Sensor Placement for Modal Parameter Subset Estimation: A Frequency Response-based Approach

Martin D. Ulrikse, Dionisio Bernal, and Lars Damkilde

1 Dept. of Civil Engineering, Aalborg University, Niels Bohrs Vej 8, 6700 Esbjerg, Denmark. mdu@civil.aau.dk & lda@civil.aau.dk
2 Dept. of Civil and Environmental Engineering, Northeastern University, 404 Cushing Hall, Boston, MA 02115, USA. D.Bernal@neu.edu.

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Abstract
The present paper proposes an approach for deciding on sensor placements in the context of modal parameter estimation from vibration measurements. The approach is based on placing sensors, of which the amount is determined a priori, such that the minimum Fisher information that the frequency responses carry on the selected modal parameter subset is, in some sense, maximized. The approach is validated in the context of a simple 10-DOF chain system by computing the variance of a set of identified modal parameters in a Monte Carlo setting for a set of sensor configurations, whose anticipated effectiveness are ranked according to the noted criterion. It is contended that the examined max-min criterion satisfies the objective of optimizing the accuracy of an identification setup more effectively than the more commonly used trace or determinant of the Fisher information matrix (FIM). It is shown that the widely used Effective Independence (EI) method, which uses the modal amplitudes as surrogates for the parameters of interest, provides sensor configurations yielding theoretical lower bound variances whose maxima are up to 30% larger than those obtained by use of the max-min approach.

1 INTRODUCTION

Identifying the modal parameters—that is, eigenvalues, damping ratios, and mode shapes—of a structural system is central in several engineering applications, for instance, structural (re)design, structural health monitoring (SHM), and structural control. Obviously, one is interested in identifying the modal parameters with as much precision as possible, and one item that has an influence on the level of variance-related error (resulting from noise in the measurements) is the deployment of the available sensors. Numerous approaches have been proposed for placing sensors to obtain as good estimates of certain modal parameters as possible, see [1] for a brief literature review. Especially information theory-based methods, in which the sensor locations are chosen to reduce the estimate uncertainties, have gained much attention. Typical approaches are to maximize some metric of the Fisher information matrix (FIM), see, for example, [2, 3], or to minimize the information entropy, see, for example, [4, 5]. Examination shows that the information entropy is linked to the determinant of the FIM, suggesting that minimization of the former is tantamount to maximization of the latter.

The FIM provides a measure of the amount of information an observable random variable carries on an unknown parameter vector. The sensor placement approach proposed in this study employs frequency response functions (FRFs) as the observable variables, which—due to the assumptions of linearity and classical damping—provide simple analytical expressions for the sensitivities required to compute the FIM. Additionally, the use of FRFs also facilitates truncation of the frequency interval, as the modes decouple such that only a limited range, bounded by the lowest and highest modes of interest, needs to be included. The task of sensor
placement is formulated as a discrete optimization problem, in which a selected metric of the FIM is the cost function, while the locations of the sensors are the variables. Unlike most FIM-based methods, where the FIM trace or determinant constitutes the metric, we employ a max-min metric in which the minimum Fisher information on the selected modal parameter subset carried in the frequency responses is maximized.

It is opportune to note that the sensor placement task is a two-step problem; addressing what to optimize and deciding how the optimization is to be carried out, given that the exhaustive search solution scheme is often computationally forbidden. This study focuses on the first part of this duo and, for this reason, uses a numerical example (a 10-DOF system) where an exhaustive search optimization is feasible. The application example focuses on the identification of the damping since this parameter is often affected by large variance errors. It is shown that the max-min approach provides sensor placements whose performance is more pleasing than that realized by maximizing the trace or the determinant of the FIM. An examination of the performance of the widely used Effective Independence (EI) method, which is based on maximizing the linear independency of the available mode shapes [3], is also included.

The paper is organized as follows: in Section 2, a brief overview of the theoretical structural and statistical modeling composing the basis of the proposed sensor placement method is provided. Subsequently, in Section 3, the cost function is formulated, followed by the aforementioned application example in Section 4. Lastly, some concluding remarks are given in Section 5.

2 FISHER INFORMATION ON MODAL PARAMETERS CONTAINED IN FREQUENCY RESPONSE FUNCTIONS

We assume that the system of interest can be adequately idealized as linear, time-invariant with \( n \) degrees of freedom (DOF), such

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = r(t),
\]

where \( K, C, M \in \mathbb{R}^{n \times n} \) are the stiffness, damping, and mass matrices, \( x, \dot{x}, \ddot{x} \in \mathbb{R}^{n \times 1} \) are the nodal displacement, velocity, and acceleration vectors, and \( r \in \mathbb{R}^{n \times 1} \) is the nodal load vector.

The transfer matrix, \( G(s) \in \mathbb{C}^{n \times n} \), of the system in Equation (1) is given as

\[
G(s) = (Ms^2 + Cs + K)^{-1} \Rightarrow X(s) = G(s)R(s),
\]

with \( X(s) = \mathcal{L}(x(t)) \) and \( R(s) = \mathcal{L}(r(t)) \) being the Laplace transforms of the output and the input, respectively. Evaluating \( G(s) \) along \( s = j\omega \) yields the frequency response matrix (FRM)

\[
H(j\omega) = (-M\omega^2 + Cj\omega + K)^{-1},
\]

which, by assuming linearity and classical damping, can be decoupled using the undamped mode shapes to yield

\[
H(j\omega) = \sum_{i=1}^{n} \frac{\Phi_i\Phi_i^T}{\bar{\omega}_i^2 - \omega^2 + 2j\zeta_i\bar{\omega}_i\omega},
\]

with \( \bar{\omega}_i, \zeta_i, \) and \( \Phi_i \) being, respectively, the eigenfrequency, damping ratio, and mass-normalized, undamped eigenvector/mode shape of the \( i \)th mode.
2.1 The Fisher information matrix

The Fisher information, \( I \), is the inverse of the minimum covariance that any unbiased estimator of the parameters that are to be estimated can achieve. In the scalar case, it is the reciprocal of the smallest variance that can be attained, and in the multivariate one, it is such that the matrix \( I - Q \) is positive semi-definite, where \( Q \) is the inverse of the covariance attained by any unbiased estimator.

Letting \( Y = [Y_1 \ldots Y_N]^T \) be an observable random variable that carries information on a \( p \)-dimensional parameter vector, \( \theta = [\theta_1 \ldots \theta_p]^T \), then, in terms of the likelihood function \( f \), the FIM has entries [6]

\[
I_{a,b} = -E \left( \frac{\partial^2 \ln f(Y; \theta)}{\partial \theta_a \partial \theta_b} \right) \quad \forall a, b \in [1, p], \tag{5}
\]

which—given that \( E \) is the expectation operator—shows that \( \text{diag}(I) \) is the curvature of the expected value of the log-likelihood evaluated at the true parameters. If one assumes that \( Y \sim \mathcal{N}(\mu(\theta), \Sigma(\theta)) \), it can be shown, see, for example, [6], that Equation (5) becomes

\[
I_{a,b} = \left( \frac{\partial \mu}{\partial \theta_a} \right)^H \Sigma^{-1} \left( \frac{\partial \mu}{\partial \theta_b} \right) + \frac{1}{2} \text{tr} \left( \Sigma^{-1} \frac{\partial \Sigma}{\partial \theta_a} \Sigma^{-1} \frac{\partial \Sigma}{\partial \theta_b} \right), \tag{6}
\]

in which \( \text{tr} \) and superscript \( H \) denote, respectively, the trace and complex conjugate transpose, while

\[
\frac{\partial \mu}{\partial \theta_a} = \left[ \frac{\partial \mu_1}{\partial \theta_a} \frac{\partial \mu_2}{\partial \theta_a} \ldots \frac{\partial \mu_N}{\partial \theta_a} \right]^T \tag{7}
\]

and

\[
\frac{\partial \Sigma}{\partial \theta_a} = \begin{bmatrix}
\frac{\partial \Sigma_{1,1}}{\partial \theta_a} & \ldots & \frac{\partial \Sigma_{1,N}}{\partial \theta_a} \\
\vdots & \ddots & \vdots \\
\frac{\partial \Sigma_{N,1}}{\partial \theta_a} & \ldots & \frac{\partial \Sigma_{N,N}}{\partial \theta_a}
\end{bmatrix}, \tag{8}
\]

If, in addition to normality, \( \Sigma(\theta) = \Sigma \), the second term in Equation (6) vanishes, and the FIM expression reduces to

\[
I_{a,b} = \left( \frac{\partial \mu}{\partial \theta_a} \right)^H \Sigma^{-1} \left( \frac{\partial \mu}{\partial \theta_b} \right). \tag{9}
\]

2.2 Evaluation using the sensitivity of the frequency response

For the case with \( Y = H_{k,l} + w \), where \( w \sim \mathcal{N}(0, \sigma^2) \) is additive Gaussian noise, the sensitivity in Equation (9) becomes the derivative of the \( (k,l) \)-entry in the FRM with regard to the selected parameters constituting \( \theta \). With reference to Equation (4), the sensitivity is thus

\[
\frac{\partial \mu}{\partial \theta_a} = \frac{\partial H_{k,l}}{\partial \theta_a} = \frac{\partial}{\partial \theta_a} \left( \sum_{i=1}^{n} \Phi_{k,i} \Phi_{l,i} \right) \tag{10}
\]

for the \( a \)’th parameter in \( \theta \). To exemplify this principle, we let \( \theta = \zeta_1 \), which yields the following sensitivity:

\[
\frac{\partial H_{k,l}}{\partial \zeta_1} = \frac{\partial}{\partial \zeta_1} \left( \frac{\Phi_{k,1} \Phi_{l,1}}{\omega_1^2 - \omega^2 + 2j\zeta_1 \omega_1} \right) = -\frac{2j\zeta_1 \omega \Phi_{k,1} \Phi_{l,1}}{(\omega_1^2 - \omega^2 + 2j\zeta_1 \omega_1)^2} \tag{11}
\]

It is noticed that the assumption of \( w \sim \mathcal{N}(0, \sigma^2) \) is maintained throughout the remaining of the present paper.
2.3 Correlation of frequency response samples

Consider the case of a single sensor. If the covariance in this case is, as often done, taken as $\Sigma = \sigma^2 I$, then the Fisher information would grow unbounded as the number of frequencies considered increases, which is contrary to the fact that the modal identification variance does not decrease monotonically by fitting FRFs using more and more points. The contradiction is resolved by recognizing the correlation in the error of the FRF values if the frequencies are very close and the signals are of finite duration. To consider this matter, we postulate that the correlation decays exponentially with distance between frequencies, that is, we assume that the correlation at any sensor (we have one FRF per sensor since we operate on the premise of a single input) is

$$\beta_k = e^{-\alpha \omega_k} \quad \forall k \in [1, N].$$

To select a reasonable value for $\alpha$, we map the Fisher information on the modal parameter subset contained in frequency response realizations at resonance to the analogous Fisher information contained in the corresponding pole, $\lambda_i$. With the assumption of classical damping, this pole is given as

$$\lambda_i = -\zeta_i \omega_i + \omega_i \sqrt{\zeta_i^2 - 1},$$

from which we can define the observable variable $Y = [\Re(\lambda_i) \ \Im(\lambda_i)]^T$ and the parameter vector $\theta = [\zeta_i \ \omega_i]^T$. Hereby, the Jacobian

$$J = \begin{bmatrix} -\omega_i & -\zeta_i \\ \frac{\zeta_i \omega_i}{\sqrt{\zeta_i^2 - 1}} & \sqrt{\zeta_i^2 - 1} \end{bmatrix}$$

is obtained, and the FIM can be computed from Equation (9) with a suitable covariance matrix, which—as demonstrated in Subsection 4.1.1—can be derived from system identification simulations. Subsequently, a suitable correlation value in Equation (12) can be found as $I_{\alpha,a}(\alpha) = I_{\lambda,a}^{(\lambda)}$, where $a = 1$ or $2$ depending on whether the damping ratio or eigenfrequency is analyzed. This approach will be demonstrated in the application example; more specifically, in Subsection 4.1.

3 SENSOR PLACEMENT FORMULATION

The merit of maximizing some metric of the FIM becomes obvious if referring to the Cramér-Rao lower bound (CRLB), which states that the variance of any unbiased estimator, $\hat{\theta}_a$, satisfies

$$\sigma^2_{\theta_a} \geq \frac{1}{I(\theta_a)}.$$ 

Thus, by maximizing some metric of the FIM, one basically tries to minimize the theoretical lower-bound variances associated with estimation of the selected modal parameter subset.

By inspection of Equations (10) and (11), it is evident that the sensitivities, and, thus, the FIM, depend on the eigenvectors, whose entries are governed by the sensor locations. Consequently, the principle behind the sensor placement approach is to, a priori, select the single-input location plus a number of sensors to employ, and then find the sensor placements by solving a discrete optimization problem. Here, the cost function is a chosen metric of the FIM and the
variables are the locations of sensors, which are integers corresponding to DOF and, equivalently, rows in the FRM. The input is assumed fixed and, in this study, single at the \( l \)th DOF. A theoretical inspection suggests that for a single input, the ranking of the sensor deployments should not be affected notably by the specific position of this input; a hypothesis that has been verified on the basis of the application example treated in Section 4.

In the sensor placement formulation, Equation (9) generalizes such that multiple sensors can be directly employed. In the case of a single input kept fixed in DOF \( l \), along with \( m \) output sensors and a \( p \)-dimensional \( \theta \), the sensitivity becomes the Jacobian matrix

\[
\mathcal{J} = \left[ \text{vec} \left( \frac{\partial H_{k,l}}{\partial \theta_1} \right) \ldots \text{vec} \left( \frac{\partial H_{k,l}}{\partial \theta_p} \right) \right] \in \mathbb{C}^{mN \times p}
\]  

(16)

for \( k = 1, \ldots, m \), thus vec is the vectorization operator that stacks the sensitivities for all \( m \) sensors into one column vector and \( N \) is the number of samples in each frequency response. Likewise, the covariance matrix generalizes to \( \Sigma \in \mathbb{R}^{mN \times mN} \), hence the FIM becomes

\[
\mathcal{I} = \mathcal{J}^H \Sigma^{-1} \mathcal{J} = \sum_{i=1}^{m} \mathcal{J}_i^H \Sigma_i^{-1} \mathcal{J}_i,
\]  

(17)

in which we take the Fisher information from different sensors as being additive.

As noted previously, the typical choice of metric for sensor placement when using the FIM is the trace or the determinant. Examination shows that the FIM is a diagonal heavy matrix, and it is thus possible, without incurring undue error, to idealize it as being diagonal. Let, for this purpose, \( \mathcal{I}_{a,b} = 0 \ \forall a \neq b \), then the determinant is given as

\[
|\mathcal{I}| = \prod_{i=1}^{p} \frac{1}{\sigma^2_{\hat{\theta}_i}},
\]  

(18)

where \( \sigma^2_{\hat{\theta}_i} = \mathcal{I}^{-1}(\hat{\theta}_i) \) is the CRLB. Since the logarithm is a monotonic function, maximizing \( |\mathcal{I}| \) is the same as maximizing \( \ln |\mathcal{I}| \), so

\[
\ln |\mathcal{I}| = \ln \left( \prod_{i=1}^{p} \frac{1}{\sigma^2_{\hat{\theta}_i}} \right) = - \sum_{i=1}^{p} \ln \left( \sigma^2_{\hat{\theta}_i} \right),
\]  

(19)

hereby implying that maximization of the FIM determinant is equivalent to minimizing the mean of the logarithmic of the parameter variances. For the trace, one gets

\[
\text{tr}(\mathcal{I}) = \sum_{i=1}^{p} \frac{1}{\sigma^2_{\hat{\theta}_i}} = \frac{\sum_{i=1}^{p} \left( \prod_{\sigma_{\theta}^2 \in S_i} \sigma_{\theta}^2 \right)}{\prod_{i=1}^{p} \sigma^2_{\hat{\theta}_i}},
\]  

(20)

and, analogously,

\[
\ln(\text{tr}(\mathcal{I})) = \ln \left( \sum_{i=1}^{p} \left( \prod_{\sigma_{\theta}^2 \in S_i} \sigma_{\theta}^2 \right) \right) - \ln \left( \prod_{i=1}^{p} \sigma^2_{\hat{\theta}_i} \right) = \ln \left( \sum_{i=1}^{p} \left( \prod_{\sigma_{\theta}^2 \in S_i} \sigma_{\theta}^2 \right) \right) - \sum_{i=1}^{p} \ln \left( \sigma^2_{\hat{\theta}_i} \right),
\]  

(21)

where \( S_1 = \{ \sigma_{\theta_2}^2 \ldots \sigma_{\theta_p}^2 \} \), \( S_2 = \{ \sigma_{\theta_1}^2 \sigma_{\theta_3}^2 \ldots \sigma_{\theta_p}^2 \} \), and so forth. Unlike the determinant metric, the trace one does not seem to have a simple interpretation.
In the context of FIM metric, we consider, as an example, a two-parameter situation where sensors in position A give \( \text{diag}(\Sigma) = [0.4, 0.6] \), and in position B give \( \text{diag}(\Sigma) = [0.2, 0.9] \). Between these two alternatives, one could argue that A is more attractive, since the largest variance is 0.6, but both the determinant and trace criterion would select B. Consequently, we propose employment of the max-min metric, in which the minimum diagonal term of the FIM is maximized; or, equivalently, such that the highest CRLB is minimized. Obviously, the three metrics provide the same result if \( \theta \) is a scalar, thus we will refer to cases where \( \theta = [\theta_1 \ldots \theta_p] \), with \( p \geq 2 \).

It is noted that whether A or B, in general terms, is preferable depends on the function that translates the error in the parameters to “consequences”. Defining the cost of the error in each parameter explicitly and replacing the variance of the estimates with the limits imposed by the Fisher information removes all ambiguity on the selection of “the best metric”, but we do not pursue this path in the present paper.

4 APPLICATION EXAMPLE

To contrast the results that are obtained with the max-min criterion and those from the determinant and the trace, we consider sensor placements for estimation of the first and third damping ratios, \( \zeta_1 \) and \( \zeta_3 \), of the 10-DOF chain system seen in Figure 1. Here, all springs have equal stiffness of \( k_i = 1000 \text{ N/m} \), all masses have magnitude of \( m_i = 1 \text{ kg} \), and classical damping is assumed such each mode has a damping ratio of \( \zeta = 5 \% \). In all analyses, it is assumed that a single input is applied at DOF 7. In the simulations, this input is chosen as a colored signal taken from an earthquake record.

4.1 Computing the correlation value

A suitable correlation value, \( \alpha \), is computed via the procedure described in Subsection 2.3 on the basis of mode/pole 1. The covariance matrices of the frequency response realizations at resonance and the pole are estimated from a Monte Carlo procedure composed of 500 simulations, with the earthquake excitation applied at DOF 7 and 5 \% white Gaussian noise added to the output. To estimate the pole itself, a subspace-based system identification technique, see, for example, [7], is used for the single-input/single-output case where the noise-contaminated displacements in DOF 7 are selected.

In Figure 2, the uncertainties of the frequency response at resonance and the pole are plotted. Clearly, both set of uncertainties seem to form a circular region around the true value, hence it is not unreasonable to assume that each of the two covariance matrices is diagonal with a constant value along the diagonal (implying equal variance for the real and imaginary part). This finding for the pole is in agreement with findings in a damping estimation study presented in [8].

With the computed covariance matrices, a suitable value for \( \alpha \) can be found directly from \( I_{1,1}(\alpha) = I^{(\lambda_1)} \). This is illustrated in Figure 3, where \( \alpha = 22 \) is selected for the coming analyses.

![Figure 1: Considered 10-DOF chain system.](image)
Figure 2: Uncertainties of 10-DOF system with earthquake excitation and 5 % white Gaussian noise added to the output, with true values indicated by black boxes. (a) $H_{7,7}(j\omega_1)$. (b) $\lambda_1$.

Figure 3: Selecting the correlation parameter, $\alpha$, from $I_{1,1}(\alpha) = I_{1,1}^{(\lambda_1)}$.

4.2 Selecting sensor placements

As mentioned in Section 3, the sensor placement formulation constitutes a discrete optimization problem, in which the discrete variables are the locations of sensors corresponding to the $n$ DOF. The solution space to this discretization problem with $m$ sensors is composed of $n!/(m!(n-m)!)$ possible configurations, thus an exhaustive search procedure is only feasible for small-scale problems. For more comprehensive structures, in terms of number of DOF, a more advanced optimization algorithm, which only treats a subset of the solution space, must be employed. However, due to the simple nature of the present application example, such advanced approach is beyond the scope of this paper and will therefore not be addressed further. For more information on the topic, the reader is referred to studies such as [4].

Based on the three different FIM metrics and the EI method, sensor placements for estimating $\zeta_1$ and $\zeta_3$ of the considered 10-DOF system are computed for $m = 1, \ldots, 10$. The results are presented in Figure 4, where it is seen how the determinant-based FIM approach and the EI method, in this particular application, provide the same sensor locations. The trace-based FIM approach, whose sensor placement results are plotted in Figure 4b, is predominantly governed by the Fisher information on $\zeta_1$, as $\mathcal{J}_{\zeta_1} >> \mathcal{J}_{\zeta_3}$ when using the receptance FRM. Therefore, the approach is, in this case, more or less blind to the nodal point of the third mode (located in the
vicinity of DOF 4). Contrary, it is seen in Figures 4a, 4c, and 4d how the remaining approaches all select DOF 4 as the last one to be populated with a sensor.

### 4.3 System identification simulation-based verification

The utility of the predicted sensor placements is verified through Monte Carlo simulations of the 10-DOF system with different sensor configurations. Here, we present results obtained when placing four sensors in three different configurations; namely, #133 which is the one selected from \( \max(\min(I_{a,a})) \), #210 which is based on \( \max(\text{tr}(I)) \), and #208 which is the configuration suggested by both \( \max|Z| \) and the EI method.

For the three different sensor placement configurations, \( I_{a,a}^{-1} \) is plotted in Figure 5. Evidently, the CRLBs on \( \hat{\zeta}_1 \) are relatively low (compared to those for \( \hat{\zeta}_3 \)) and quite similar for all configurations, whereas the CRLBs on \( \hat{\zeta}_3 \) vary with a factor of two from the best configuration, #133, to the worst, #210. From a theoretical point of view, this suggests that we should see distinct differences in the precision with which \( \zeta_3 \) is estimated in the system identification simulations, whereas the estimates of \( \zeta_1 \) should be, for all practical purposes, somewhat similar for the sensor configurations. It is, however, noted that the CRLB is merely a theoretical lower bound, thus the quantitative relative differences presented in Figure 5 should not necessarily be expected from simulations as well—only the qualitative ranking.

For each of the three configurations, 500 simulations are carried out with the earthquake excitation applied at DOF 7 to yield displacement output samples, which are corrupted with 5% white Gaussian noise to emulate real experimental findings. Subsequently, a subspace-
Figure 5: Normalized bound on the variance of the damping for the configurations selected “best” by the min-max (#133), trace (#210), and determinant (#208) criteria. (a) $\sigma^2_{\zeta_1}$. (b) $\sigma^2_{\zeta_3}$.

Figure 6: Probability distributions for modal parameter subset estimates. (a) $f(\hat{\zeta}_1)$. (b) $f(\hat{\zeta}_3)$.

Based system identification technique is used to identify estimates of $\zeta_1$ and $\zeta_3$. In Figure 6, the resulting probability distributions based on the different sensor configurations are presented for both damping ratios. Evidently, the configurations provide estimates of $\zeta_1$ that, for all practical purposes, are equally good. However, for $\zeta_3$, configuration #133—that is, the one predicted by max(min($\mathcal{I}_{a,a}$))—clearly yields the best estimate with a standard deviation of $\hat{\sigma}_{\zeta_3} = 0.0085$. The configuration predicted by max(tr($\mathcal{I}$)) provides $\hat{\sigma}_{\zeta_3} = 0.0525$, while the one based on max $|\mathcal{I}|$ and the EI method give $\hat{\sigma}_{\zeta_3} = 0.0148$. So, although the orders of magnitude by which the standard deviations vary are higher than the deviations obtained from the CRLBs, see Figure 5, the mutual performance ranking according to the CRLBs coincides explicitly with the simulation-based findings for both $\zeta_1$ and $\zeta_3$.

5 CONCLUSIONS

This paper deals with the task of placing a pre-determined number of sensors in configurations that are optimal in the restricted sense that the maximum CRLB—estimated using a formulation based on the sensitivity of the FRFs—is minimized. In the development, the correlation at nearby frequencies is considered showing, as one intuitively expects, that the information is asymptotic as the sampling of the frequency response decreases.
It is recognized that maximizing a metric of the FIM is a simplification of the true problem, since the actual issue is to minimize the cost of the deviations in the estimated parameters from their true values; and the Fisher information only provides a lower bound for the covariance of the probability distribution of the parameters. Although it is not possible, in a strict sense, to claim that one metric of the FIM is better than any other (since any definition could outperform the other for some definition of the cost of errors), it is nonetheless reasonable to operate on the premise that minimizing the largest variance is a good “practical target”. This is evidenced in an application example with a 10-DOF system subjected to a single input. Here, the performance, in the context of estimating the first and third damping ratios ($\zeta_1$ and $\zeta_3$), of the max-min criterion is compared to those of the other Fisher information-based approaches and the EI method. For $\zeta_1$, the different approaches yield sensor configurations providing CRLBs implying that one should expect to obtain estimates that, for all practical purposes, are equally precise. This theoretical finding is confirmed through Monte Carlo simulations in which the two damping ratios have been estimated. Regarding $\zeta_3$, it is found theoretically, and subsequently verified by the aforementioned Monte Carlo simulations, that the proposed max-min approach yields a sensor configuration that clearly outperforms the other. In fact, the second best of the examined sensor configurations, namely, the one found from the determinant-based FIM approach and the EI method, provides a simulated standard deviation of $\zeta_3$ that is approximately 74% higher than what is found when using the sensor configuration predicted by the max-min approach.

REFERENCES


