S}_a\begin{bmatrix}-\mathbf{M}^{-1}\mathbf{K}(\boldsymbol{\theta}) & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}\ \text{and}\ \mathbf{D}_c=\mathbf{S}_a\mathbf{M}^{-1}\mathbf{P} \tag{4}$$

for some matrix $\mathbf{S}_a$ that maps the states to the measured DOFs. Discretization of Eq. 2 at a sampling period T_s (s), assuming constant intersample behaviour of the inputs, implies

$$\boldsymbol{\xi}[k+1]=\mathbf{A}_d(\boldsymbol{\theta})\boldsymbol{\xi}[k]+\mathbf{B}_d\mathbf{f}[k] \tag{5a}$$

$$\mathbf{y}[k]=\mathbf{H}_d(\boldsymbol{\theta})\boldsymbol{\xi}[k]+\mathbf{D}_d\mathbf{f}[k] \tag{5b}$$

for $\mathbf{A}_d(\boldsymbol{\theta})=\mathbf{I}_n+T_s\mathbf{A}_c(\boldsymbol{\theta})+0.5T_s^2\mathbf{A}_c^2(\boldsymbol{\theta})$, $\mathbf{B}_d=T_s\mathbf{B}_c$, $\mathbf{H}_d(\boldsymbol{\theta})=\mathbf{H}_c(\boldsymbol{\theta})$ and $\mathbf{D}_d=\mathbf{D}_c$.

3. DERIVATION OF THE STATE-INPUT-PARAMETER OBSERVER

In a first step, two fictitious equations that correspond to $\mathbf{f}[k]$ and $\boldsymbol{\theta}$, respectively, are introduced as

$$\mathbf{f}[k+1] = \mathbf{f}[k] + \mathbf{w}_f[k] \tag{6a}$$

$$\boldsymbol{\theta}[k+1] = \boldsymbol{\theta}[k] + \mathbf{w}_\theta[k] \tag{6b}$$

where $\mathbf{w}_f[k]$ and $\mathbf{w}_\theta[k]$ are zero mean Gaussian processes of covariance matrices $\boldsymbol{\Sigma}_{ff}$ and $\boldsymbol{\Sigma}_{\theta\theta}$, respectively. An augmented state vector is accordingly defined as $\bar{\boldsymbol{\xi}}[k] = (\boldsymbol{\xi}[k], \boldsymbol{\theta}[k])^T$ and a corresponding state–space model is formulated from Eqs. 5, 6b as

$$\bar{\boldsymbol{\xi}}[k+1] = \underbrace{\begin{bmatrix} \mathbf{A}_d(\boldsymbol{\theta}) & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}}_{\mathbf{A}(\boldsymbol{\theta})} \bar{\boldsymbol{\xi}}[k] + \underbrace{\begin{bmatrix} \mathbf{B}_d \\ \mathbf{O} \end{bmatrix}}_{\mathbf{B}} \mathbf{f}[k] + \underbrace{\begin{bmatrix} \mathbf{w}_\xi[k] \\ \mathbf{w}_\theta[k] \end{bmatrix}}_{\mathbf{p}[k]} \equiv f\big(\bar{\boldsymbol{\xi}}[k], \mathbf{f}[k]\big) + \mathbf{p}[k] \tag{7a}$$

$$\mathbf{y}[k] = \underbrace{\begin{bmatrix} \mathbf{H}_d(\boldsymbol{\theta}) & \mathbf{O} \end{bmatrix}}_{\mathbf{H}(\boldsymbol{\theta})} \bar{\boldsymbol{\xi}}[k] + \underbrace{\mathbf{D}_d}_{\mathbf{D}} \mathbf{f}[k] + \mathbf{r}[k] \equiv g\big(\bar{\boldsymbol{\xi}}[k], \mathbf{f}[k]\big) + \mathbf{r}[k] \tag{7b}$$

In Eq. 7a the quantity $\mathbf{w}_\xi[k]$ denotes zero mean Gaussian process noise of covariance matrix $\boldsymbol{\Sigma}_{\xi\xi}$, uncorrelated with $\mathbf{w}_\theta[k]$, that has been superimposed to Eq. 5a. The process noise $\mathbf{p}[k]$ of the augmented state equation is in turn characterized by zero mean and covariance matrix $\boldsymbol{\Sigma}_{pp} = \mathrm{diag}\{\boldsymbol{\Sigma}_{\xi\xi}, \boldsymbol{\Sigma}_{\theta\theta}\}$. In Eq. 7b the quantity $\mathbf{r}[k]$ denotes zero mean Gaussian measurement noise of covariance matrix $\boldsymbol{\Sigma}_{rr}$.

3.1 Input estimation: the DKF

From Eqs. 6a and 7b it follows that a new state–space model can be considered, in which $\mathbf{y}[k]$ is the observed quantity, $\mathbf{f}[k]$ is the unknown state and $\bar{\boldsymbol{\xi}}[k]$ plays the role of a known input to the system, when an estimate becomes available through the UKF (see Sec. 3.2):

$$\mathbf{f}[k+1] = \mathbf{f}[k] + \mathbf{w}_f[k] \tag{8a}$$

$$\mathbf{y}[k] = \mathbf{H}(\theta)\bar{\boldsymbol{\xi}}[k] + \mathbf{D}\mathbf{f}[k] + \mathbf{r}[k] \tag{8b}$$

Thus, through the implementation of the standard Kalman filter, an online estimation of $\mathbf{f}[k]$ can be obtained. To this, the measurement update step calculates the input gain, mean and covariance as,

$$\mathbf{K}_f[k] = \Big(\mathbf{D}\mathbf{P}_f[k|k-1]\mathbf{D}^T + \boldsymbol{\Sigma}_{rr}\Big)^{-1} \mathbf{P}_f[k|k-1]\mathbf{D}^T \tag{9a}$$

$$\mathbf{f}[k|k] = \mathbf{f}[k|k-1] + \mathbf{K}_f[k]\Big(\mathbf{y}[k] - \mathbf{H}(\theta)\bar{\boldsymbol{\xi}}[k|k-1] - \mathbf{D}\mathbf{f}[k|k-1]\Big) \tag{9b}$$

$$\mathbf{P}_f[k|k] = \mathbf{P}_f[k|k-1] + \mathbf{K}_f[k]\mathbf{D}^T\mathbf{P}_f[k|k-1] \tag{9c}$$

where $\mathbf{f}[k|k-1]$, $\mathbf{P}_f[k|k-1]$ and $\bar{\boldsymbol{\xi}}[k|k-1]$ denote one–step ahead predictions of the input mean, input covariance and state mean at time k using data up to and including time $[k-1]$, respectively. Accordingly, during the time update step, the input mean and covariance predictions for time $k+1$ are provided by

$$\mathbf{f}[k+1|k] = \mathbf{f}[k|k] \tag{10a}$$

$$\mathbf{P}_f[k+1|k] = \mathbf{P}_f[k|k] + \boldsymbol{\Sigma}_{ff} \tag{10b}$$

in accordance to Eq. 8a.

3.2 State and parameter estimation: the UKF

The UKF provides a solution to the joint state and parameter estimation problem, which, by the state matrix of Eq. 7a, includes bilinear products between the original state $\boldsymbol{\xi}[k]$ and the unknown parameter vector $\boldsymbol{\theta}$. A number of filter parameters are initially defined [22]

$$\begin{aligned}
&\alpha = 1,\ \ \beta = 2,\ \ \kappa = 0,\ \ \lambda = \alpha^2(n_\xi + \kappa) - n_\xi,\ \ c = \alpha^2(n_\xi + \kappa)\\
&W_m^0 = \lambda/(n_\xi + \lambda),\ \ W_m^i = 1/2(n_\xi + \lambda),\ \ i = 1,2,\dots,2n_\xi\\
&W_c^0 = \lambda/(n_\xi + \lambda) + (1 - \alpha^2 + \beta),\ \ W_c^i = W_m^i,\ \ i = 1,2,\dots,2n_\xi\\
&\boldsymbol{\mu}_\xi = [W_m^0\ \dots\ W_m^{2n_\xi}]^T\\
&\mathbf{M}_\xi = \big(\mathbf{I} - [\boldsymbol{\mu}_\xi\ \dots\ \boldsymbol{\mu}_\xi]\big) \times diag(W_c^0\ \dots\ W_c^{2n_\xi}) \times \big(\mathbf{I} - [\boldsymbol{\mu}_\xi\ \dots\ \boldsymbol{\mu}_\xi]\big)^T
\end{aligned} \tag{11}$$

Then, given measurement data at time k, $\mathbf{y}[k]$ and the input estimate $\mathbf{f}[k|k]$ from the DKF, a set of sigma points is calculated

$$\overline{\boldsymbol{\Xi}}[k|k-1] = \Big[\bar{\boldsymbol{\xi}}[k|k-1]\ \dots\ \bar{\boldsymbol{\xi}}[k|k-1]\Big] + \sqrt{c}\Big[\mathbf{0}\ \sqrt{\mathbf{P}_\xi[k|k-1]}\ \dots\ -\sqrt{\mathbf{P}_\xi[k|k-1]}\Big] \tag{12}$$

and directed to Eq. 7b to calculate a set of output vectors

$$\widehat{\mathbf{Y}}[k|k] = g\big(\overline{\boldsymbol{\Xi}}[k|k-1], \mathbf{f}[k|k]\big) \tag{13}$$

Accordingly, the output mean and covariance, as well as the cross covariance between the state and the output are calculated by,

$$\hat{\mathbf{y}}[k] = \widehat{\mathbf{Y}}[k|k]\boldsymbol{\mu}_\xi \tag{14a}$$

$$\widehat{\boldsymbol{\Sigma}}_{yy}[k] = \widehat{\mathbf{Y}}[k|k]\mathbf{M}_\xi\widehat{\mathbf{Y}}^T[k|k] + \boldsymbol{\Sigma}_{rr} \tag{14b}$$

$$\widehat{\boldsymbol{\Sigma}}_{\xi y}[k] = \overline{\boldsymbol{\Xi}}[k|k-1]\mathbf{M}_\xi\widehat{\mathbf{Y}}^T[k|k] \tag{14c}$$

respectively, while the UKF filter gain is calculated by

$$\widehat{\boldsymbol{\Sigma}}_{yy}[k]\mathbf{K}_\xi[k] = \widehat{\boldsymbol{\Sigma}}_{\xi y}[k] \tag{15}$$

The augmented state mean and covariance matrix are then updated as

$$\bar{\boldsymbol{\xi}}[k|k] = \bar{\boldsymbol{\xi}}[k|k-1] + \mathbf{K}_\xi[k]\Big(\mathbf{y}[k] - \hat{\mathbf{y}}[k]\Big) \tag{16a}$$

$$\mathbf{P}_\xi[k|k] = \mathbf{P}_\xi[k|k-1] + \mathbf{K}_\xi[k]\widehat{\boldsymbol{\Sigma}}_{yy}[k]\mathbf{K}_\xi^T[k] \tag{16b}$$

In the time update step a new set of sigma points is calculated

$$\overline{\boldsymbol{\Xi}}[k|k] = \Big[\bar{\boldsymbol{\xi}}[k|k]\ \dots\ \bar{\boldsymbol{\xi}}[k|k]\Big] + \sqrt{c}\Big[\mathbf{0}\ \sqrt{\mathbf{P}_\xi[k|k]}\ \dots\ -\sqrt{\mathbf{P}_\xi[k|k]}\Big] \tag{17}$$

and directed to Eq. 7a to calculate a set of state vectors

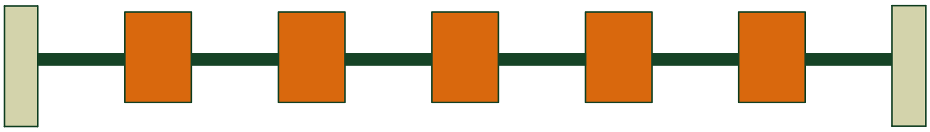
$$\widehat{\boldsymbol{\Xi}}[k+1|k] = f\big(\overline{\boldsymbol{\Xi}}[k|k], \mathbf{f}[k+1|k]\big) \tag{18}$$

Then, a prediction of the augmented state mean and covariance for time $k+1$ is obtained by

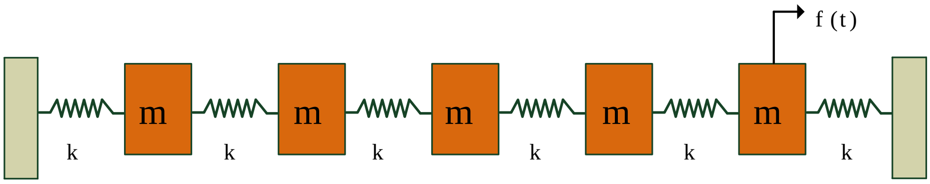
$$\bar{\boldsymbol{\xi}}[k+1|k] = \widehat{\boldsymbol{\Xi}}[k+1|k]\boldsymbol{\mu}_\xi \tag{19a}$$

$$\mathbf{P}_\xi[k+1|k] = \widehat{\boldsymbol{\Xi}}[k+1|k]\mathbf{M}\widehat{\boldsymbol{\Xi}}^T[k+1|k] + \boldsymbol{\Sigma}_{pp} \tag{19b}$$

with the former being forwarded to Eq. 10 for the calculation of the unknown force vector during the next iteration of the process.



(a) Masses connected with bars



(b) N-DOF equivalent

Figure 1 : Spring-mass chain-like model

4. FATIGUE DAMAGE

By applying the theory of structural mechanics, the stresses that are induced by the application of the unknown excitation forces can be calculated by the estimated state vector via [1]

$$\boldsymbol{\sigma}[k] = \mathbf{T}\boldsymbol{\xi}[k] \tag{20}$$

with $\mathbf{T}$ denoting a transformation matrix that associates the state vector to the desired stresses in the entire structure. The calculated stresses can then be used in conjunction with an appropriate fatigue accumulation scheme, in order to predict the fatigue damage and the remaining useful life of the structure. A common such scheme is described in Eurocode 3 [23] and pertains to the implementation of the Palmgren–Miner rule [24, 25], according to which the damage accumulation at a point in the structure that is amenable to stresses of variable amplitude is defined as

$$D = \sum_{j=1}^{k} \frac{n_j}{N_j} \tag{21}$$

where n_j and N_j correspond to the number of cycles at a stress level $\Delta\sigma_j$ and the number of cycles required for failure at the same stress level, respectively, and k is the number of stress levels considered for fatigue damage assessment. The quantities n_j and $\Delta\sigma_j$ are usually estimated using the rainflow counting method (ASTM) and correspond to a specific fatigue detail category. The interesting reader is referred to Eurocode 3 [23] for further details.

5. APPLICATION STUDY

The 5 DOF structure of Fig. 1 is now considered for the assessment of the proposed method. The system consists of a series of masses connected through bars (Fig. 1(a)). Each bar has length L, cross–sectional area A and modulus E, and it is characterized by unknown/uncertain equivalent stiffness $k = EA/L$ [1] (Fig. 1(b)), while every structural mode is attributed with 1% modal damping. The structure is excited at its far right mass by an unknown external force. Table 1 displays the numerical values of all the associated structural and geometric parameters.

The structural system is accordingly simulated (via its discrete–time state–space representation, see Eq. 5) at a sampling period $T_s = 0.005$ s (sampling frequency $F_s = 200$ Hz) for approximately 5 s. A zero mean Gaussian white noise process with a variance of 10^5 N is used as excitation. The vibration acceleration responses of the first (from the left), the second and the fifth mass are considered as the available data, the final length of which is $N = 10000$ values. Based on the estimated state vector and

Table 1 : Structural and geometric parameters of the system.

Parameter	m	L	A	E	k
Value	30	0.3	$6\pi \times 10^{-3}$	2.1×10^{11}	1.98×10^{7}
Unit	kg	m	m^2	$\mathrm{N/m}^2$	N/m

Table 2 : Adopted numerical values for the parameters of the DKF–UKF filter.

Quantity	**Value**	**Notes**
$\boldsymbol{\xi}[0]$	$\mathbf{0}_{1\times 10}$	initial value for the state vector of Eq. 5a
$\mathbf{f}[0]$	0	initial value for the input vector
$\boldsymbol{\theta}[0]$	2.3×10^{7}	initial value for the parameter vector $\boldsymbol{\theta}$
$\bar{\boldsymbol{\xi}}[0]$	$(\boldsymbol{\xi}[0], \boldsymbol{\theta}[0])^T$	initial value for the augmented state vector
$\mathbf{P}_f[0]$	10	initial values for the covariance matrix of $\mathbf{f}[0]$
$\mathbf{P}_\xi[0]$	$\mathrm{diag}(10^{-15}, \ldots, 10^{-15}, 10)$	initial values for the covariance matrix of $\bar{\boldsymbol{\xi}}_0$
$\boldsymbol{\Sigma}_{ff}$	10	process noise covariance matrix associated with $\mathbf{f}$
$\boldsymbol{\Sigma}_\theta$	10^{-1}	process noise covariance matrix associated with $\boldsymbol{\theta}$
$\boldsymbol{\Sigma}_{\xi\xi}$	$10^{-14} \times \mathbf{I}_{10}$	process noise covariance matrix associated with $\boldsymbol{\xi}$
$\boldsymbol{\Sigma}_{pp}$	$\mathrm{diag}(\boldsymbol{\Sigma}_{\xi\xi}, \boldsymbol{\Sigma}_\alpha)$	augmented process noise covariance matrix
$\boldsymbol{\Sigma}_{rr}$	$\mathrm{diag}(1,1,1) \times 10^{-6}$	measurement noise covariance matrix

by considering that the bars are amenable only to horizontal stresses, fatigue life estimation is based on Eqs. 20–21 using a transformation matrix that is given by

$$\mathbf{T} = \frac{E}{L} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

while the adopted numerical values for the DKF–UKF filter are shown at Tab. 2.

The results of the simulation are presented in Figs. 2–6 and they are very encouraging, as the estimated quantities follow the true ones with a high degree of accuracy. The unknown input estimate (Fig.2) follows the real force already from the start of the simulation, while similar performance is observed in the displacement and velocity states (Figs.3–4). The unknown stiffness parameter has also been successfully identified (Fig. 5) with rapid convergence to the vicinity of the true value. Finally, the estimated remaining fatigue life of the bars has resulted close to their theoretical counterparts (Fig. 6), returning percentage errors that vary from about 2% to 12% (Tab. 3). These discrepancies are rather attributed to the convergence of the state vector, yet they deserve further attention, especially in respect to the proper tuning of the filter.

Table 3 : Percentage difference between actual and estimated fatigue life.

Bar no.	1	2	3	4	5	6
Error (%)	8.39	5.76	8.97	4.64	12.45	1.76

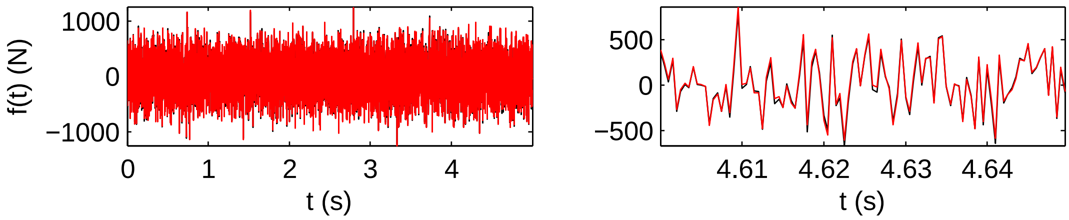


Figure 2 : Actual (black) versus estimated (red) input force. Left figure: total simulation time. Right figure: approximately 50ms detail.

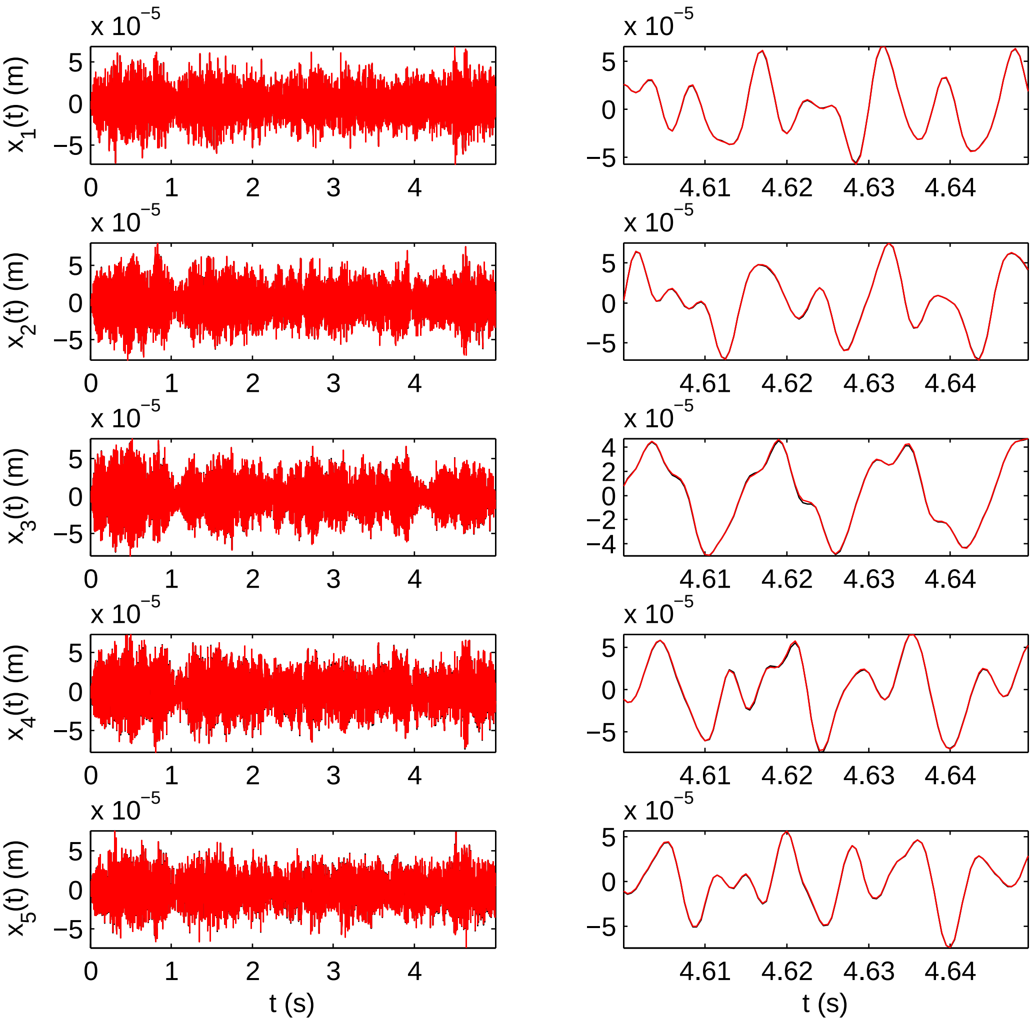


Figure 3 : Actual (black) versus estimated (red) displacements. Left figure: total simulation time. Right figure: approximately 50ms detail.

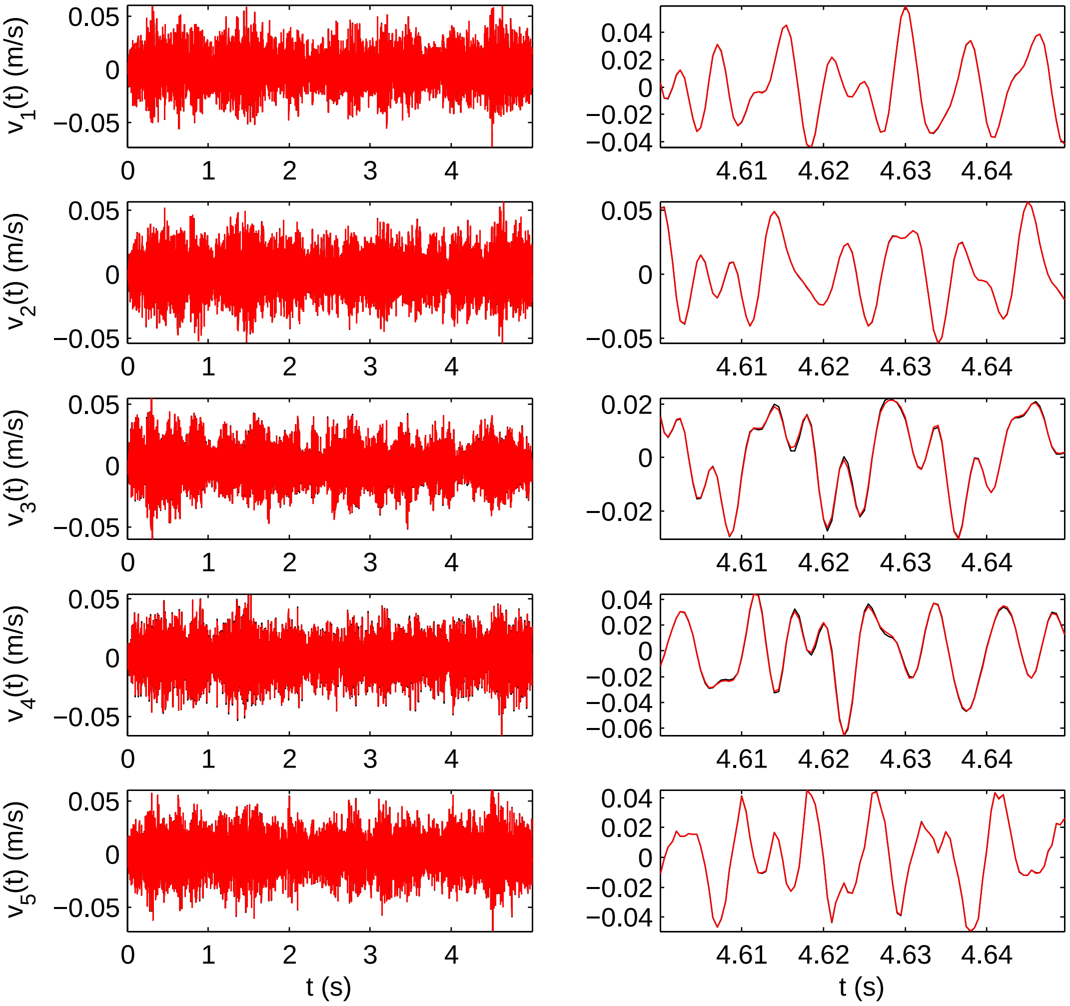


Figure 4 : Actual (black) versus estimated (red) displacements. Left figure: total simulation time. Right figure: approximately 50ms detail.

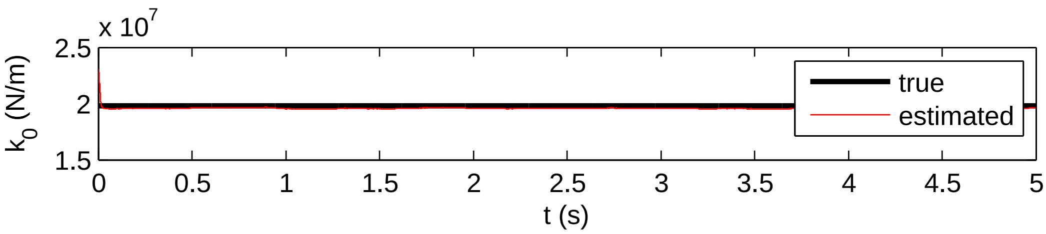


Figure 5 : Actual (black) versus estimated (red) stiffness.

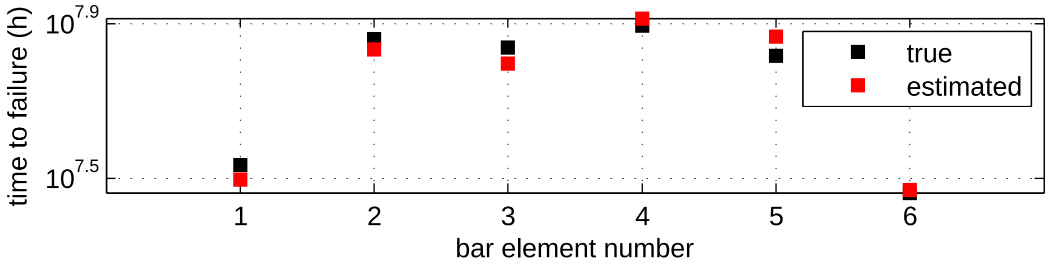


Figure 6 : Actual (black) versus estimated (red) remaining fatigue life of each bar.

6. CONCLUSIONS

This paper presents a fatigue lifetime assessment scheme under standard assumptions regarding the availability of structural and data information. It was showed that the fatigue estimation problem can be viewed as joint input-state-parameter estimation problem of a structural system with uncertain properties that can be treated using limited, noise-corrupted observations. To this end, a DKF-UKF framework was developed and applied to a spring–mass chain structural model amenable to white noise excitation. The proposed scheme introduces two fictitious process equations that aim at resembling the unknown dynamics of the structural excitation and the evolution of the uncertain parameter vector, respectively. Then, a DKF is established for the measurement and time update of the unknown input. This input is forwarded to an augmented state-space model, the state vector of which contains both the original states of the structure and the vector of unknown structural parameters. Since the latter two quantities are nonlinearly related, the corresponding state-parameter estimation problem is handled by the UKF.

Further investigation of the induced observer performance is currently being undertaken by the authors, in respect also to a number of important issues that deserve further investigation. Indeed, the robustness of the fictitious equations is studied, in order to account for a wider class of inputs, which range from purely sinusoidal to severely nonstationary, as well as uncertain parameters with arbitrary functional relationships to the structural matrices. Stability, observability and convergence analysis is additionally perfromed, in an effort to formulate a rigorous and robust framework for fatigue prediction of structures under realistic conditions.

ACKNOWLEDGEMENTS

This research has been partially implemented under the ARISTEIA Action of the Operational Programme Education and Lifelong Learning and was co-funded by the European Social Fund (ESF) and Greek National Resources.

REFERENCES

[1] C. Papadimitriou, C. P. Fritzen, P. Kraemer, and E. Ntotsios. Fatigue predictions in entire body of metallic structures from a limited number of vibration sensors using kalman filtering. *Structural Control and Health Monitoring*, 18(5):554–573, 2011.

[2] Lars J. L. Nordstrm and T. Patrik Nordberg. A time delay method to solve non-collocated input estimation problems. *Mechanical Systems and Signal Processing*, 18(6):1469–1483, 2004.

[3] D. Bernal and A. Ussia. Sequential deconvolution input reconstruction. *Mechanical Systems and Signal Processing*, 50-51:41–55, 2015.

[4] Tao Lai, Ting-Hua Yi, and Hong-Nan Li. Parametric study on sequential deconvolution for force identification. *Journal of Sound and Vibration*, 2016.

[5] A. Kazemi Amiri and C. Bucher. Derivation of a new parametric impulse response matrix utilized for nodal wind load identification by response measurement. *Journal of Sound and Vibration*, 344(0):101–113, 2015.

[6] Ramin Madarshahian, Juan M Caicedo, and Diego Arocha Zambrana. *Evaluation of a time reversal method with dynamic time warping matching function for human fall detection using structural vibrations*, pages

171–176. Springer International Publishing, 2014.

[7] P. K. Kitanidis. Unbiased minimum-variance linear state estimation. *Automatica*, 23(6):775–778, 1987.

[8] C. S. Hsieh. Robust two-stage kalman filters for systems with unknown inputs. *IEEE Transactions on Automatic Control*, 45(12):2374–2378, 2000.

[9] S. Gillijns and B. De Moor. Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough. *Automatica*, 43(5):934–937, 2007.

[10] E. Lourens, E. Reynders, G. De Roeck, G. Degrande, and G. Lombaert. An augmented kalman filter for force identification in structural dynamics. *Mechanical Systems and Signal Processing*, 27(1):446–460, 2012.

[11] Eleni Chatzi and Clemente Fuggini. Online correction of drift in structural identification using artificial white noise observations and an unscented kalman filter. *Smart Structures and Systems*, 16(2):295–328, in press.

[12] E. Lourens, E. Reynders, G. De Roeck, G. Degrande, and G. Lombaert. An augmented kalman filter for force identification in structural dynamics. *Mechanical Systems and Signal Processing*, 27(1):446–460, 2012.

[13] F. Naets, J. Cuadrado, and W. Desmet. Stable force identification in structural dynamics using kalman filtering and dummy-measurements. *Mechanical Systems and Signal Processing*, 5051(0):235–248, 2015.

[14] S. Eftekhar Azam, E. Chatzi, and C. Papadimitriou. A dual kalman filter approach for state estimation via output-only acceleration measurements. *Mechanical Systems and Signal Processing*, 60:866–886, 2015.

[15] S.; Eftekhar Azam, E.; Chatzi, C.; Papadimitriou, and A. W.; Smyth. Experimental validation of the kalman-type filters for online and real-time state and input estimation. *Journal of Vibration and Control*, 2015.

[16] Ramin Madarshahian, Juan M Caicedo, and Zhuoxiong Sun. Direct inverse finite element model updating, 2012.

[17] S. Eftekhar Azam, M. Bagherinia, and S. Mariani. Stochastic system identification via particle and sigma-point kalman filtering. *Scientia Iranica*, 19(4):982–991, 2012.

[18] E. N. Chatzi and A. W. Smyth. The unscented kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing. *Structural Control and Health Monitoring*, 16(1):99–123, 2009.

[19] S. Eftekhar Azam, A. Ghisi, and S. Mariani. Parallelized sigma-point kalman filtering for structural dynamics. *Computers and Structures*, 92-93:193–205, 2012.

[20] S. Eftekhar Azam and S. Mariani. Dual estimation of partially observed nonlinear structural systems: A particle filter approach. *Mechanics Research Communications*, 46:54–61, 2012.

[21] E. N. Chatzi and A. W. Smyth. Particle filter scheme with mutation for the estimation of time-invariant parameters in structural health monitoring applications. *Structural Control and Health Monitoring*, 20(7):1081–1095, 2013.

[22] S. Särkkä. *Recursive Bayesian inference on stochastic differential equations*. PhD thesis, Helsinki University of Technology, Helsinki, Finland, 2006.

[23] European Committee for Standardization. En 1993- eurocode 3. design of steel structures - part 1-9: fatigue, 2004.

[24] A. Palmgren. Die lebensdauer von kugallagern. *VDI-Zeitschrift*, 68(14):416–425, 1924.

[25] M. A. Miner. Cumulative damage in fatigue. *ASME Applied Mechanics Transactions*, 12(3):159–164, 1945.