Guided Wave-based Damage Localization in Isotropic Structures with Smoothly Varying Thickness

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Abstract

Modern technical structures, e.g. in the aviation and maritime industry, aim at an increased material efficiency and optimized weight. Therefore, the structural thickness changes locally according to the particular load case. At the same time the varying thickness represents a challenge for active and passive structural inspection by means of guided elastic waves, because the phase and group velocity of each wave mode changes with the thickness of the structure. As a result, conventional beamforming techniques for damage and acoustic source localization are inherently affected by errors, because the average group velocity and not the actual group velocity is used in the localization procedures.

In this paper, a generalized delay-and-sum beamforming approach for damage localization in isotropic structures with smoothly varying cross section will be presented. This formalism takes the spatially varying group velocity of the adiabatic wave motion into account. Damage localization results based on the fundamental symmetric wave mode in a structure with a linear and quadratic thickness profile will be presented and discussed.

1 INTRODUCTION

Modern technical structures, e.g. in the aviation and maritime industry, aim at an increased material efficiency and optimized weight. Therefore, the structural thickness changes locally according to the particular load case. Guided wave propagation in a structure with nonuniform thickness is complex, because the phase and group velocity of each propagating wave mode changes along the nonuniform waveguide [1]. Examples of studies that consider this adiabatic wave motion can be found for example in Refs. [2]–[5].

The varying thickness represents a challenge for active and passive approaches for structural inspection by means of guided elastic waves. As a result, conventional beamforming techniques for damage or acoustic source localization are inherently affected by errors, because the average group velocity and not the actual group velocity is used in the localization procedures. A performance comparison of different damage localization techniques in a structure with constant thickness can be found in [6]. The authors compared variants of the time of arrival method (also called ellipse method or delay-and-sum algorithm), time difference of arrival method (also called hyperbola method), RAPID-method, energy of arrival and total product method. Damage localization in a flat anisotropic structures has been demonstrated in [7]. More recently, model-based localization techniques have been proposed that solve a sparsity problem [8], [9].
The novelty of this paper is to present a generalized delay-and-sum beamforming technique that takes the locally varying thickness variation of the structure into account. The underlying signal model and mathematical description will be presented in Section 2 together with a brief discussion on guided wave propagation in isotropic structures with changing thickness. Damage localization results based on the fundamental symmetric wave mode in structures with linear and quadratic thickness profile will be presented in Section 3 and discussed in terms of the corresponding localization error. Finally, conclusions are drawn at the end.

2 MATHEMATICAL DESCRIPTION

2.1 Signal Model

In the following, we will assume \( N \) transmitters and the same number of receivers arranged in a spatially distributed sensor network. Each element acts in turn as transmitter which leads to a total number of \( N(N-1)/2 \) sensor signals. The scattered signal that is measured at the \( m \)-th receiver can be expressed as

\[
Y_m(\omega) = \sum_{n=1}^{N} U_n(\omega) e^{i k (\omega, d(x,y)) r_{mn}} + N(\omega)
\]  

(1)

In this equation, \( U_n(\omega) \) is the spectrum of the excitation signal sent by the \( n \)-th transmitter. This can be e.g. a chirp \([10]\) or spread spectrum signal \([11]\). Moreover, \( N(\omega) \) is the spectrum of the superimposed measurement noise. The Euclidian distance from the actuator to the damage and from the damage to the sensor is given by \( r_{mn} \). Interesting here is that the wavenumber is not only a function of angular frequency \( \omega \), but also a function of the plate thickness \( d(x,y) \).

In this work, we will consider symmetric wave motion. The dispersion relationship for the \( S_0 \)-mode for different thickness values is illustrated in Figure 1. A maximum change in group velocity can be observed at a carrier frequency of 350kHz. This is a frequency at which common piezoelectric transducers have a high mode purity of the fundamental symmetric wave mode \([12]\). This carrier frequency will also be studied in the results, subsequently.

![Figure 1: Dispersion curves for \( S_0 \)-Mode at different thicknesses: (left) phase velocity (right) group velocity (Young Modulus \( E=70 GPa \), Poisson ratio \( v=0.34 \) and material density \( \rho=2.700 kg/m^3 \)).](image-url)
2.2 Image Reconstruction in Isotropic Structures with Nonuniform Cross Section

The image reconstruction problem is illustrated in Figure 2 by means of three piezoelectric transducers $P_1$, $P_2$ and $P_3$. The phase velocity $c_{ph}=f(x,y,d)$, and hence also the wavenumber, is a function of the thickness profile.

![Image](image.png)

Figure 2: Geometry for the image reconstruction (thickness variations illustrated by color).

The reconstruction is based on a spatial discretization of the structure and each voxel has a unique velocity. It is assumed that the effect of dispersion can be neglected so that the frequency dependency of the problem is not considered. Only the differences in velocities at the carrier frequency, as shown in Figure 4, are taken into account.

The time delay $\tau_{nD}$ from the $n$-th transmitter to the voxel $D(x,y)$ and the time-delay $\tau_{Dm}$ from the voxel to the $m$-th receiver is calculated on an incremental basis. Therefore, $N_P$ points need to be defined on a straight line between the known coordinates of the transmitter/receiver and the voxel [13]. The corresponding velocity at each of the $N_P$ points needs to be interpolated from adjacent voxels.

\[
\tau_{nD} = \sum_{k=2}^{N_P} \frac{\|\tilde{x}_k - \tilde{x}_{k-1}\|}{(1/2)[c(f(\tilde{x}_k, d) + c(f(\tilde{x}_{k-1}, d))]} \tag{2}
\]

\[
\tau_{Dm} = \sum_{l=2}^{N_P} \frac{\|\tilde{x}_l - \tilde{x}_{l-1}\|}{(1/2)[c(f(\tilde{x}_l, d) + c(f(\tilde{x}_{l-1}, d))]} \tag{3}
\]

Note that the velocity in the denominator of this equation is the average velocity between two adjacent voxels on the straight line. Moreover, in contrast to conventional beamforming techniques the presented technique operates in the time domain and not in the range domain. The total time delay is defined as $\tau_D(x,y) = \tau_{nD} + \tau_{Dm}$.

In order to make the image reconstruction more robust, e.g. in terms of a better robustness against noise and jitter, Byrne et al. [14] do not sum the signals directly at the focal point $D(x,y)$. Instead they suggest to use a window of length $T_{win}$ by which the energy associated with the focal point at the structure can be expressed as
The global maximum of $I(x, y)$ represents the location of the scatterer. In this work the time window is defined by $T_{\text{win}} = 10 \mu s$ and the number of points on the straight line is $N_P = 30$.

3 RESULTS

The results section will demonstrate the performance of the generalized beamforming technique by means of three case studies.

3.1 Case A: Damage localization in a tapered isotropic plate with ten segments

The first scenario considers a $1m \times 1m$ structure equipped with nine piezoelectric transducers. Its thickness changes linearly from 3mm to 7mm. The structure consists of ten equidistant segments as shown in Figure 3(left) and the point target is placed at $x=0.3m$ and $y=0.3m$. The corresponding velocity map at the carrier frequency of the tone-burst signal at 350kHz is depicted in Figure 3(right). It is interesting to see that the phase velocity of the $S_0$-mode changes in this region from 3881m/s to 5310m/s.

The image reconstruction based on the actual velocity map is shown in Figure 4. The localization error in this case is approximately 5mm. The best assumption for the conventional delay-and-sum image processing is to use the average velocity from the velocity map shown in Figure 3(right). This leads to the localization result presented in the right part of Figure 4, where the localization error in this example is about 24mm.

![Figure 3: (left) Thickness profile of an isotropic wave guide with ten equidistant segments; (b) corresponding $S_0$-mode phase velocity at the center frequency of 350kHz.](image-url)
For a more detailed analysis the individual contributions of the transducer pairs P₄-P₆ are considered. Therefore, Figure 5 shows the case of the actual velocity distribution (top left), the average velocity (top right), the maximum velocity (bottom left) and minimum velocity (bottom right). It can be observed that only minor differences occur when the average and not the actual velocity is used. However, as shown in Figure 4(right) the average velocities lead to a remarkable localization error in the final image. More significant is the impact when the maximum velocity, i.e. 5310m/s, and the minimum velocity, i.e. 3881m/s, is used. We can see that the ellipses do not cross the actual position of the scatterer so that the localization procedure fails in that case.
3.2 Case B: Damage localization in an isotropic plate with two segments

In the second case the number of segments is reduced from ten (see previous section) to two. The wave travels now longer in the thinner or thicker part of the structure. It can be expected that the localization error further increases also for the case when the average velocity is considered. This hypothesis is confirmed in Figure 6(left) which shows the image reconstruction exploiting the actual velocity distribution. The reconstruction performance is similar to the one in Figure 4(left). On the other hand, the right part of Figure 6 illustrates the localization result when the average velocity $4596m/s$ is used. The localization error increases as expected to about $81mm$.

![Figure 6: Image reconstruction of a point target in the quadratic isotropic structure with ten equidistant segments: (left) using the actual velocity distribution (localization error: 5mm) (right) using the average velocity (localization error: 81mm).](image)

3.3 Case C: Damage localization in an isotropic plate with quadratic thickness profile

Finally, a more complex scenario will be presented using a structure with a quadratic change in thickness as shown in Figure 7(left). The corresponding phase velocity map is shown in Figure 7(right). In case the actual velocity map is used then the scatterer can be located precisely as shown in Figure 8(left). The localization error for the average velocity is about $77mm$ as shown in Figure 8(right).

![Figure 7: (left) Quadratic thickness profile of an isotropic wave guide with ten equidistant segments; (right) corresponding $S_0$-mode phase velocity at the center frequency of $350kHz$.](image)
Figure 8: Image reconstruction of a point target in the quadratic isotropic structure with ten equidistant segments: (left) using the actual velocity distribution (localization error: 21 mm) (right) using the average velocity (localization error: 77 mm).

5 CONCLUSIONS

This paper introduced a generalized delay-and-sum beamforming technique that enables the precise localization of a damage in an isotropic structure with varying thickness. This can be achieved by incorporating the spatially varying wave mode velocity into the formulation of the beamforming technique. As a result, the processing is performed in the time-domain rather than the range-domain. Accurate imaging results have been obtained in an aluminum structure based on the fundamental symmetric wave mode. Significant localization errors occur when the average velocity map and not the actual velocity map is used. Future work will focus on the investigation of the dispersion effect which has not been considered in this paper.

REFERENCES


