

Damage feature extraction from measured Lamb wave signals using a model-based approach

Christian DÜRAGER¹, Christian BOLLER², Andrew CORNISH¹

¹ SR Technics, NDT Center, Kloten Airport, CH 8058 Zurich/Switzerland

christian.duerager@srtechnics.com

² Chair of NDT and Quality Assurance (LZfPQ), Saarland University, Saarbrücken / Germany,
c.boller@mx.uni-saarland.de

Key words: Guided ultrasonic waves, Damage feature extraction, Model-based approach, Non-linear least mean square.

Abstract

Compared to conventional ultrasonic methods used in non-destructive testing (NDT) Lamb wave based NDT methods allow damages to be detected in structures of much bigger scales. Lamb waves can behave nonlinear when propagating through solid state structures. A wave may therefore change its shape while propagating and even split into several wave pulses resulting from a structure's geometry designed or damage occurred during operation. An originally well defined signal can therefore become rather complex and interpretation becomes difficult due to different features overlapping each other.

Within this paper a new approach is proposed which enables damage relevant features inside a measured Lamb wave response signal to be identified even when different features of the Lamb wave signal are overlapped by other wave pulses. The approach and algorithm proposed is specifically considering damage feature extraction and mainly consists of an approximation model of the expected Lamb wave response signal to be measured with a non-linear least mean square algorithm building the basis. The principal functionality of this algorithm is as follows: In a first step the modelled signal is compared to a measured response signal using the non-linear least mean square algorithm. The task of this algorithm is to adjust particular parameters of the modelled signal such that the measured signal can be met. In a second step, those parameters caused by the wave pulses reflected from a potential damage/imperfection are identified and the basis of these parameters is calculated such that the model originally established is corrected in the sense of an inverse problem. Each model of the reflected wave pulses contains the information related to the arrival time, the amplitude and the corresponding Lamb wave mode of the wave pulse being core parameters of the model.

In order to demonstrate the functionality of the proposed algorithm, verification tests will be explained and shown by using synthetic data, proving that the algorithm introduced allows damage relevant features inside the measured complex overlapping Lamb wave signals to be detected. Further means of improvement of the approach will be also addressed.

1 INTRODUCTION

Ultrasonic testing (UT) has a well-established performance for detection of defects and will



be used ever more in the future with a view to a predicted increase of composite structures to be used. However, testing of large structures with UT is slow because the testing region is limited to the area immediately surrounding the transducer.

An alternative and more elegant approach is the use of guided ultrasonic waves because they can be excited at one location on the structure and propagate over a long distance (e.g., [1, 2, 3]) provided the geometry (thickness) is kept constant. By analyzing the characteristic of the guided waves such as the returning echoes, change of dispersion relationships, etc., the presence of flaws may be detected. However, because of the non-linear propagation behavior and the multimode nature of guided ultrasonic waves, the interpretation of the measurement signals is more complicated and makes the localization of the flaw position in the structure less precise. Therefore, signal processing is a crucial aspect in any guided ultrasonic wave application. The objective is to extract information from the signals measured to decide if a flaw has propagated in the structure and if so, characterize it in terms of its location.

Conventional solutions to the problem of guided ultrasonic wave signal processing are usually in the form of some time-frequency representation and here two major algorithms, the short-term Fourier transform (STFT) and the wavelet transform (WT) are commonly used. The basic concept of the STFT is to break up a non stationary signal into small segments, and to apply the Fourier transform to each of the segments to ascertain the frequencies that exist in that segment. The totality of such spectra indicates how the spectrum is varying in time (e.g., [4, 5]). However, the STFT windowing process results in a trade-off between the time and the frequency resolutions and therefore accuracy cannot be simultaneously obtained in both time and frequency domains. The presence of short-duration high-frequency signal bursts which are used for guided ultrasonic wave inspection are therefore hard to detect. In contrast to the STFT, the WT uses a functional basis consisting of dilated and shifted versions of single basis function called a mother wavelet which can also be seen as a wave packet. The WT analyzing method breaks up a signal into a series of wavelets that are shifted and scaled. WT has the advantage that it can adjust the window length according to the need of the real signals. Therefore, detailed information (high frequency components) can be obtained with a narrow window and general information (low frequency components) with a large window. However, the difficulty of WT is to pick up the correct wavelet for a specific target signal and therefore the application of WT requires more knowledge and experience [6].

A different method for processing guided ultrasonic wave signals is the matching pursuit approach. Its basic idea is to iteratively project the measured guided ultrasonic wave signal onto a large and redundant dictionary of waveforms (e.g., [7]). At each step it chooses the waveform from the dictionary that is best adapted to the signal to analyze. Hong et al. [8] have explored the matching pursuit algorithm for the analysis of non-dispersive guided ultrasonic waves. Ranghavan and Cesnik [9] used a computationally efficient algorithm for matching pursuits by using a dictionary consisting of Gaussian modulated chirplet atoms for damage location and characterization.

In this work we present a new approach for extracting the damage relevant features out of the measured signal, which provides significant advantages over the damage feature extraction methods actually in use. The method developed in the framework of this work resembles the matching pursuit approach, with the difference however that the dictionary of the waveforms has been replaced by an adaptive model of the expected guided ultrasonic wave signal. One advantage of this method is its possibility to extract the damage relevant features out of dispersive multi mode signals even when these signals are overlapped by non damage relevant features such as e.g., incident wave pulses. The ability to adapt to changing

environmental conditions which may affect the measured signal is a further advantage of the present damage feature extraction process. This makes the present method very well suited for the analysis of guided ultrasonic signals and particularly for real world applications.

2 MODEL-BASED DAMAGE FEATURE EXTRACTION IN THIS WORK

The process for the extraction of the damage relevant features in this work consists of two main parts, an approximated mathematical model of the expected measured test signal and a non-linear least mean square (NLMS) algorithm for the estimation of the damage relevant parameters. The approximated model is based on the two-dimensional wave equation whereby the dispersion and the chirping of the wave pulses have been taken additionally into account.

In this section, we first explain the development of the approximated mathematical equation for the model of the expected measured signal, followed by the explanation of the model-based damage feature extraction process. The proof of concept for the proposed process and the main results are described in the subsequent sections.

2.1 Approximated mathematical wave pulse equation

The derivation of the approximated mathematical equation starts with the equation for a time-harmonic two dimensional plane wave with the angular frequency ω and propagating along the x-axis [10],

$$y(x, t) = y_0 \exp^{j(kx - \omega t)}. \quad (11)$$

Here k is the wave number and t the time of the propagating plane wave. Due to the dispersive behavior of the guided waves the wavenumber k is a function of ω , which can be approximated by a parabolic function about the angular frequency ω_0 ,

$$k = k_0 + a(\omega - \omega_0) + \frac{1}{2}b(\omega - \omega_0)^2 \quad (2)$$

where $a = c_g^{-1}$ is the invers of the group velocity $c_g = \frac{d\omega}{dk}$ at ω_0 and $b = d^2k/d\omega^2$ expresses the dispersion of the medium, i.e. the fact that the group velocity depends on ω . When we consider a wave pulse whose duration involves a fairly large number of oscillations of the field, the frequency spectrum is no longer infinitely narrow and the various frequency components travel at different speeds. However, the intensity is significant only if t/x is not too different from $a = v_g^{-1}$, because the frequency spectrum of the pulse is narrow.

When a wave pulse propagates from its origin ($t = x = 0$) to a point (t, x) the shift of the wave pulse can be by expressed by,

$$S(x, t) = k(\omega)x - \omega t \quad (3)$$

If we replace $k(\omega)$ by its expression from Equation 2 and take ω from Equation (3), we obtain after rearranging,

$$S(x, t) = k_0z - \omega_0t - \frac{1}{2}(t - ax)^2/bx \quad (4)$$

The amplitude reaching the space-time point (x, t) is,

$$y(x, t) = y_0 \exp[jS(x, t)] \quad (5)$$

We now assume in this work that the wave pulses are propagating in the shape of a Gaussian

pulse and furthermore the pulses are propagating circular away from the exciting source ($x = r$). After adding this additional information and rearranging Equation (5) we get the final approximated mathematical equation for the wave pulse for the approximated mathematical model of the expected measured signal used in this work,

$$y(r, t) = \underbrace{\left[\frac{\tau_0^4}{\tau_0^4 + (\kappa r)^2} \right]^{\frac{1}{4}}}_{\text{Amplitude wave pulse}} \underbrace{\exp \left[-\frac{(t - k_0 r^2 \tau_0^2)}{2\sqrt{\tau_0^2 + (\kappa r)^2}} \right]}_{\text{Envelope wave pulse}} \underbrace{e^{j(\omega_0 t + \frac{\psi t^2}{2} - k(\omega)r)}}_{\text{Carrying wave}}. \quad (6)$$

Here, τ_0 is the initial width of the Gaussian pulse, κ is the dispersion coefficient and ψ is assumed to be the chirping coefficient.

2.2 Damage feature extraction algorithm

In this section we describe the principle functionality of the damage feature extraction process developed in the framework of this work. The whole damage feature extraction process with its single steps is furthermore listed in Table (2) in sequential form.

- i. The measured response signal at the sensor position $y(r, t)$ is the input for the damage feature extraction process. In a first step, the envelope F of $y(r, t)$ is calculated by applying the Hilbert transform. At the same time, an approximated signal y_{est} of the expected measured signal is calculated by using the approximated mathematical equation for each expected wave pulse inside the measured signal. Subsequently the envelope F_{est} of the signal y_{est} is calculated.
- ii. In the next step, the parameter vector a^k of the approximated model of the expected measured signal is adjusted by a non-linear least mean square (NLMS) algorithm in that way that the envelope from the calculated signal F_{est} fits best the envelope of the measured signal F . The Levenberg-Marquard algorithm is used for the NLMS parameter estimation. The adjustable parameters of the approximated model are listed in Table (1).

Parameter, a^k	Description
A_n	Wave pulse amplitude
r_n	Propagation distance
κ_n	Dispersion parameter
ψ_n	Chirping parameter

Table 1: Adjustable parameters of the approximated mathematical model.

- iii. If the estimated squared error $S(a^k)$ between the envelope of the calculated signal F_{est} and the envelope of the measured signal F reaches a certain threshold the damage feature extraction algorithm stops and it is assumed that no damage features are present within the measured signal $y(r, t)$. In this case the estimated parameter vector a^k corresponds to the wave pulses contained in the signal without damage (e.g., incident wave pulses and/or reflected wave pulses from the edges of the structure).

- iv. If the NLMS algorithm stops after a defined number of runs and there will be a remaining squared error $S(a^k)$ between F and F_{est} , then it is assumed that further wave pulses are within the measured signal $y(r, t)$. For this case the approximated model will be expanded by two additional wave pulses, the supposed S_0 and A_0 reflected wave pulse from a defect to the structure. Now, step (ii) will be carried out again and after the NLMS stops, the final estimated parameter vector a^k contains the values for the reflected wave pulses from the defect.

Input signal:	
F ;	Envelope of measured input signal
Initial parameters:	
$a^0 = [a_{1n}, a_{2n}, \dots, a_{mn}]$;	Parameter vector
μ^0 ;	Initial step size
$\beta_0 = 0.25, \beta_1 = 0.75$;	Criteria for step size
ϵ_1 ;	Threshold minimum squared error
ϵ_2 ;	Threshold minimum change of
k_{max} ;	Maximal number of iterations
Step 1:	
$F_{est}(a^k) = Hilbert(y_{est}(a^k))$	Calculate envelope of model signal y_{est}
$S(a^k) = \ F - F_{est}(a^k)\ ^2$	Calculate sum of squared error $S(a^k)$
$\frac{\delta S(a^k)}{\delta a^k} = \frac{\delta \ F - F_{est}(a^k)\ ^2}{\delta a^k} = J(a^k)$	Calculation of the Jacobian
Step 2:	
$\left\ \begin{pmatrix} J(a^k) \\ \mu I \end{pmatrix} s^k + \begin{pmatrix} S(a^k) \\ 0 \end{pmatrix} \right\ \Rightarrow \min$	Minimize the linear least mean square problem
Step 3:	
$a^{k+1} = a^k + s^k$	
$S(a^{k+1}) = S(a^k + s^k)$	
$S(a^k) + J(a^k)s^k$	
Step 4:	
$\rho_\mu = \frac{\ S(a^k)\ ^2 - \ S(a^k - s^k)\ ^2}{\ S(a^k)\ ^2 - \ S(a^k) + J(a^k)s^k\ ^2}$	Calculate criteria for new step size
if $\rho_\mu \leq \beta_0$	s^k is not accepted; μ is doubled; new correction factor s^k is calculated
if $\beta_0 < \rho_\mu$	s^k is accepted; μ is retained
if $\rho_\mu \geq \beta_1$	s^k is accepted; μ is halved
Step 5:	
$\ S_{a^k}\ \leq \epsilon_1$ or	
$\ a^k - a^{k-1}\ \leq \epsilon_2(\ a^{k-1}\ + \epsilon_2)$ or	
$k \geq k_{max}$	
Result:	
$a^k = [a_{1n}, a_{2n}, \dots, a_{mn}]$	Estimated parameter vector

Table 2: Sequence of the damage feature extraction algorithm.

3 PROOF OF CONCEPT

In the context of the present work the functionality of the proposed damage feature extraction process was tested by using signals with changing wave pulse parameters and furthermore with varying overlapping ratios between the different wave pulses inside the signal. The signals used for the test were simulated based on the basic wave Equation (6) described in the previous section. The simulated test signal was once assumed without damage features and further with damage features. Both cases were tested in the validation phase but here only the case with damage features will be explained.

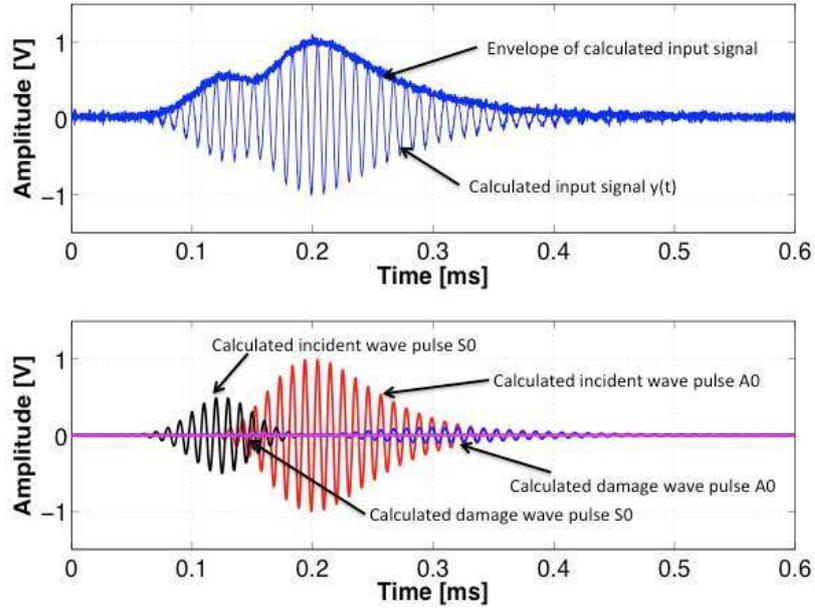


Figure 1: Example of a simulated signal used for the proof of the damage feature extraction process. (Picture above) whole simulated signal with it envelope. (Picture below) Single wave pulses inside the simulated signal.

In Figure (1) the upper graph shows an example of a simulated test signal $y(r, t)$ and its envelope used for the concept proof. The lower graph in Figure (1) shows the different wave pulses inside the simulated signal. The parameters used for the simulation are listed in Table (3).

A_{A0inc} [V]	r_{A0inc} [m]	κ_{A0} [-]	ψ_{A0} [-]	A_{S0inc} [V]	r_{S0inc} [m]	κ_{S0} [-]	ψ_{S0} [-]	A_{A0dam} [V]	r_{A0dam} [m]
1	0.13	1e-4	1e-8	0.5	0.05	0	0	0.1	0.18
A_{S0dam} [V]	r_{S0dam} [m]								
0.03	0.08								

Table 3: Wave pulse parameters for the simulated test signal.

In order to demonstrate a more realistic test case additional noise was added to the test signal. The values for the initial parameters used for the damage feature extraction process are listed in Table (4) below.

A_{A0inc} [V]	r_{A0inc} [m]	κ_{A0} [-]	ψ_{A0} [-]	A_{S0inc} [V]	r_{S0inc} [m]	κ_{S0} [-]	ψ_{S0} [-]	A_{A0dam} [V]	r_{A0dam} [m]
0.2	0.1	0.001	0.001	0.1	0.01	0	0	0.01	0.13
A_{S0dam} [V]	r_{S0dam} [m]								
0.001	0.06								

Table 4: Initial wave pulse parameters.

4 RESULTS

The result of the damage feature extraction process is the estimated parameter vector a^k . It contains the values about the respective wave pulse amplitude A_n and the propagation distance r_n for each single wave pulse contained in the simulated test signal. For a better illustration the envelope of the estimated signal, and here only the damage relevant wave pulses, is calculated based on the estimated wave pulse parameters and displayed in Figure (2). There, the envelopes of the simulated test signal (damage relevant wave packets) are compared to the estimated envelope described above.

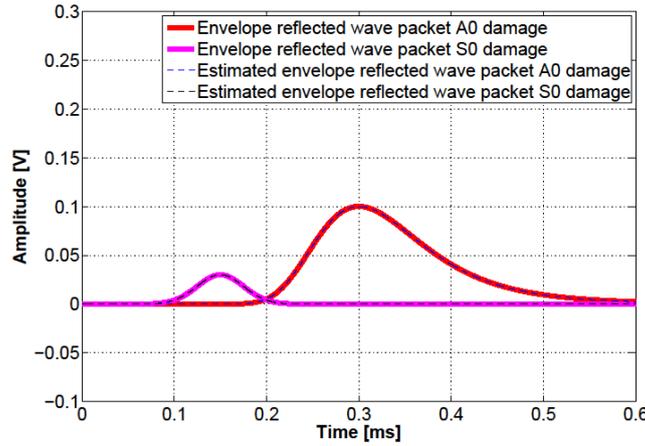


Figure 2: Example of the resulting envelope signal after applying the damage feature process.

In order to test the stability of the proposed process the A_0 incident wave pulse was stepwise overlapped by the reflected A_0 wave packet from the damage. The overlapping was done in a ratio range between 0% (no overlapping) to 100% (full overlapping). Furthermore, the amplitude ratio between the two wave pulses was varied between 0.01 (low amplitude of the A_0 reflected wave packet from the damage) and 0.1. In Figure (3) the results for the test with different overlapping ratios and varying wave pulse amplitudes are shown.

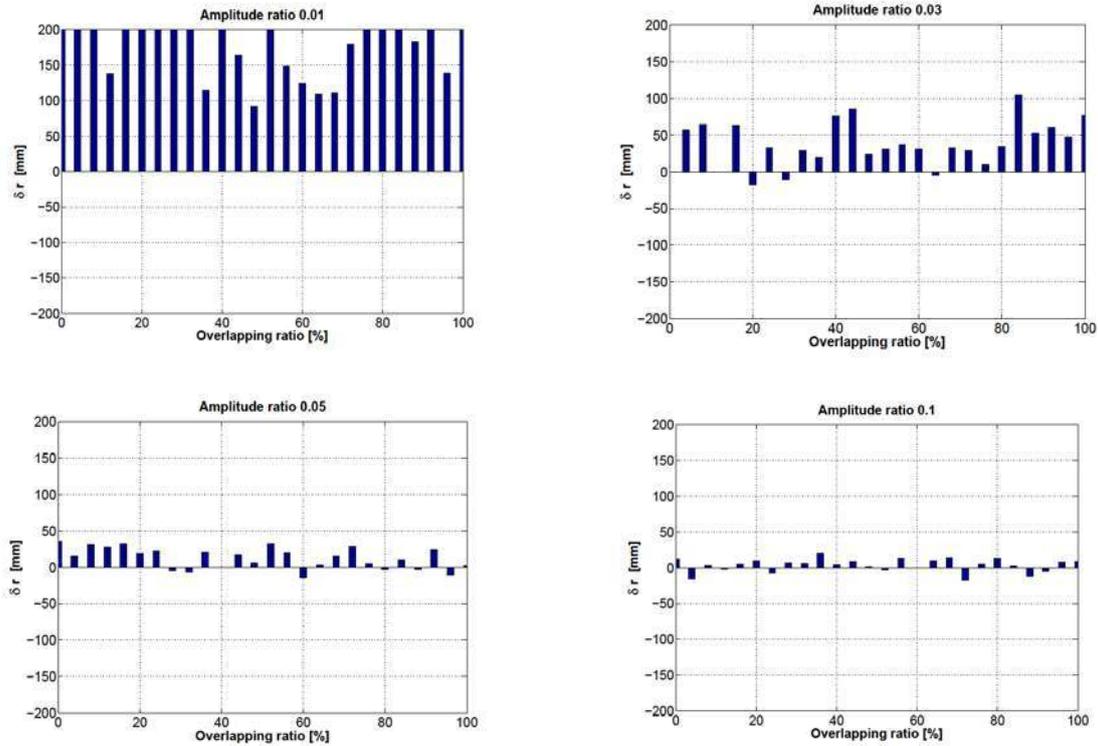


Figure 3: Results after applying the damage feature extraction process to a simulated signal with different overlapping ratios between wave pulses and changing wave pulse amplitude.

It can be seen in Figure (3) that with a relatively small amplitude ratio between the incident and the reflected wave packet the error δr between the estimated and simulated propagation distance of the reflected wave packet from the damage is relatively high (>200 mm). This is because of the additional noise inside the simulated signal, which makes an estimation of the damage relevant features with low amplitude not possible. With increasing amplitude of the reflected A_0 wave packet from the damage the error δr decreases and for an amplitude ratio between incident and reflected signal the average error is approximately 5 mm which is a reasonable value for the estimation of the damage position.

5 SUMMARY AND CONCLUSIONS

In this work a new process for the identification of damage relevant features within signals measured from propagating guided ultrasonic waves in solid structures is present. The proposed process is very similar to the matching pursuit approach with the main difference that an approximated model of the expected measured signal is adaptively adjusted to the real measured signals instead of using a dictionary of possible signals as used for the matching pursuit approach. This enables the use of the proposed process for a wide range of measured signals coming from the propagating guided waves inside solid structures. Furthermore, the proposed process enables to identify damage features inside a measured signal even when they are overlapped by other wave pulses.

REFERENCES

- [1] Y. Cho, D.D. Hongerholt, J.L. Rose, Lamb wave scattering analysis for reflector characterization. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **44**, 44-52, 2002
- [2] J.L. Rose, M.J. Avioli, P. Mudge, R. Sanderson, Guided wave inspection potential of defects in rail, *NDT S E International*, **37**, 153-161, 2004
- [3] Z. Su, L. Ye, Y. Lu, Guided lamb waves for identification of damage in composite structures: A review, *Journal of Sound and Vibration*, **295**, 753-780, 2006
- [4] G. Beresford-Smith, I.M. Mason, A parametric approach to the compression of seismic by frequency transformation, *Geophysical Prospecting*, **28**, 551-571, 1980
- [5] A.K. Booer, J. Chambers, I.M. Mason, Fast numerical algorithm for the recompression of dispersed time signals, *IEEE Electronic Letters*, **13**, 453-456, 1980
- [6] H. Joeng, Y-S. Jang, Wavelet analysis of plate wave propagation in composite laminates, *Composite Structures*, **49**, 443-450, 2000
- [7] B. Xu, V. Giurgiutiu, L. Yu, Lamb Wave Decomposition and Mode Identification Using Matching Pursuit Method, *Report Department of Mechanical Engineering, University of South Carolina*, page 12, 2009
- [8] J-C. Hong, K.H. Sun, Y.Y. Kim, The matching pursuit approach based on the modulated Gaussian pulse for efficient guided wave inspection, *Smart Materials and Structures*, **14**, 548-560, 2005
- [9] A. Raghavan, C.E.S Cesnik, Guided-wave signal processing using chirplet matching pursuits and mode correlation for structural health monitoring, *Smart Materials and Structures*, **16**, 355-366, 2007
- [10] D. Royer, E. Dieulesaint, *Elastic Waves in Solids I, free and guided propagation*, Springer- Verlag Berlin Heidelberg New York, ISBN 3-540-65932-3, 1996