Removal of temperature-induced strain variations for fatigue crack growth detection in a real aeronautical structure

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Abstract
Many structural health monitoring techniques suffer from high sensitivity to operational and environmental variations that often hide the effect of anomalies in the measured signals. This may result in a high number of false alarms caused by natural, contingent variations of load, temperature and humidity, just to name a few. Cointegration, originally developed in the econometrics field, proved to be a promising data-driven method for the identification and removal of long-term trends in time series data. Later, applications of cointegration to structural health monitoring data have demonstrated the ability to remove trends caused by naturally occurring changes of the baseline structure’s dynamic or quasi-static response in the monitored features. This work presents the application of cointegration to a full-scale aeronautical stiffened structure, consisting of a tail boom of a military helicopter subject to fatigue crack growth. A constant-amplitude load caused fatigue crack propagation from an artificial notch at a riveted joint. During the full-scale test, a fatigue crack monitoring system based on real-time distributed strain measures by a fiber Bragg grating (FBG) sensor network was dedicated to the real-time damage identification. However, strain measures by FBGs are known to be extremely sensitive to temperature variations thus strongly affecting the robustness of any selected feature dependent on the strain field. The objective of this paper is to show the effectiveness of cointegration in working as environmental compensator in a realistic, though rather simplified, case study. The effect of temperature change is filtered from the strain signals acquired during the test thus emphasizing the effect of damage on the strain field pattern and providing a more robust feature for damage identification.

1 INTRODUCTION
The exploitation of structural health monitoring (SHM) systems remains limited in industry, despite extensive research on the topic carried out during the last decades. Several complications make the practical application of SHM tools on real structures non-trivial, and among them, the variability of environmental and operational conditions is one of the most complex issues that has to be solved to implement SHM in effective ways [1]. The operational load can be variable in nature for many structures, e.g., wind-induced vibrations of aircraft wings, bridges and high-rise buildings. Also, maneuver loads, which are intrinsically random,
induce low-frequency, high-amplitude stresses in the structure of fixed- and rotary-wing vehicles. In addition to operative and contingent loads, the environmental conditions can markedly affect the structure's mechanical behavior as well as the feature extraction from the recorded signals, which is carried out to perform a system's diagnosis. The unpredicted, or un-modeled, change of signal features might induce the SHM system to detect a damage that is actually non-existing, producing a false alarm. Also, the changing of the environmental conditions can alter the damage assessment stage: the changing of the signal features caused by environment can produce an overestimation (or worse, underestimation) of the damage size. Recently, some papers showed a potential solution to environmental and operational variations affecting SHM by introducing cointegration [1][3], a data-driven method initially developed in econometrics [4].

In SHM, the objective of cointegration is the filtering of innocuous, environmental-induced variations of the signals recorded through sensor networks. Such benign variations must be previously recorded in a baseline signal categorized as 'normal condition'. Once a new signal is acquired, cointegration will highlight any novelty hidden in the signal. It would work as damage detector by recognizing if (and when) the signal feature(s) deviates from the learned normal condition [1].

In order to make cointegration a state-of-the-art methodology, its promising contribution to SHM must be verified and validated in a number structural damage scenarios highlighting its strengths, limitations and drawbacks. Therefore, this paper presents the application of cointegration as an anomaly detector in a real aeronautical structure subject to fatigue degradation and large, naturally-induced temperature variations in a lab environment. The aeronautical structure is a helicopter tail equipped with a fiber Bragg grating (FBG) sensor network for strain recording installed on the inner stringers. The strains were recorded during the experimental activity, and the high fluctuations of temperature (because of day-night transitions and sun hitting the structure) produced large variations of the recorded signals, thus hiding the presence of anomalies. The application has proven how cointegration can remove the effect of such temperature-induced strain variations and detect a crack nucleating from a rivet hole in a full-scale fatigue crack growth (FCG) test.

The paper is organized as follows: Section 2 describes the experimental activity emphasizing the sensitivity of the strain features to the temperature variations, Section 3 summarizes the cointegration method and Section 4 shows the resulting detection performance. Section 5 concludes the paper with a summary and critical analysis of the results.

2 FULL-SCALE FATIGUE CRACK GROWTH TEST

The FCG test was performed on the tail of a dismissed Mi-8/17 helicopter belonging to the Air Force Institute of Technology of Poland. The tip of the tail carrying the tail rotor was removed, and the root of the tail was connected to a rigid frame by means of a dedicated carbon fiber-reinforced polymer structure. The latter was calibrated with the stiffness of the central fuselage, thus reproducing realistic boundary conditions. The fatigue load was applied on the free end, transversally to the tail axis, to simulate the effect of the rotor and induce bending and torsion at the root of the tail. A series of 45 FBGs was mounted on the stringers: 9 sensors × 5 stringers. Figure 1 shows the test rig, while table 1 summarizes the test features. The crack was induced by an artificial notch around 15 mm-long made on a rivet of the stringer 7 (see Figure 1), where two crack gauges were also applied to monitor the crack evolution.

A number of LVDTs, crack gauges and traditional strain gauges were also installed on the structure to monitor the global displacement of the tail and its structural integrity during the
entire test. The temperature was measured and a temperature compensator was also installed on the structure. However, data from the thermometer and temperature compensator have not been used in this application.

2.1 FBG strain-signal relationship

FBGs are optical fibers with a periodic modulation engraved within the core of the fiber, which is the Bragg grating [5], and they are considered one of the state-of-the-art sensor technologies for SHM applications [6]. When the light passes through the optical fiber, the Bragg grating reflects a specific wavelength band $\lambda_b$ back, which changes if the Bragg is stretched because of tensile load or temperature. The relationship among mechanical strain, temperature and wavelength of the optical fiber can be expressed as:

$$\frac{\lambda_b - \lambda_{b,0}}{\lambda_{b,0}} = k\varepsilon + \alpha_\delta \Delta T,$$

where $\lambda_b$ is the reflected wavelength in the deformed condition, $\lambda_{b,0}$ is the reference wavelength, $k$ is the strain sensitivity of the Bragg grating, $\alpha_\delta$ is the change of the refraction index and $\Delta T$ is the temperature variation in K [7]. The total strain $\varepsilon$ is the sum of the strain caused by mechanical loads and temperature.

The temperature effect must be filtered out to obtain the strain due to mechanical loads, so

<table>
<thead>
<tr>
<th>Load shape</th>
<th>Sinusoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load frequency</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Maximum load ($F_{\text{max}}$)</td>
<td>8 kN</td>
</tr>
<tr>
<td>Load ratio ($R$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Damage type</td>
<td>Skin crack</td>
</tr>
<tr>
<td>Damage location</td>
<td>Rivet hole</td>
</tr>
<tr>
<td>Damage initiation</td>
<td>Artificial notch, 15 mm</td>
</tr>
<tr>
<td>Skin material (driving FCG)</td>
<td>D-16 (equivalent to Al2024-T2)</td>
</tr>
</tbody>
</table>

Table 1: Test features.
FBGs need a temperature compensator. However, even if the compensator would remove the wavelength shift caused by temperature correctly, one should still apply advanced signal processing algorithms to detect the damage from the strain signals. Cointegration has a clear advantage in this anomaly detection process; assuming that the temperature variation would introduce a common trend in the measured signals, the application of cointegration would automatically remove such temperature effect (by removing the common trends) highlighting the presence of anomalies in the recorded signals so, potential damages in the structure.

In this application, the strain peaks caused by sinusoidal, fatigue loads have been picked as sensitive features to the crack propagation phenomenon. Therefore, cointegration has been applied to the strain peaks that change during the FCG. The reference wavelength $\lambda_{b,0}$ was estimated by acquiring ‘baseline’ signals without loads on the structure. Once $\lambda_{b,0}$ is available, the maximum strain or strain peak caused by the fatigue load can be calculated using the following relation:

$$\varepsilon_{\text{max}} = \frac{\lambda_{b,\text{max}} - \lambda_{b,0}}{K}$$

which obviously include any temperature effect. Here, $\lambda_{b,\text{max}}$ is the reflected wavelength corresponding to the peak of a load cycle, so $\lambda_{b,\text{max}} = \lambda_b(F = F_{\text{max}})$. The resulting deformation is therefore the maximum strain recorded during the load cycle. Figure 2 presents the plot of the 45 strain peaks overlapped with the crack propagation induced by the fatigue load. The high-amplitude fluctuations with low frequency are caused by the temperature changing during day-night transitions and sun passing through the lab windows and hitting the structure. Figure 2 also highlights the trend of increasing strains during the end of the test, when the crack becomes notable. Here, the large fluctuations of the signals may jeopardize the performance of strain-based diagnostic systems.

The strain peaks changed during the FCG because of: (i) temperature variations and (ii) the crack propagating from the rivet hole, which is the effect that one wants to identify to assess the health condition of the structure.

Figure 2: Strain history and crack propagation during the test. The left y-axis refers to the micro-strains acquired by the FBG sensor network, while the right y-axis shows the semi-crack length.
3 COINTEGRATION

Here, the advantage of cointegration is the combined removal of the temperature effect and the highlighting of anomalies within the strains by comparing a feature coming from the baseline signal (acquired in a healthy or normal condition) with the signal recorded during the crack propagation. Given the mathematical complexity of cointegration, this section summarizes the concept and the main equations only. The interested reader can refer to [3] for finer details.

Let us consider two time series \( y_1, y_2 \in \mathbb{R}^{N \times 1} \). Cointegration finds whether a cointegrating vector \( \eta \) exists, i.e. whether a linear combination of the time series is stationary,

\[
y_1 - \eta y_2 = u,
\]

where \( u \) is a stationary process. In SHM, the monitored variables, coming from the same system and driven by the same process, more than likely share common trends [3]. The purpose of cointegration in SHM is to find when the time series (representing the features or monitored variables) do not share the same trend, which occurs when the linear combination of such features is no longer stationary. The deviation from the expected trend means that the system is not operating in its 'normal condition' (the condition recorded earlier), so the presence of an anomaly in the structure is likely.

The cointegration method proposed here, which is based on the augmented Dickey-Fuller (ADF) test and the Johansen procedure, can be successfully applied on condition that:

- \( y_1 \) and \( y_2 \) are both integrated of order one, \( y_1, y_2 \sim I(1) \), so their first difference is a stationary process, and
- they are linearly correlated (i.e., \( y_1 \) vs. \( y_2 \) is approximately linear).

These requirements must be satisfied by all the time series included in the analysis. Therefore, a pre-check must be carried out to make sure that the hypothesis of cointegration are fulfilled. The procedure in extensively discussed in [3] and it is briefly summarized below.

3.1 Augmented Dickey-Fuller test

The ADF test [8] enables the evaluation of the integration order of each time series by performing a unit-root modified t-test statistics on the following time series model:

\[
\Delta y_i = \rho y_{i-1} + \sum_{j=1}^{p-1} b_j \Delta y_{i-j} + \epsilon_i.
\]

Here, the subscript \( i \) is the \( i \)-th observation, \( \Delta y_{i-j} = y_{i-j} - y_{i-j-1} \) and \( \epsilon_i \) is the model error that becomes a white noise process by including a sufficient number of lags \( p \). The unit root test is carried out by a proper substitution of the model parameters \( \rho \) and \( b_j \), which brings to the characteristic equation of an auto-regressive process of order \( p \), and by eventually ensuring that

\[
t_\rho = \frac{\hat{\rho}}{\sigma_\rho} < t_\alpha
\]

is less than \( t_\alpha \) (where \( \alpha \) is a specific confidence level). If the hypothesis is accepted, the time series has a unit root and is integrated of order one, thus satisfying one of the requirements
If all the time series are \( I(1) \), the Johansen procedure can be carried out to find the cointegrating vector and evaluate the residuals of cointegration.

### 3.2 Application of Johansen procedure

The Johansen procedure [9] is a likelihood method to estimate the parameters of a vector autoregressive (VAR) model written in the error-correction form

\[
\{\Delta y_i\} = \{\Pi\}\{y_{t-1}\} + \sum_{j=1}^{p-1} [B_j]\{\Delta y_{t-j}\} + \{\phi\}\{D(t)\} + \epsilon_t. \tag{6}
\]

The vector \( \{y_i\} \) contains the \( n \) time series that are monitored, so is a \( n \)-dimensional vector. The matrices \( \{\Pi\} \), \( \{B_j\} \) and \( \{\phi\} \) contain the model parameters, the vector \( \{D(t)\} \) describes a deterministic trend of the time series (if one exists) and \( p \) is the model order.

The matrix \( \{\Pi\} \) describes the long-run equilibrium between the variables, and contains the cointegrating vector that has to be found [9]. Assuming that (6) is a true error-correction model and all the time series are \( I(1) \), the parameter matrix must be rank deficient, with rank \( r \), and can be decomposed as \( \{\Pi\} = [\alpha]\{\beta\}^T \), where \( \{\beta\} \in \mathbb{R}^{n \times r} \) is the matrix containing the cointegrating vectors. If the time series are cointegrated, then the model error must be stationary and defined by

\[
\{\epsilon_i\} \sim \mathcal{N}(0, \Sigma), \tag{7}
\]

where \( 0 \) is a zero vector and \( \Sigma \) is the covariance matrix. The likelihood of observing a model error like the one in (7) is therefore the product of probabilities coming from a Normal distribution:

\[
\mathcal{L} = \prod_{i=1}^{N} p(\epsilon_i) = ((2\pi)^{n/2} |\Sigma|)^{-N/2} \exp \left( -\frac{1}{2} \sum_{i=1}^{N} \{\epsilon_i\}^T \Sigma^{-1} \{\epsilon_i\} \right), \tag{8}
\]

where \( p(\epsilon_i) \) is the probability of \( \epsilon_i \). The objective is to select the parameter vector that maximizes the likelihood \( \mathcal{L} \), which corresponds to the maximization of the determinant of the covariance matrix \( |\Sigma| \). Therefore, the error covariance matrix must be expressed as a function of \( \{\beta\} \). The entire process requires the introduction of further notations and several equations, so it is not reported for the sake of brevity. The reader is referred to [3],[9] for details on the Johansen procedure. Eventually, the estimation of the most likely cointegrating vector requires the solution of a generalized eigenvalue problem

\[
(\lambda[N] - [M])\{v\} = 0, \tag{9}
\]

where the solutions compose of the eigenvalues \( \lambda_j, j = 1, ..., r \) and the corresponding eigenvectors \( \{v_j\}, j = 1, ..., r \). The two matrices \( [N] \) and \( [M] \) come from the definition of the error \( \{\epsilon_i\} \) as a function of the cointegrating vectors \( \{\beta\} \), and are defined in [3]. The best cointegrating vector (among the \( r \) available) will be the one linked to the largest eigenvalue.

As reported in [3], the cointegrating vector is calculated using a training set, and then is used to fit new features coming from further recorded signals. The residuals from the new fitting will continue to be stationary if the structure will remain in its normal condition. If (or when) any anomaly will be present in the signal, the resulting residuals will deviate from the previous, stationary condition.
4 NOVELTY DETECTION RESULTS

The cointegration method described in Section 3 is applied here to the strain peaks acquired during the FCG test presented in Section 2. Therefore, the peaks extracted from the strain signals in figure 2 are the features or monitored variables that have to be cointegrated. The purpose of cointegration is the detection of the crack propagating on the skin by detecting an anomalous deviation of the cointegration residuals, according to the procedure hereafter.

Part of the strain history has been used as training set, which contains the strain peaks representing the normal condition of the structure without any crack. The cointegrating vector has been estimated using such training set, and a confidence boundary of the stationary residuals has been defined as $\pm 2\sigma$, where $\sigma$ is the standard deviation of the residuals. Figure 3 shows the residuals of cointegration performed on all the recorded variables. They refer to the training set, which has been selected as the first $200 \cdot 10^3$ load cycles, when the crack did not yet nucleate. The length of the training set remains the same in all of the runs henceforth. The amount of data that should be used for training is also a parameter that could condition the performance of the method, but it has not been investigated at this stage of the research.

The spikes visible in figure 3 are in reasonable agreement with the spiky signals in figure 2, that as mentioned in Section 3, the spikes in the original signals were generated by day-night transitions and the hit of the sun on the structure. The remaining strain history has been fitted using the cointegrating vector that has generated the residuals in figure 3. The residuals of the remaining strain history have been monitored to check if they fall outside of the confidence boundary visible in figure 3. All the recorded strain peaks ($n = 45$) are potential features to detect the presence of anomalies in the structure. The cointegrating vector obtained from the training set has been used to fit the 45 signals, and figure 4 shows the marked trend appearing in the residuals when the crack modifies the strain pattern.

The residuals begin deviating from the stationary condition at an early stage, when the crack is still relatively small. Since the spikes can generate a considerable number of false alarms by
making the residuals temporarily fall outside the ± 2σ confidence boundary, a moving average of the residuals with \( l = 500 \) lags has been adopted as anomaly detector. The triangle in figure 4 (a) shows when the moving average falls outside the threshold, obtained by cointegrating all the 45 strain signals recorded by the sensor network. According to the cointegrated signal, an anomaly appears around 410000 load cycles, when the semi-crack is 9-10 mm long. It should be noted that the semi-length of the initial notch at the rivet hole was 7.5 mm long. The residuals move from the stationary condition with a rather monotonic drift, in accordance with the crack propagation. As a matter of fact, the increasing crack length introduces growing alterations of the strain pattern in the material, so larger anomalies in the strain peaks.

Given the high number of sensors installed on the panel, a preliminary sensitivity analysis of the method has been carried out to understand the detection capabilities of cointegration with respect to the distance from the damage. This sensitivity analysis has involved a limited number of sensors only. The sensors applied on stringer 7, which is the stringer where the crack propagated, have been cointegrated and the time instant when the residuals have exceeded the confidence boundary has been recorded. Cointegration has been performed using sensor pairs (i.e., two sensors), starting from the farthest sensors (named \{28, 36\}) towards the sensors closest to the crack (named \{32, 33\}). So, in this stage, only two time series per run have been used to find the cointegrating vector and detect the damage. The drift of the residuals should appear late in the test when using far sensors (\{28, 36\}), while should appear early when the cointegration is performed on close sensors (\{32, 33\}).

Figure 5 shows the detection results with respect to the crack propagation measured with a manual caliper. The graph reports the semi-crack length against load cycles: the crack was not visible during the first 400000 load cycles, so the initial data refer to the artificial notch whose semi-length was around 7.5 mm. Then, the crack started propagating and it was monitored with a manual caliper. A scheme of the stringer 7 shows the sensor locations and the location of the crack, on the rivet in between the sensors 32 and 33. The triangles refer to the time instant (or number of load cycles) when the moving average of the cointegration residuals has exceeded the confidence boundary, and so the crack has been detected by cointegration. As visible, the
closer the sensor pairs, the earlier the detection of the anomaly in the strain peaks.

5 CONCLUSIONS

Cointegration has been recently proposed in SHM as effective signal processing techniques for the removal of benign variations of the monitored features. The work presented in this paper constitutes a further, promising result of the application of cointegration in the SHM domain. The results refer to a relevant structure in a laboratory environment, where the large temperature fluctuations caused by day-night transitions and the hitting of the sun on the structure affected the strain recorded by a FBG-based sensor network. The strain peaks recorded during the application of a sinusoidal fatigue load have been used as input features. As visible from the results in Section 4, cointegration seems able to filter out the common trends caused by temperature fluctuations, and highlights the presence of a crack without any additional smart algorithm for anomaly detection. The analysis performed using the sensors on stringer 7 has shown an intuitive result: the closest the sensors, the better the damage detection performance. At the same time, it provides a qualitative indication of the sensitivity of the method in the presented scenario. The detection performance of cointegration obtained in this work is in reasonable agreement with the current damage tolerance regulation indicated in [10] using both the entire sensor network (figure 4) and only two sensors on the stringer closest to the damage (figure 5). A limitation of the method is the linearity required to cointegrate time series. As already stressed in some papers, if the variables are not linearly related, the proposed cointegration method cannot be applied. Focusing on the work presented in this paper, the strain peaks may not work as an effective feature when the fatigue load applied on the structure has a random spectrum, since the strain peaks would change at each load cycle. Therefore, the use of such a feature is questionable in real environments where aerostructures are subject to random loads, and the use of other features should be investigated. Nevertheless, this
application corroborates the successful outcomes and the potentiality of cointegration presented in previous papers and regarding more simplified case studies.

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