STUDY OF BRIDGE STRUCTURAL STATE EVOLUTION BASED ON NONLINEAR VIBRATION

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Abstract
This paper analyzes bridge nonlinear vibration generating mechanism, Modeling the related Bridge Resilience to structural state and damage evolution. Firstly, obtain resilience expressions that contain squared item, cubic item, etc. nonlinear term by Energy Method. Secondly, creating a nonlinear vibration equation of the system by "three assumptions", constructing of structural damage evolution model combined with the relationship between the Restore Force and the structural state. Finally using Runge-Kutta numerical method analysis system dynamic characteristics, through the phase space trajectory, Poincaré section, Lyapunov exponent, Poincaré section radius based on Euclidean distance, the relationship between structural damage and its dynamics behavior were established respectively from the qualitative and quantitative. After analysis of damage evolution model. With deterioration of structural state obtained that with Phase space trajectory expanding, Poincaré section expanding gradually; Lyapunov exponent showed monotonically increasing or decreasing trend; Radius of the Poincaré section presents exponential increment state.

Keywords: Nonlinear Vibration, Restoring Force, Phase Space Trajectory, Poincaré Section, Lyapunov Exponent

1. INTRODUCTION
Currently, Non-linear Vibration Theory, Complex Systems Theory and Computer Numerical Analysis Techniques in bridge structural analysis, structural health monitoring and safety evaluation has been studied. In 2012, Zhenhua Nie [1] and Hui Zheng[2] proposed structural damage detection method based on phase space reconstruction, structural dynamic information analysis by using the phase space reconstruction techniques are studied from different aspects of structural vibration response of the nonlinear features. At the same year, the Professor Jianxi Yang[3] doctoral thesis which based on analysis of existing bridge state of chaos in nonlinear dynamics theory reflects the structural dynamic response information contains information about the structural non-linear to a certain extent, and the structural of the state can be characterized by a complex system characteristic parameters. In 2011, Zhigang Zeng[4] studied nonlinear dynamics nanocomposites buckled beam parametric
excitation system, noting that such a system present an extremely rich and complex dynamical behavior, such as Bifurcation, Fractal and Chaos characteristics etc.

Thus, this paper further analyzes relationship between structure of non-linear, nonlinear vibration, characteristics of complex systems and structural state. Theoretically revealed nonlinear vibration generating mechanism of the bridges, constructed status evolution nonlinear vibration model and detailed analysed of the system vibration phase space trajectory, Poincaré section properties.

2. STUDY OF STRUCTURE NONLINEAR VIBRATION AND DAMAGE MECHANISM

In this paper, use the Energy Method[5]analyzed structure of nonlinear vibration mechanism. Hooke's law of elasticity in the system is proportional to the elastic potential energy and second power of displacement.

\[ U(x) = \frac{1}{2} \omega^2 x^2 \]  

(1)

It proved that bridge structure system, bridge pier, cables, towers and other structures are not subject to such a simple rule of [5,6] as formula (1). Its elastic potential should take the following general formula:

\[ U(x) = \frac{1}{2} kx^2 + \frac{1}{3} \lambda x^3 + \frac{1}{4} \mu x^4 + \cdots \]  

(2)

In this formula, \( k \) , \( \lambda \) and \( \mu \) are constant coefficients, the structure Restoring force should take the following form:

\[ f = -\frac{dU}{dx} = -kx - \lambda x^2 - \mu x^3 + \cdots \]  

(3)

Such equations of motion obtained by \( U \) and \( f \) are non-linear. According to Newton's second law to establish the structure of the system dynamics parametric equation formula (4) as follows:

\[ \ddot{x} + c \dot{x} + kx + \lambda x^2 + \mu x^3 + \cdots = F(t) \]  

(4)

According to the literature [7] noted that the resilience reflects nonlinear of the materials, and shows us the load-displacement hysteresis curve; literature[8]pointed out that the specific values \( n \) of the slope of restoring force curve is related to structural properties; literature[9] pointed out that the cause of non-linear restoring force are basically in two ways: first, the larger displacement of the quality, it beyond the scope of small deformation and resulting in geometric nonlinearity; second, the structural material properties and strength properties of the components have beyond the elastic range and resulting in physics nonlinear. Thus, with the intensification of structural damage, the structure restoring force curve will changed naturally. By changing the \( k \) , \( \lambda \) and \( \mu \), etc. coefficients. You can simulate the structure damage and the extent of it. By changing the coefficients of \( k \) , \( \lambda \) and \( \mu \) etc, you can simulate the structure damage and the its’ degree. Thus, the study of nonlinear behavior of the evolution of structural vibration mode by establishing the nonlinear restoring force of the structure damage model combined with non-linear vibration theory, nonlinear dynamics,
numerical analysis methods, it provides a theoretical basis for bridge structural health monitoring.

3. BRIDGE NONLINEAR VIBRATION MODEL

3.1 Basic Assumptions
Nonlinear vibration model in this study are as the following basic assumptions:
1) Symmetry elastic potential energy

\[ U(x) = \frac{1}{2} kx^2 + \frac{1}{4} \mu x^4 \] (5)

2) Structural systems are hard elastic (that is \( k > 0, \mu > 0 \));
3) The damping keeps the same size in different structural state, and the mass of the structure is approximately equal to 1.

3.2 Structural Damage Model
Bridge structure system dynamics model can be abstracted into a mathematical model of mass, damping and stiffness matrices of mechanical systems[10], when beam structure subjected to external force and the damped forced by vibration, the vibration equation can be written as (6):

\[ [M]\dddot{x} + [C]\dot{x} + [K]x = \{F(t)\} \] (6)

Where \([M]\), \([C]\), \([K]\) are Mass Matrix, Damping Matrix, Stiffness Matrix; \([\dddot{x}]\), \([\dot{x}]\), \([x]\) are Acceleration Vector, Velocity Vector, Displacement Vector; \([F(t)]\) is Load Vector (Excitation Matrix).

In summary, build SDOF beam structure model (Figure one) based on three basic assumptions.

![Figure 1: SDOF model of a simply supported beam](image)

Establishing structural system dynamics parameters model (7) according to Newton’s second law as follows:

\[ \dddot{x} + c \dot{x} + kx + \mu x^3 = F(t) \] (7)

Where \(k\) and \(\mu\) are decided by structural material, the structure mass is approximately equal to 1.

The structural state is correspondent to structural restoring force curve[11,12,13], so it is possible to simulate structural damage condition and the degree by changing the size of \(k\) and \(\mu\). Select more suitable restoring force parameters, construct a model and three kinds
of damage cases. The structural parameters $k$ and $\mu$ in each condition are shown in Table 1, the obtained structural recovery force curve shown in Figure 2.

Table 1: System equation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Cases</th>
<th>Structural Health</th>
<th>Damage cases 1</th>
<th>Damage cases 2</th>
<th>Damage cases 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td></td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 2: Structure Restoring Force Curve

4. NUMERICAL ANALYSIS

Then using Fourth-order Runge-Kutta method for solving nonlinear differential equations under different conditions by dynamics model and damage model established above. In this paper, the following in-depth analysis the phase space trajectory, Poincaré section changes and build Lyapunov exponent and Poincaré section radius safety assessment factor combined with Complex Systems Theory and Nonlinear Vibration Theory

4.1 Structural State Evaluation Method based on The Phase Space Trajectory Exception.

Considering the Damping and the influence of external incentives, the initial value equation set as (8), the damping coefficient is 0.2, using the harmonic load as external excitation, then obtained nonlinear vibration system of equations as (9).

$$\ddot{x} + 0.2\dot{x} + 20x + \mu x^3 = 6\cos(t) \quad (9)$$

Then using Fourth-order Runge-Kutta method to obtain four conditions phase trajectory combined with phase space theory [14], as shown in Figure 3.
Figure 3: damping, external incentives Phase trajectory case
(a) when $\mu=0$ phase trajectory; (b) when $\mu=4$ phase trajectory;
(c) when $\mu=8$ phase trajectory; (d) when $\mu=12$ phase trajectory;
(e) four kinds of conditions comprehensive phase trajectory;

From above chart, with decreasing nonlinear restoring force parameters (increasing of the
degree of structural damage), structural system in phase space trajectory shows an outward
expansion trend. Thus, the analysis and assessment of the structural state evolutionary trend
could by use od structural vibration phase trajectory spectrum. Then, calculate Lyapunov
exponent and quantitatively reflect the expansion phase space trajectory.
Combined with Lyapunov exponent calculation theory [14], the system equations of this paper written in the form of autonomous systems as (10):

\[
\begin{align*}
x &= y \\
y &= 6 \cos z - 0.2 y - kx - \mu x^3 \\
z &= o \end{align*}
\]  

(10)

Corresponding Jacobian Matrix:

\[
Df = \begin{bmatrix}
0 & 1 & 0 \\
-20 - 3 \mu x^2 & -0.2 & -6 \sin z \\
0 & 0 & 0
\end{bmatrix}
\]  

(11)

Obtained Lyapunov exponent under various conditions shown in Figure 3 by different $\mu$ values numerical analysis, the values shown in Table 2:

![Figure 4: Lyapunov Exponent Spectrum](image)

(a) when $\mu=0$ Lyapunov exponent spectrum; (b) when $\mu=4$ Lyapunov exponent spectrum; (c) when $\mu=8$ Lyapunov exponent spectrum; (d) when $\mu=12$ Lyapunov exponent spectrum;

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
</tr>
</thead>
</table>

Table 2: Lyapunov exponent size
Obtained the Lyapunov exponent development trend from the table, shown in Figure 5.

<table>
<thead>
<tr>
<th></th>
<th>-0.03</th>
<th>-0.05</th>
<th>-0.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>1.2</td>
<td>-2</td>
</tr>
<tr>
<td>8</td>
<td>0.9</td>
<td>2.4</td>
<td>-3.5</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>4.5</td>
<td>-6</td>
</tr>
</tbody>
</table>

Figure 5: Lyapunov exponent development trends

It can be drawn that with structural damage degree increased, the Lyapunov exponent showed monotonically increasing or decreasing, Lyapunov exponent in different directions have the same monotony. Thus, the Lyapunov exponent can quantitatively reflect structural state on the damage evolution to assess the structural damage trend.

4.2 Structural State Evaluation Method based on Changes Poincaré Section.
Using Fourth-order Runge-Kutta method combined with Poincaré map theory [14] to obtain four conditions of Poincaré section, as shown in Figure 6.

(a) (b)
The numerical analysis shows that Poincaré section in the innermost (green represents the area) of the image under healthy case. With increasing degree of injury, its section gradually expand outward, the red area represents the most serious structural damage state is the outermost of the image. Thus, the Poincaré section can qualitatively reflect the trend of deterioration of structural damage and assess the development direction of the structural state.

In order to assess Poincaré section trends more intuitive, this paper constructs Poincaré section radius based on Euclidean distance and use it as the assessment of the structural state of the development trend factor (Structure State Evolution Radius Difference, SSERD). Specific methods are as follows:

Obtain n data amount from the database of system equations and calculated d for each condition.

$$d = \frac{\sum_{i=1}^{n} \sqrt{x_i^2 + y_i^2}}{n}$$  \hspace{1cm} (12)

Establish the structural state evolution radius difference (Structure State Evolution Radius Difference, abbreviation: SSERD).
The results shown in Table 2:

<table>
<thead>
<tr>
<th>Working conditions</th>
<th>$\mu=0$</th>
<th>$\mu=4$</th>
<th>$\mu=8$</th>
<th>$\mu=12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSERD</td>
<td>0.495</td>
<td>0.508</td>
<td>0.521</td>
<td>0.545</td>
</tr>
</tbody>
</table>

The SSERD evolutionary trend as shown in Figure 7.

The numerical results show that: with increasing degree of structural damage, the structural state of the development trend factor present exponentially rising trend, this factor can quantitatively assess the structural state of the development trend.

5. CONCLUSIONS

This paper reveals the bridge nonlinear vibration generating mechanism and established system nonlinear vibration equation, constructed the nonlinear dynamics model for bridge damage evolution, analog different structural damage conditions by a single degree of freedom model of a simply supported beam, then obtained Phase Trajectory, Lyapunov Exponent, Poincaré Section, Poincaré Section Radius from healthy and injuries conditions by numerical analysis. It can be qualitatively and quantitatively assess the structural state development trends.

The results of this paper enriching and developing Non-Linear, Non-Linear Dynamics and Complex Systems Theory in health monitoring and safety evaluation of bridge. However, many questions need further study, such as how to transform dynamic response information from actual bridge into a theory nonlinear vibration model, the nonlinear dynamic behavior of large span, multi-degree of freedom coupling, the construction of bridges nonlinear vibration model and so on.

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REFERENCES


