8th European Workshop On Structural Health Monitoring (EWSHM 2016), 5-8 July 2016, Spain, Bilbao
www.ndt.net/app.EWSHM2016

Vibration Based Damage Detection Techniques for Small to Medium Span Bridges: A Review and Case Study

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Abstract

Overtime, the structural condition of bridges tends to decline due to a number of degradation processes, such as; creep, corrosion and cyclic loading, among others. Considerable research has been conducted over the years to assess and monitor the rate of such degradation with the aim of reducing structural uncertainty. Traditionally, vibration-based damage detection techniques in bridges have focused on monitoring changes to modal parameters and subsequently comparing them to numerical models. These traditional techniques are generally time consuming and can often mistake changing environmental and operational conditions as structural damage. Recent research has seen the emergence of more advanced computational techniques that not only allow the assessment of noisier and more complex data, but also allow research to veer away from monitoring changes in modal parameters alone. This paper presents a review of the current state-of-the-art developments in vibration based damage detection in small to medium span bridges with particular focus on the utilization of advanced computational methods that avoid traditional damage detection pitfalls. A case study of the S101 Bridge is also presented to test the damage sensitivity a chosen methodology.

Key words: Structural Health Monitoring, Damage Detection,

1 INTRODUCTION

The identification of structural damage in bridges is a research topic that has generated significant attention in recent years. The primary reason for its surge in popularity is an aging road and rail infrastructure, which is subjected to traffic loading conditions that far surpass original design criteria. This unprecedented increase in loading accelerates structural fatigue, which in turn reduces service-life. Additionally, as bridge infrastructure continues to age and deteriorate, the frequency of inspection must increase to counteract the reduction in safety of these structures. This task is made more difficult due to its sheer enormity, as Europe's highway bridge count is circa one million, and of Europe's half a million rail bridges, 35% are over 100 years old [1]. This has lead to a considerable surge of research in how to efficiently manage their maintenance and upkeep [2]. Most proliferous, however, is the study of vibration based damage detection and identification techniques.

This paper is the product of an initial research review constructed by the first author to determine the most suitable and effective methodology for damage detection in bridges from vibration data so that further work can be conducted by the authors. Suitable methodologies will be chosen through a process of elimination based on the following criteria; robustness under varying conditions, suitability for long term monitoring applications, proven field-based performance.

2 DEVELOPMENT OF MODAL-BASED DAMAGE DETECTION TECHNIQUES

As a first pass, the authors investigated the traditional modal-based damage detection techniques, as their theory and application were well known. The following section outlines some traditional modal based methods and aims to highlight their advantages and disadvantages.

The concept of using measured vibrations to discern damage in structures has been employed for some time. Various modal parameters, such as natural frequency shifts and other modal properties such as mode shapes, damping ratios and modal curvatures have been traditionally used to detect damage. Mode shapes are particularly advantageous as they are less influenced by environmental effects than natural frequencies and also contain both local and global information, which can aid damage localization. Numerous mode shape monitoring techniques have been developed over the years, such as the Modal Assurance Criterion (MAC) [3], which measures mode shape changes over the entire structure by taking advantage of eigenvector orthogonality. Kim et al. [4] later advanced MAC to develop the Coordinate Modal Assurance Criterion (COMAC), which monitors modal node displacement to detect and locate damage. Eq. (1) shows how COMAC can be applied to a node i, by measuring the normalised difference of mode shape vectors of the undamaged ($\varphi^{u}_{i,j}$) and damaged ($\varphi^{d}_{i,j}$) conditions. Application of MAC and COMAC in bridge structures found that the methods could detect most structural changes and locations, but also indentified spurious damage as well [5].

$$COMAC_{i,j} = \frac{\left[\sum_{j=1}^{m} \varphi^{u}_{i,j} \quad \varphi^{d}_{i,j}\right]^2}{\sum_{j=1}^{m}\left(\varphi^{u}_{i,j}\right)^2 \sum_{j=1}^{m}\left(\varphi^{d}_{i,j}\right)^2} \tag{1}$$

Pandey et al. [6] expanded COMAC's theory further to focus on the monitoring of mode shape curvatures (mode shapes' second derivative) in a technique known as the Modal Curvature Method (MCM). Its hypothesis is based on the relationship between modal curvature and flexural stiffness, as presented in Eq. (2) where modal curvature (φ'') is a function of cross-sectional bending moment (M) and cross-sectional flexural stiffness (EI). The premise of the MCM is that by using this relationship, one can monitor stiffness variations and detect damage, as cracks will reduce cross-sectional stiffness, resulting in a larger curvature value. Eq. (3) shows that the MCM simply uses the absolute difference between the damaged curvature ($\varphi''_{d,j}$) and undamaged curvature ($\varphi''_{u,j}$) values to detect damage. This can be conducted for single mode or for cumulative multi-mode, depending on application. This methodology demonstrated a high level of damage sensitivity and produced good results when tested [7]. The MCM has some drawbacks however; its results are dependent on the number of modes considered [8], inherent errors in curvature calculation from vibration data, usually through the central difference method, reduce the MCM's robustness. Furthermore, the MCM also requires a large quantity of sensors to ensure sufficient accuracy, particularly for higher modes, which thus reduces its practicality for mass application.

$$\varphi'' = \frac{M}{EI} \tag{2}$$

$$\Delta\varphi'' = \sum_{j=1}^{m}\left(\varphi''_{d,j} - \varphi''_{u,j}\right) \tag{3}$$

Modal curvatures have formed part of numerous damage detection methodologies since introduced. Most notable is the Damage Index Method (DIM) [9], which uses modal curvatures to calculate and monitor the modal strain energy between two adjacent nodes (Eq. (4)), where $\beta_{i,j}$ indicates a damage feature value for the ith mode at location j, while $\varphi^{u''}$ and $\varphi^{d''}$ are the curvatures of the undamaged and damaged mode shapes, respectively, L is the element length, a and b are the limits of the evaluated element. As the DIM is based on modal curvatures, it therefore suffers from the same drawbacks as the MCM. This is particularly emphasized during the differentiation process, which amplifies high-frequency noise and can thus increase the variance of the subsequently extracted damage features.

$$\beta_{i,j} = \frac{\left[\int_a^b (\varphi^{d''})^2 dx + \int_0^L (\varphi^{d''})^2 dx\right]}{\left[\int_a^b (\varphi^{u''})^2 dx + \int_0^L (\varphi^{u''})^2 dx\right]} \cdot \frac{\int_0^L (\varphi^{u''}) \quad dx}{\int_0^L (\varphi^{d''}) \quad dx} \tag{4}$$

A comparative study of many of the aforementioned traditional, modal-based damage detection techniques was conducted by Talebinejad et al. [10]. The COMAC, MCM and DIM were all applied to a cable-stayed bridge FE model under varying vibration intensities and noise contamination levels. The study found that only high intensity damages were detectable through the application of these methods, and that they were quite sensitive to noise contamination and that they indentified numerous false damage events. Overall, modal based damage indicators require considerable data normalisation to improve their sensitivity to actual damage events.

3 RECENT ADVANCEMENTS

3.1 Common challenges: environmental and operational variability

A common challenge for many damage detection methodologies is insuring that detected damage events are truly damage and not benign system variations. Bridges are monitored over long periods of time and are subjected to large temperature fluctuations, harsh storms and numerous traffic scenarios. These varying conditions affect changes to a bridge's stiffness and mass in a non-linear manner, which in turn alters the bridge's modal properties. This is evident in Peeters & De Roeck's [11] assessment of the Z-24 Bridge in Switzerland, where significant variation in the bridge's natural frequency was observed when the ambient temperature dropped below freezing point (see Figure 1(a)). The cause of this bi-linear behaviour was attributable to the newly solidified ice in the bridge deck contributing to its stiffness.

Small changes in natural frequency due to temperature variation can often be mistaken for structural damage and, in some cases, can also hide actual damage events, as Farrar [12] discovered when investigating the suitability of frequency variation for structural damage detection in bridges by incrementally introducing damage to a bridge girder. The expected results were that the induced damage would reduce the girder's stiffness and thus reduce its natural frequency; instead, the girder's natural frequency rose for the first two damage scenarios before falling, as can be seen in Figure 1(b). It was subsequently revealed that the ambient temperature caused the initial increase in the girder's frequency.

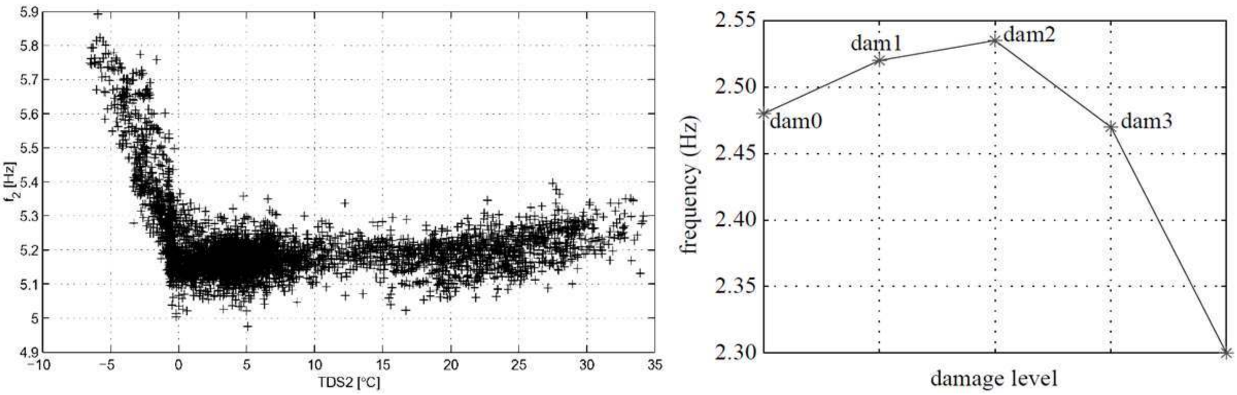

Figure 1 (a) Z-24 Bridge - Frequency v Temperature [11]   (b) Frequency vs. Damage Sequence [12]

3.2 Separating Environmental Variation from Damage

As mentioned in Section 3.1, environmental and operational variations have considerable influence on a bridge's dynamic behaviour which may be mistaken for damage. Data normalisation techniques help determine a bridge's baseline response under a range of normal environmental and operational conditions Generally, for data normalisation to be achieved, additional information is required relating to traffic and environmental conditions, usually temperature and wind speed. The process of data normalisation can be a challenging in itself due to the non-linear, multivariate nature of a bridge's behaviour and due to the quantity of data required.

Dervilis et al. [13] produced a study on the Tamar and Z24 Bridge to test a novel regression-based methodology for damage detection in changing environment and operational conditions. The algorithms used for the initial regression analysis and subsequent outlier detection in the vibration data were the Least Trimmed Squares (LTS) regression algorithm and the Minimum Covariance Determinant (MCD) estimator. The LTS regression algorithm is an adaptation on the popular least squares method of regression that minimises the sum of squared residual errors, however, instead of being applied to a full data set, it is applied to subsets, or clusters. This allows it to create a more robust fit to the data as it has a lower sensitivity overall to outliers when compared to many other regression techniques.

The MCD estimation method is applied to the LTS residual data. It is a multiple outlier detection method which expands on the classic Mahalanobis Squared Distance (MSD) method for outlier detection [14], where outliers are measured from the centre of a baseline data cluster, relative to the cluster size. The traditional MSD method has the disadvantage of potentially masking outliers, as the training data used to calculate the baseline cluster centre may already contain damage and erroneous data, resulting in an inaccurate baseline centre point. This would subsequently compromise the method's effectiveness for detecting future outliers. However, the advantage of the MCD estimation method is that it actively searches for and removes the inherent masking effect by identifying outliers in the training phase and ignoring them when calculating the cluster centre. This allows subsequent outlier detections to be unaffected by the presence of erroneous data in the trained algorithm. It achieves this by finding the subset of data points (must be over half of total number) whose covariance matrix has the lowest possible determinant to that of the whole set. This process takes multiple iterations to be completed. The MCD baseline centre point is then computed from the final minimum covariance subset only.

Dervilis et al.'s main objective of the study is to explain that different forms of outliers give distinct and different characteristics with respect to environmental and operational variations and damage. This is achieved by plotting the LTS residuals against the MCD index and superimposing thresholds that define the change point in outlier characteristics.

The Z24 Bridge vibration data was used to test the methodology's robustness in differentiating outlier differences. An example result plot of LTS residuals for temperature and first natural frequency versus MCD distance is presented in Figure 2. As can be seen, all six regions contain some data points. Region 3 contains normal behaviour data, while vertical regions 1 and 5 contain temperature induced outliers. Horizontal regions that cross the MCD threshold contain damage outliers. In the example presented the methodology was successfully able to discern the data points 1201-1500 as temperature induced variations and data points 2496-3932 as damage.

The benefit of employing this technique is that it clearly differentiates environmental induced variability from actual damage events.

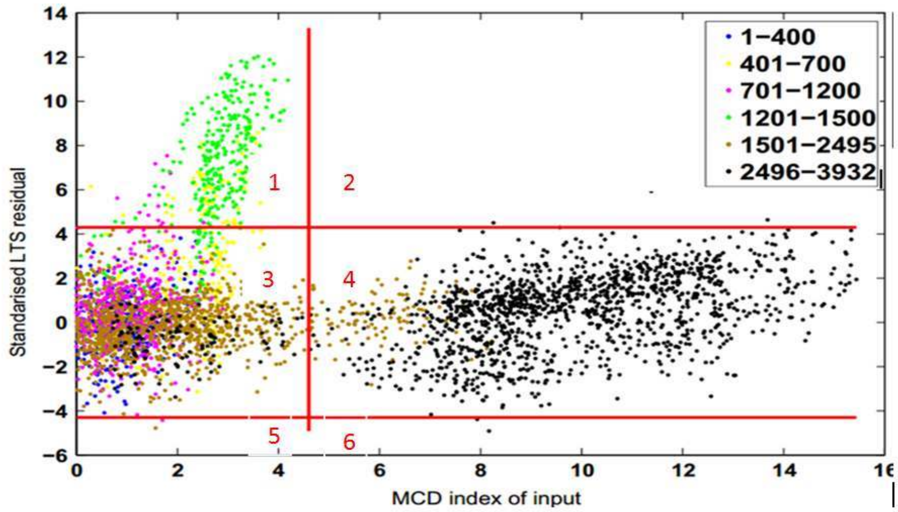

Figure 2 LTS residual vs. MCD distance - after [13]

Chatzi & Spiridonakos [15] proposed time-series based damage detection model that also attempts to infer a functional dependence between vibration data and environmental data through incorporation of a full numerical model of the structure in question. The method employed to achieve this is known as Polynomial Chaos Nonlinear AutoRegressive with eXogenous input (PC-NARX), as originally proposed in [16].

The PC-NARX model requires vibration data and temperature data as inputs so that the NARX portion of the methodology can fit a nonlinear relationship between the two in a training phase, which subsequently allows natural frequencies to be produced as an output. The Polynomial Chaos Expansion (PCE) allows parameters to be characterised as random variables, for example; acceleration time histories are represented by their PDF parameters so that measured vibration data can be handled as a set of random variables. This speeds up subsequent runtimes considerably, as large acceleration data sets can be reduced to a few representative values. Likewise, this methodology caters for the inclusion of structural uncertainty by allowing structural properties and dimensions to be included in the numerical model as PDF variables also. By having the structural properties and applied excitations, the PC-NARX can predict the dynamic response of a structure through a process called metamodeling. Chatzi & Spiridonakos demonstrate the efficiency of their proposed metamodeling of uncertain nonlinear systems subjected to stochastic excitation by comparing their predictions and simulation rates to that of a FE model. The results of which showed that the PC-NARX model achieved excellent accuracy, producing a normalised one-step-ahead prediction error of 0.0074%. Additionally, the PC-NARX based simulation rate was 100 times faster than that of the FE model. The ability of the PC-NARX methodology to accurately predict the dynamic response of a structure under varying environmental conditions implies that it should also be able to discern damage events by monitoring the magnitude of its prediction errors, as these are assumed to be normally distributed.

The PC-NARX was tested on the well known Z-24 bridge problem. Chatzi & Spiridonakos proposed a prediction error threshold of three standard deviations (99.7% confidence interval). Apart from isolated outliers, values of error outside this threshold will signify damage. The results of the Z-24 verification test showed that the PC-NARX was capable of reliably discerning damage in a bridge subjected to varying temperature.

The PC-NARX methodology demonstrated great promise as it combines deterministic and probabilistic processes to produce an accurate and efficient tool for detecting abnormal changes in structural behaviour. However, the outputs can only be as precise as the inputs, so it is essential that the training phase includes a full seasonal cycle of environmental variables to allow the model to learn the structure's full spectrum of normal behaviour. This implies that application of the PC-NARX should be reserved for long-term monitoring purposes only.

3.3 Non-Modal Based Approaches to Damage Detection

As discussed, modal based damage detection techniques contain a number of inherent drawbacks when applied to bridges. These drawbacks have led many researchers to investigate alternative procedures that circumvent the need for modal parameters. Dilena et al. [17] tested one such non-modal based technique, known as the Interpolation Damage Detection Method (IDDM), on a single span reinforced concrete bridge. The IDDM does not require a numerical model either; instead it defines a damage index in terms of deformed shapes to track changes in bridge condition. Reference deformed shapes are calculated from Frequency Response Functions (FRFs) of the undamaged structure and are used as a baseline condition for subsequent deformed shapes that are calculated during the testing and monitoring phase. By using deformed shapes as a damage indicator, one can take advantage of concentrated vibration amplitude irregularities to detect and locate damage [18]. Dilena et al. extends this base theory by incorporating a cubic polynomial spline interpolation function to the deformed shapes to extenuate behavioural variation without the need to calculate deformed shape curvatures. The detected abnormalities are denoted in IDDM by an interpolation error, which is simply the difference between the recorded and interpolated FRF profiles. Figure 3 presents a graphical explanation of the interpolation procedure conducted at a point Zl along a beam axis Z. The term E(zl) denotes the interpolation error, calculated as the distance between the recorded signal (Hg(z)) and the spline interpolation value (Hs(z)).

Higher interpolation errors signify a higher likelihood of damage. In this way, the IDDM is a probabilistic method of damage detection whereby only interpolation errors that are greater than a pre-determined threshold value are deemed as probable damage events. This decision criterion means that there will be a certain amount of false damage and missed damage events due to some interpolation errors falling on the incorrect side of the threshold value. For this reason, the threshold value should be determined though an optimisation, or cost/benefit analysis to minimise false and missed detections. An example of the probabilities of false and missed damage events due to IDDM is portrayed in Figure 4, where ET represents the location of the threshold value between the baseline/undamaged distribution (PE,0) and in the possibly damaged distribution (PE,d). The hatched areas Pm and Pf represent the probability of missed damage events and false damage events respectively. Note that threshold value is determined from the mean of the undamaged distribution (μEo) by the distance υσEo, which is the variance of PE,0 multiplied by a scaling factor (υ).

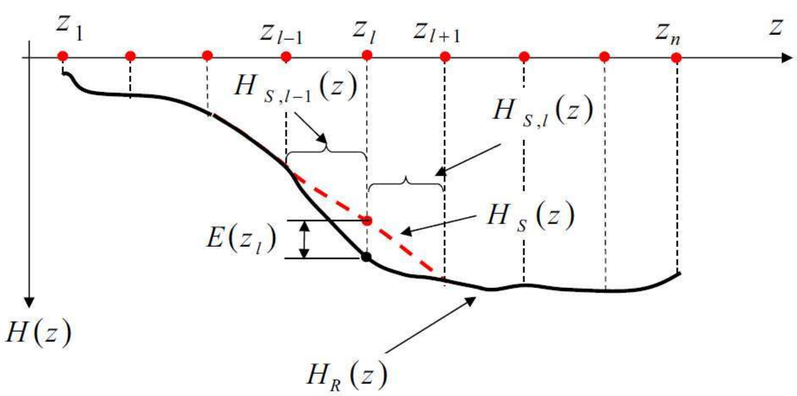

Figure 3 Spline interpolation of FRF - after [17]

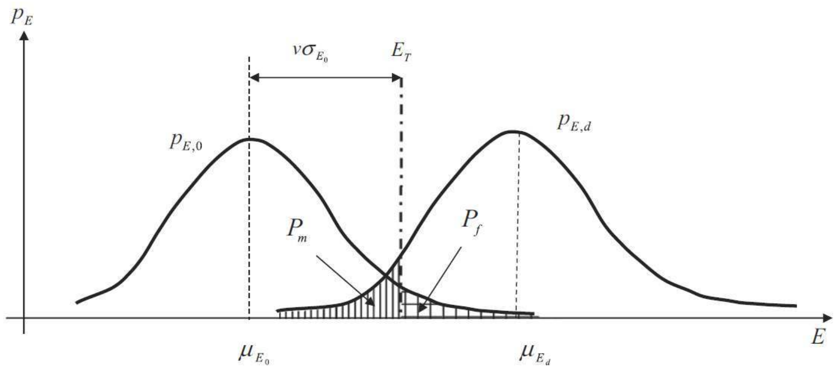

Figure 4 Threshold value and probabilities of false and missing damage events - after [17]

It should be noted that, accurate and detailed data is required for the undamaged state so that damage events can be confidently detected during the monitoring phase. However, if no undamaged data is available then a proposed variation on the original method will allow unsupervised damage detection to be conducted. It firstly assumes that, for an undamaged state, all sources of vibration will equally cause all locations to produce the same interpolation error variation. Conversely, if some locations produce significantly higher interpolation errors, then damage is confirmed at these locations. Again, to be deemed as damage, the interpolation error must surpass a predetermined threshold value. As the interpolation errors are assumed to be normally distributed for undamaged behaviour, the threshold value is thus calculated in terms of the average (μΔE) and variance (σΔE) of the damage parameter ΔE(zl). Eq. (15) shows how the threshold value is calculated using a variance multiplier (υ), which can achieve false damage detection probabilities of 15%, 2% & 1% for υ = 1, 2, & 3, respectively, as normal distribution is assumed. Conversely, increasing the variance multiplier (υ) will also increase the probability of missed damage events.

$$\Delta\mathrm{E}(z_l) = \mu_{\Delta\mathrm{E}} + \nu\sigma_{\Delta\mathrm{E}} \tag{5}$$

Dilena et al. tested the performance of the IDDM on a single span RC bridge under forced harmonic vibration. Numerous damage events were introduced to the bridge in different locations during testing. The results of the IDDM were compared to those of the MCM, which was also tested. The results showed that the IDDM is capable of detecting and locating damage consistently; however, its performance is dependent on the threshold value chosen and on the geometry of sensors. The experiment also showed that IDDM is capable of tracking the evolution of damage, which was tested by incrementally increasing the severity of the manually induced damage events. Damage localisation did not improve by increasing the number of vibration modes in the FRF range. When compared to the MCM results, the IDDM fairs quite well. The MCM demonstrated good sensitivity to damage for the first two vibration modes, but became less accurate thereafter. This is most probably due to the requirement of a denser array of sensors for accurate modal curvatures at higher modes. The IDDM requires fewer sensors than the MCM and, overall, has shown that the IDDM can reliably detect and locate damage without modal parameters as a damage indicator. A disadvantage of the IDDM is that its assumption that for an undamaged state, all sources of vibration will equally cause all locations to produce the same variation in interpolation error will not be suitable to all bridge applications.

3 S101 CASE STUDY

3.1 S101 Background

The flyover S101 was a post-tensioned three-span bridge in Austria that was constructed in the early 1960's. The main span had a length of 32 m, the two side spans were 12 m long. The cross-section was 7.2 m wide and was designed as a double-webbed t-beam, whose webs had a width of 0.6 m. The height of the beam varied from 0.9 m in the mid-span to 1.7 m over the piers (see Figure 5 & Figure 6) [19]. It was decided to replace it due to insufficient carrying capacity and its maintenance condition. Additionally, the bridge did not meet the current requirements, as the structure did not fit into the overall traffic and infrastructure concept anymore.

A progressive damage test was conducted on the S101 Bridge in 2008. The stages of the progressive damage test are presented in Table 1. The damage was applied in two main stages, with the first comprising of a simulated pier foundation settlement and the second comprising of a stiffness reduction through the severing of four tendons. Vibration data was recorded by numerous accelerometers located on the bridge deck, with a sample rate of 500Hz. The bridge was closed to traffic during the progressive damage test, so ambient vibration due to environmental excitations are prominent, however, one traffic lane beneath the bridge was open throughout the test which resulted in vibrations transmitted though the foundations. It is worth noting that prior to the undamaged vibration data being collected, the deck at the location of the pier chosen of damage was supported by a temporary supporting pier which was hydraulically loaded to the original pier's supporting force of 120t.

Figure 5 S101 Bridge [19]

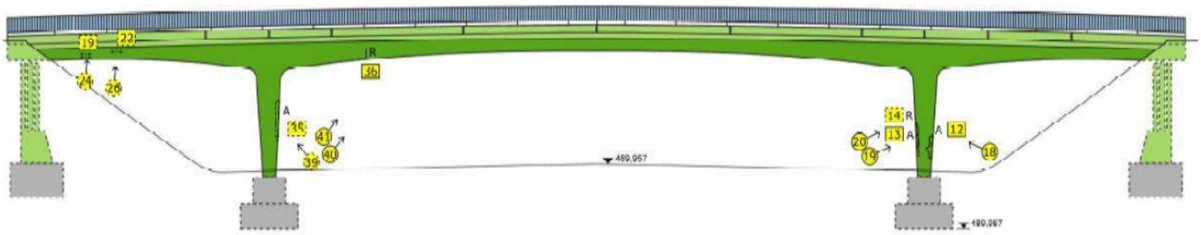

Figure 6 S101 Bridge cross-section [19]

Table 1 List of measurements recorded during the S101 Bridge progressive damage test

Damage State	Start time	End time	Stages of progressive damage test
1 [(1)]	10.12.2008 05:16 PM	11.12.2008 07:13 AM	undamaged structure
2	11.12.2008 07:13 AM	11.12.2008 10:21 AM	the north-western column was cut through
3[(1)]	11.12.2008 10:21 AM	11.12.2008 11:49 AM	first step of lowering the column (1cm)
4[(1)]	11.12.2008 11:49 AM	11.12.2008 01:39 PM	second step of lowering the column (2cm)
5	11.12.2008 01:39 PM	11.12.2008 02:45 PM	third step of lowering the column (3cm)
6	11.12.2008 02:45 PM	12.12.2008 05:52 AM	compensating plates are inserted
7	12.12.2008 08:04 AM	12.12.2008 01:12 PM	column returned in original position
8	12.12.2008 01:12 PM	12.12.2008 03:03 PM	first tendon intersected
9	12.12.2008 03:03 PM	13.12.2008 05:44 AM	second tendon intersected
11	13.12.2008 05:44 AM	13.12.2008 10:08 AM	third tendon intersected
12	13.12.2008 10:08 AM	13.12.2008 11:14 AM	fourth tendon partially intersected

Note: 1. Data sets marked were chosen for damage detection assessment as part of this paper's work

3.2 S101 Assessment

From the review of the above methodologies and additional studies, it can be determined that non-modal based, output only techniques offer robustness in varying conditions, ease of application and a high level of damage sensitivity. To analyse this opinion, the S101 Bridge data was used in an ARMA model for system identification and the Mahalanobis Distance outlier detection algorithm was then employed for damage detection based on AR coefficient variation. The assessment considered an undamaged stage to train the algorithm and two damaged states that consisted of successive pier settlement stages (pier lowered by 1cm and pier lowered by 2cm). A confidence interval of 95% was chosen as the damage threshold in the Mahalanobis Distance algorithm. The appropriate AR model order was determined through the partial autocorrelation function (PAF) algorithm prior to the damage detection phase.

Figure 7 presents the damage detection results based on the ARMA and Mahalanobis Distance algorithms. As can be seen, the damaged data set surpasses the damage threshold for all damage cases. Moreover, it is clear that the ARMA based system identification is sensitive enough to distinguish between the two damage states.

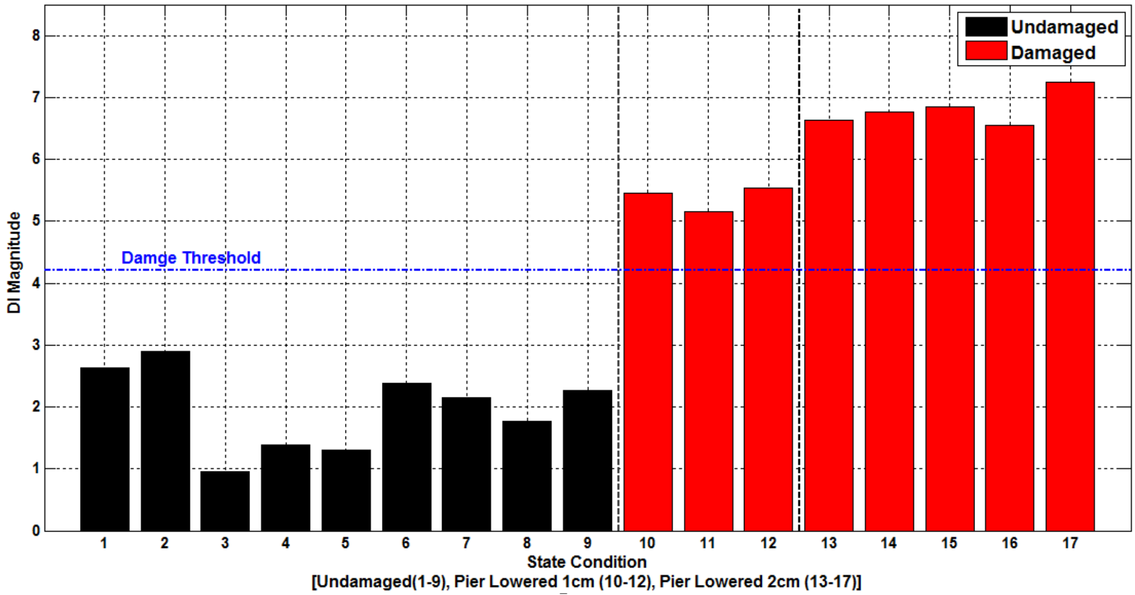

Figure 7 S101 damage detection results (ARMA & Mahalanobis Distance)

CONCLUSIONS

Overall, it can be concluded that there is no outright consensus among researchers regarding which vibration based damage indicator or damage detection method is most suited to bridges structures. However, it can be determined that non-modal based, output only techniques offer robustness in varying conditions, ease of application and a high level of damage sensitivity. Additionally, non-modal based damage indicators, such as vibration intensity are also viable for damage detection and it is the author's intention to advance with further work in this area.

ACKNOWLEDGMENTS

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 642453.

REFERENCES

1. MAINLINE. Maintenance, renewal and improvement of rail transport infrastructure to reduce economic and environmental impacts. (2013) Deliverable D1.1: "Benchmark of new technologies to extend the life of elderly rail infrastructure" European Project. 7th Framework programme. European Commission.
2. Casas, J. R. (2010) "Assessment and monitoring of existing bridges to avoid unnecessary strengthening or replacement". International IABMAS Conference on Bridge Maintenance, Safety and Management. Philadelphia: CRC Press, 2010, p. 2261-2269.
3. Allemang, R.J. and Brown, D.L. (1982), “A correlation coefficient for modal vector analysis”, Proceedings of the 1st SEM International Modal Analysis Conference, Orlando, FL, November.
4. Kim, J.H., Jeon, H.S. & Lee, C.W. (1992) "Application of the modal assurance criteria for detecting and locating structural faults", Proceedings of the 10th International Modal Analysis Conference, San Diego, CA, pp. 536–540.
5. Salawu, O.S. and Williams, C. (1995), “Bridge assessment using forced-vibration testing”, J. Struct. Eng., 121(2), 161-173.
6. Pandey, A.K., Biswas, M. & Samman, M. (1991) "Damage detection from changes in curvature mode shapes", J. of Sound & Vib 145(2) 321–332.
7. Abdel Wahab, M. M. & De Roeck, G. (1999), "Damage detection in bridges using modal curvatures: Application to a real damage scenario", Journal of Sound and Vibration, 226(1), 217–35.
8. Farrar, C.R. and Worden, K. (2013). ‘‘Structural health monitoring - a machine learning perspective,’’ 1st edn, John Wiley & Sons, Ltd. Chichester, West Sussex, UK
9. Stubbs, N., Kim, J.T. & Topole, K. (1992) “An efficient and robust algorithm for damage localization in offshore platforms”, in Proceedings ASCE 10th Structures Congress, pp. 543-546.
10. Talebinejad, I., Fischer, C. & Ansari, F. (2011) "Numerical Evaluation of vibration-based methods for damage assessment of cable-stayed bridges" Computer-aided civil and infrastructure eng, 26, 239-251.
11. Peeters, B & Roeck, G.D. (2001) " One-year monitoring of the Z24-Bridge: environmental effects versus damage events ". Earthquake Engineering and Structural Dynamics, 30, 149-171.
12. Farrar, C. R., Baker, W. E., Bell, T. M., Cone, K. M., Darling, T. W., Duffey, T. A., Eklund, A. & Migliori, A. (1994). "Dynamic characterization and damage detection in the I-40 bridge over the Rio Grande". Los Alamos National Laboratory Report: LA-12767-MS.
13. Dervilis, N., Worden, K., & Cross, E.J. (2015) "On robust regression analysis as a means of exploring environmental and operational conditions for SHM data" Journal of Sound and Vibration 347 279–296
14. Mahalanobis, P.C. (1936) "On the generalized distance in statistics", Proc. Nat. Inst. Sci. India 2 (1) 49–55.
15. Chatzi, E.N. & Spiridonakos, M.D. (2015) "Dealing with uncertainty in the monitoring of dynamically evolving systems" Proceeding of the 7th International Conference on Structural Health Monitoring of Intelligent Infrastructure (SHMII). Torino, Italy, July 2015.
16. Spiridonakos, M.D. & Chatzi, E.N. (2015) "Metamodeling of structural systems with parametric uncertainty subject to stochastic dynamic excitation" Int. J. For Earthquake Eng & Earthquake Effects on Structures. 8(4), pp 915-934
17. Dilena, M., Limongelli, M.P. & Morassi A. (2015) "Damage localization in bridges via the FRF interpolation method" Mechanical Systemsand Signal Processing 52-53, 162–180.
18. Zhang, Y., Lie, S.T. & Xiang, Z. (2013) "Damage detection method based on operating deflection shape curvature extracted from dynamic response of a passing vehicle", Mech. Syst. Signal Process. 35 238–254.
19. Vienna Consulting Engineers (2009) "Progressive damage test S101 Flyover Reibersdorf" Report Nr. 08/2308.