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Damage localization using the Modal Interpolation Method

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Abstract

The Interpolation Damage Detection Method (IDDM) has been extensively applied for damage localization purposes to both beam like and plate like structures. The method is based on the detection of localized reductions of smoothness in the Operational Shapes of the structure. If the structural responses are strongly corrupted by noise, the estimation of the OS may be inaccurate leading to false and/or missing alarms when the IDDM is applied. At resonance the effect of noise is usually lower with respect to other frequency values hence the relevant ODS are estimated with higher reliability. In order to overcome the drawbacks related to noise in recorded signals a modification of the method based on the use of modal shapes instead of Operational Shapes is herein investigated. In the paper the comparison between the results of the IDDM in its actual version and in the modal version is reported for a numerical example previously studied in literature. Several levels of noise in structural responses are considered in order to discuss the influence of this parameter on the results.

1 INTRODUCTION

Several methods have been proposed in literature to localize damage based on structural response to vibrations [1]-[2]. A number of methods rely on modal parameters, other on operational shapes retrieved from Frequency Response Functions. Both families of methods present advantages and drawbacks.

Modal parameters are easier to interpret thank to their clear physical meaning and their estimation is today quite reliable thanks to the development of the experimental modal analysis technique. Modal shapes can be identified with an higher level of uncertainty with respect to modal frequencies but may give information about the location of damage.

The principal aim of most methods of damage localization base don modal or operational shapes, is to find an irregularity (presumably induced by damage) in the deformed shapes. The Interpolation Damage Detection Method (IDDM) detects discontinuities of curvature exploiting the Gibbs phenomenon for splines. The method, that has been extensively applied for damage localization purposes to both beam like and plate like structures [3]-[5] is based on the detection of localized reductions of smoothness in the Operational Shapes (OSs) of the structure. A damage feature is defined in terms of the interpolation error related to the use of a spline function in modeling the OSs retrieved form Frequency Response Functions or from Transmissibility Functions: significant variations of the interpolation error between two successive inspections of the structure indicate the onset of damage.

The IDDM can be applied to any type of structure provided the OSs are estimated accurately

in the original and in the damaged configurations. If the latter circumstance fail to occur, for example if the structural responses are strongly corrupted by noise, both false and missing alarms appear when the IDDM is applied to localize a concentrated damage.
In order to overcome these drawbacks a modification of the method based on the use of modal shapes instead of operational shapes (OS) is herein investigated. An OS is the deformed shape of a structure subjected to a harmonic excitation: at resonances the OSs are dominated by the relevant mode shapes. At resonance the effect of noise is usually lower with respect to other frequency values hence the relevant OSs are estimated with higher reliability using one of the several methods have been proposed to reliably estimate modal shapes in case of both known and unknown input. Herein, in order to reduce or eliminate the drawbacks related to the inaccurate estimation of the operational shapes, is investigated a modified version of the IDDM based on the use of modal shapes only. The modified version is based on a damage feature calculated considering the interpolation error relevant only to the modal shapes and not to all the operational shapes in the significant frequency range. Using a numerical example previously used in literature, herein will be reported the comparison between the results of the IDDM in its actual version (with the interpolation error calculated summing up the contributions of all the operational shapes) and in the new proposed version (with the estimation of the interpolation error limited to the modal shapes). Several levels of noise in structural responses will be considered in order to discuss the influence of this parameter on the results given by the two different versions of the method.

2 THE MODAL INTERPOLATION METHOD

Localized damage in a beam can induce a reduction of the flexural stiffness hence a localized increase of curvature in the deformed shapes of the beam. This variation of the curvature reduces the smoothness of the shape and, if a smooth function is used to interpolate the deformed shape both in the undamaged and in the damaged configurations, an increase of the interpolation error occurs at the damaged location. This phenomenon is particularly important for cubic spline interpolating functions due to the so-called Gibbs phenomenon for splines that occurs when the spline interpolates functions with a discontinuity in the second derivative. The spline interpolation oscillates near the point of discontinuity where it has an overshoot and this causes a sharp increase of the interpolation error at the discontinuity section. This phenomenon can be used as a sort of ‘curvature discontinuity detector’ and is at the base of the Interpolation Damage Detection Method. In the original version of the IDDM the spline interpolation is applied to Operational Shapes (OSs) and the damage feature at each instrumented location is defined as the sum of the interpolation errors all over the frequency range of interest. This formulation is effective if the OSs can be reliably estimated from recorded responses or from Frequency Response Functions (FRFs) or from Transmissibility Functions in case of unknown input. This is usually the case if measurement noise in recorded responses is not too high.
At resonance the effect of noise is usually lower with respect to other frequency values hence the modal shapes are usually estimated with a higher reliability with respect to operational shapes at other frequencies.
More details on the IDDM can be found in reference [4]. Herein the formulation of the damage localization method in terms of modal shapes is reported. Assume that the beam is instrumented with n_s sensors allowing to identify n_s components of N modal shapes.
The spline interpolation $\hat{\varphi}_l^{(i)}$ of the i-th modal shape at the l-th location z_l (l=1,…n_s) can be

calculated using the following relationship:

$$\hat{\varphi}_l^{(i)} = \sum_{j=0}^{3} c_{j,l}^{(i)} \left(z_l - z_{l-1} \right)^j \tag{1}$$

Where the coefficients $c_{j,l}^{(i)}$ are calculated from the n_s-1 values of the modal shape $\varphi^{(i)}$ at locations z_k $(k \neq l)$ imposing continuity of the spline function and of its first and second derivative at all the n_s locations:

$$c_{j,l}^{(i)} = g\left(\varphi_k^{(i)}\right) \qquad k \neq l \tag{2}$$

The interpolation error of the i-th mode at location z_l (Figure 1) is defined as the magnitude of the difference between the real and the interpolated l-th components of the i-th modal shape:

$$E_i\left(z_l\right) = \left|\varphi_l^{(i)} - \hat{\varphi}_l^{(i)}\right| \tag{3}$$

If N modes are identified for the considered beam, the total interpolation error at location z_l is given by the norm:

$$E\left(z_l\right) = \sqrt{\sum_{i=1}^{N} E^2\left(z_l\right)} = \sqrt{\sum_{i=1}^{N} \left|\varphi_l^{(i)} - \hat{\varphi}_l^{(i)}\right|^2} \tag{4}$$

The values of $E\left(z_l\right)$ at all the locations z_l of the structure where sensors are available (that is at all locations where the components of the modal shapes are identified), characterize the current status of the structure that is are the ‘signature’ of the beam in a given configuration.
A variation of this function at a certain location z_l, reflects a change in the structural condition (variation of curvature) : the higher the change, the higher the variation of the error $E\left(z_l\right)$.

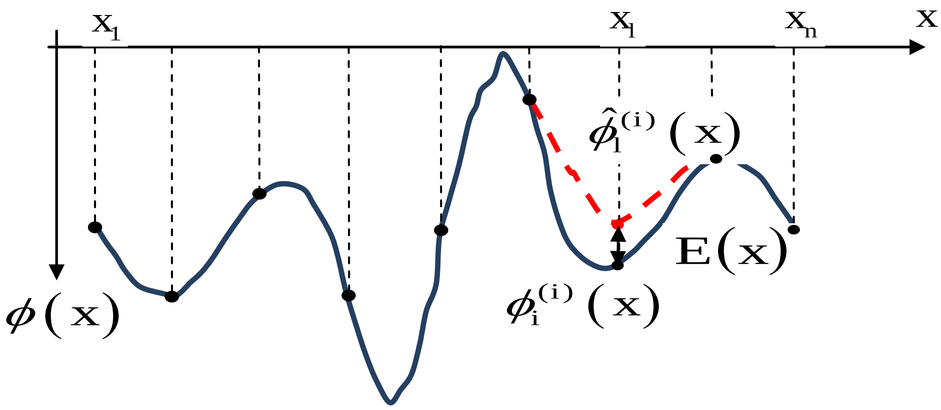

Figure 1: The cubic spline interpolation error for the i-th modal shape

The interpolation error is thus assumed as a damage parameter and its variations between an inspection (I) phase and a reference (R) phase allow to estimate the damage feature at each instrumented location z_l :

$$\Delta E\left(z_l\right) = E_I\left(z_l\right) - E_R\left(z_l\right) \tag{5}$$

In real world conditions, due to several sources of variability influencing recorded responses, the modal shapes, hence the damage parameter $E\left(z_l\right)$ can change even if no damage occurs

or, viceversa, it can exhibit no changes when damage exist, leading to false or missing detection of damage.
For short term monitoring, that is if only data retrieved from one test in the undamaged and one test in the inspection phases are available, a statistical analysis of the damage feature is not possible. In this case a threshold can be defined in terms of a given percentile of the distribution of the damage feature at all the instrumented locations. Namely the threshold E_T can be defined as:

$$\Delta E_T = \mu_{\Delta E} + \nu \cdot \sigma_{\Delta E} \qquad (6)$$

being μ_E and σ_E mean and standard deviation of the distribution in the inspection phase and ν the value $Z_{1-\alpha}$ of the standard Normal distribution, corresponding to the percentile α. This corresponds to asume as damaged locations the ones where the variations of the interpolation error are among the highest of all the instrumented locations.
The classification of a certain z_l as a "damaged location" is thus carried out basing on the comparison of the current value of the damage feature $\Delta E(z_l)$ with the threshold E_T:

if $\Delta E(z_l) - \Delta E_T > 0$ damage at z_l

if $\Delta E(z_l) - \Delta E_T < 0$ no damage at z_l

3 NUMERICAL APPLICATION

Without loss of generality, the structure described by Choi and Stubbs (see Ref.[6]) has been considered to verify the proposed method. Figure 2 reports the numerical model considered for the application. The system properties and the considered damage locations and intensities are detailed in reference [6] and are reported herein for the sake of completeness.

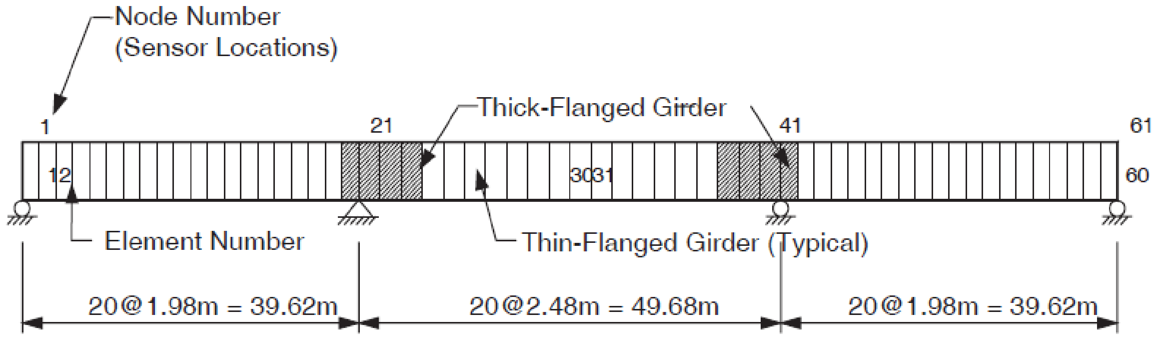

Figure 2: Numerical model of the beam (from ref. [6])

According to the approach followed in reference [6], responses in terms of absolute acceleration have been assumed to be recorded at the all 61 nodes of the finite element model. In a real world application such a dense distribution of sensor is not usually deployed. However, since in this paper the aim is to describe the modal interpolation method and to compare results to those obtained with the original version of the method (IDDM), the impact of a more sparse distribution of sensors has not been specifically addressed even if a lower resolution in damage detection should be expected in that case.
In order to simulate response to ambient vibrations, the numerical model of the beam was subjected to white-noise random excitations of low intensity at its supports. Responses have been assumed to be sampled at 100Hz, hence the Nyquist frequency is of 50Hz. The interpolation error has been calculated in the frequency range [0-50Hz] for the IDDM and

considering all the modes up to a frequency of 50Hz for the application of the MIM. Damage is simulated as a reduction of the elastic modulus of one or more elements of the model. The same damage scenarios considered in reference [6] are considered. The comparison between the IDDM and the MIM will be carried out assuming that the excitation is known and that Operational Shapes are calculated from the Frequency Response Functions (FRFs). For the application of the MIM the modal shapes have been retrieved from the numerical model. In order to simulate real world conditions, both the responses for the estimation of the FRFs and the modal shapes have been polluted with uncorrelated white noise.

Table 1. Simulated damage scenarios

Damage scenario	Elements damaged			Severity %		
1	5			6		
2	32			16		
3	54			9		
4	20			3		
5	56			7		
6	15	10		7	9	
7	12	37		16	7	
8	7	21		3	5	
9	26	31		8	4	
10	3	42		4	2	
11	33	11	47	3	5	12
12	42	34	8	7	3	4
13	50	15	4	3	12	5
14	53	9	16	7	8	13
15	6	35	40	2	5	8

Noise is added to the responses of the numerical model in the form proposed in reference [7]:

$$y_i(t) = y_i^0(t) + \beta_R \lambda \left| y_i^0(t) \right|_{max}$$

where $y_i(t)$ and $y_i^0(t)$ are the noisy and non-noisy responses of the i-th sensor at time t and λ is the random parameter with a Gaussian Normal distribution. The parameter β_R fixes the level of noise.

Noise is introduced in modal shapes using a similar model:

$$\phi_i = \phi_i^0 + \beta_M \lambda \phi_i^0$$

where ϕ_i and ϕ_i^0 are respectively the noisy and unpolluted i-th component of the modal shape and β_M is the parameter that fixes the level of noise.

Following the approach proposed in [7], for the values of the noise parameters it has been assumed $\beta_R = 5\beta_M$ considering three different levels of noise corresponding to $\beta_M = 1\%,\ 2\%$ and 3%.

4 DISCUSSION OF RESULTS

A selection of results is reported in Figure 3 to Figure 5; similar patterns have been found for all the other scenarios but results are not reported here due to space limitations. The values of the difference $\Delta E(z_i) - \Delta E_T > 0$ are reported for damage to respectively one, two and three elements. The value of the threshold in equation (6) has been calculated assuming ν=1.64 corresponding to a percentile of 5% (that is 5% of the values of the damage parameter exceed the threshold). In the figures the location of damage is indicated by the title: D_12_37 indicates a reduction of the elastic modulus at the elements number 12 and 37.

Results of the application of the IDDM are reported in Figure 3. For 5% noise even very small damages are correctly detected in most cases: for example a stiffness reduction of 7% at location 37 in scenario D_12_37, or damages of 7% and 8% at locations 53 and 9 in scenario D_53_09_16. At the increase of noise from 5% to 10% appear a number of false alarms (see for example at locations 32 and 50 for scenario d12_37). Also missing alarms occur in scenarios not reported herein. At the further increase of noise from 10% to 15% more false and missing alarms appear (location 28 in D12_37 and location 4 and 29 in scenario D_53_09_16). Results of the application of the MIM are reported in Figure 4 for the same damage scenarios. The damage parameter in equation (4) is calculated considering the first 32 modes that fall in the frequency range [0-50Hz]. Also in this case, as already seen for the results of the IDDM, at the increase of the level of noise from 1% to 2% to 3% an increasing number of both missing (location 37 in scenario D32) and false (several locations in scenarios D12_37 and D53_09_16) alarms appear. As a general result, the reliability of both the IDDM and the MIM increases with the severity of damage and decreases with the level of noise. Location 32 with a 16% reduction of stiffness in scenario D32 is always correctly identified as well as location 12 with a 16% reduction of stiffness in scenario D12_37 and location 16 in scenario D53_09_16 with a 13% of stiffness reduction. Locations with lower damages may be missed for higher levels of noise both by the IDDM and by the MIM. The comparison between results of the IDDM and the MIM shows that the latter could be less reliable for higher levels of noise, particularly for multiple damage scenarios. This is probably due to the fact that the MIM is applied in a given frequency range taken into account only a limited number of shapes, the modal ones. In the application of the IDDM, the detection of the damaged locations is carried out considering all the OSs in a given frequency range. This allows in the IDDM to reduce the global effect on the damage feature of noise that randomly affects the deformed shapes at different frequency values. If a larger number of modes is considered, the same effect leads to an improvement of the results obtained with the MIM (see Figure 5): considering 45 modes and the higher level of noise (that is 3%), in all cases damage is correctly detected and no false or missing alarms occur. If the IDDM is applied considering the entire range of frequencies corresponding to the 45 modes there is not a sensible improvement in the identification of the damaged locations. This is probably due to the fact that operational shapes far from resonance are more affected by noise. The highest modes are scarcely excited by the considered input hence enlarging the frequency range for the computation of the damage parameter does not improve damage localizations. It must be underlined that actually in this case also the identification of the modal shapes in the same frequency range would be affected by a higher uncertainty. This means that better results could be obtained by the MIM provided modal shapes can be identified basing on tests able to excite all the modes in the frequency range thus allowing a reliable identification of the relevant modal shapes.

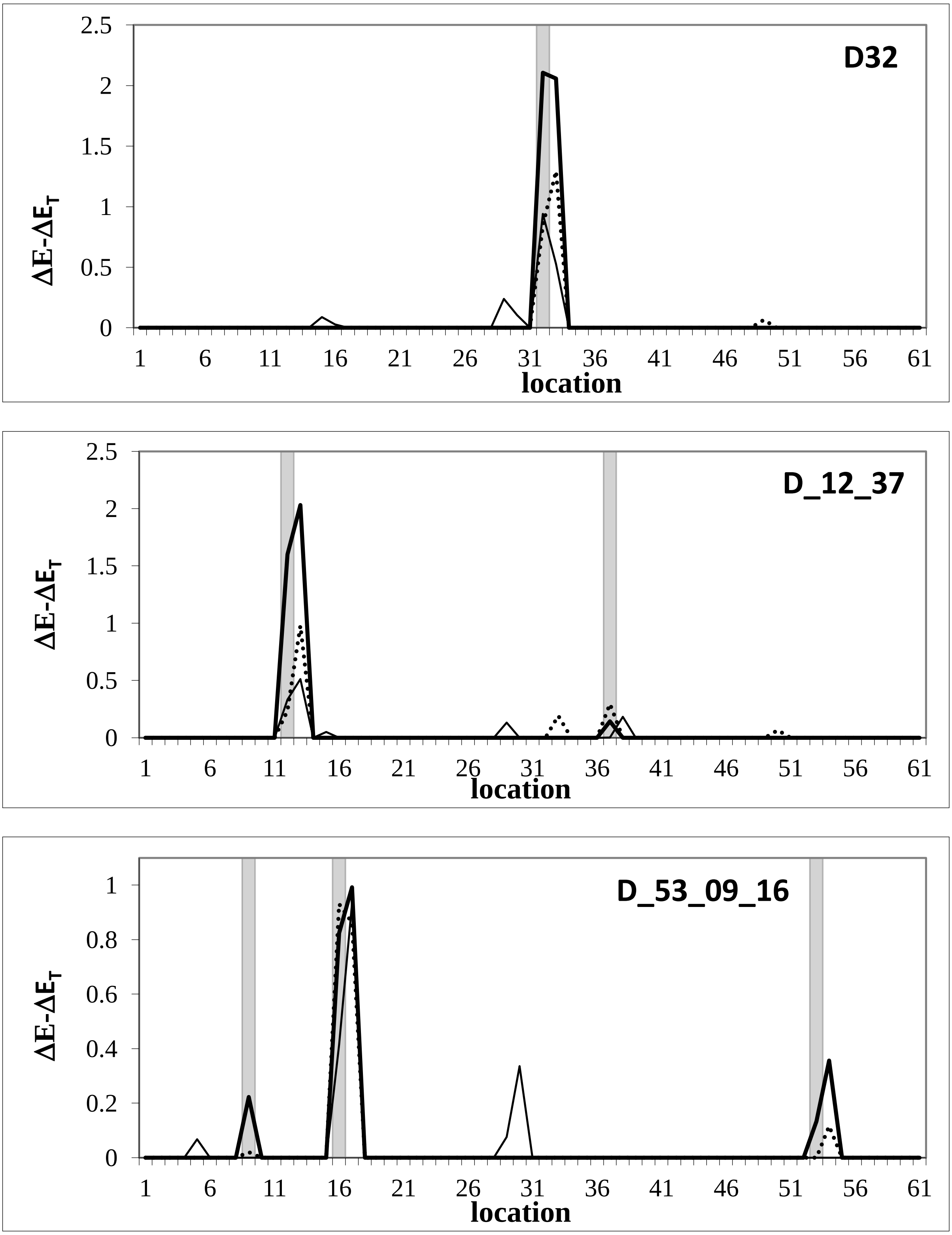

Figure 3: Results of the application of the IDDM : noise 5%; noise 10%; noise 15% damage location

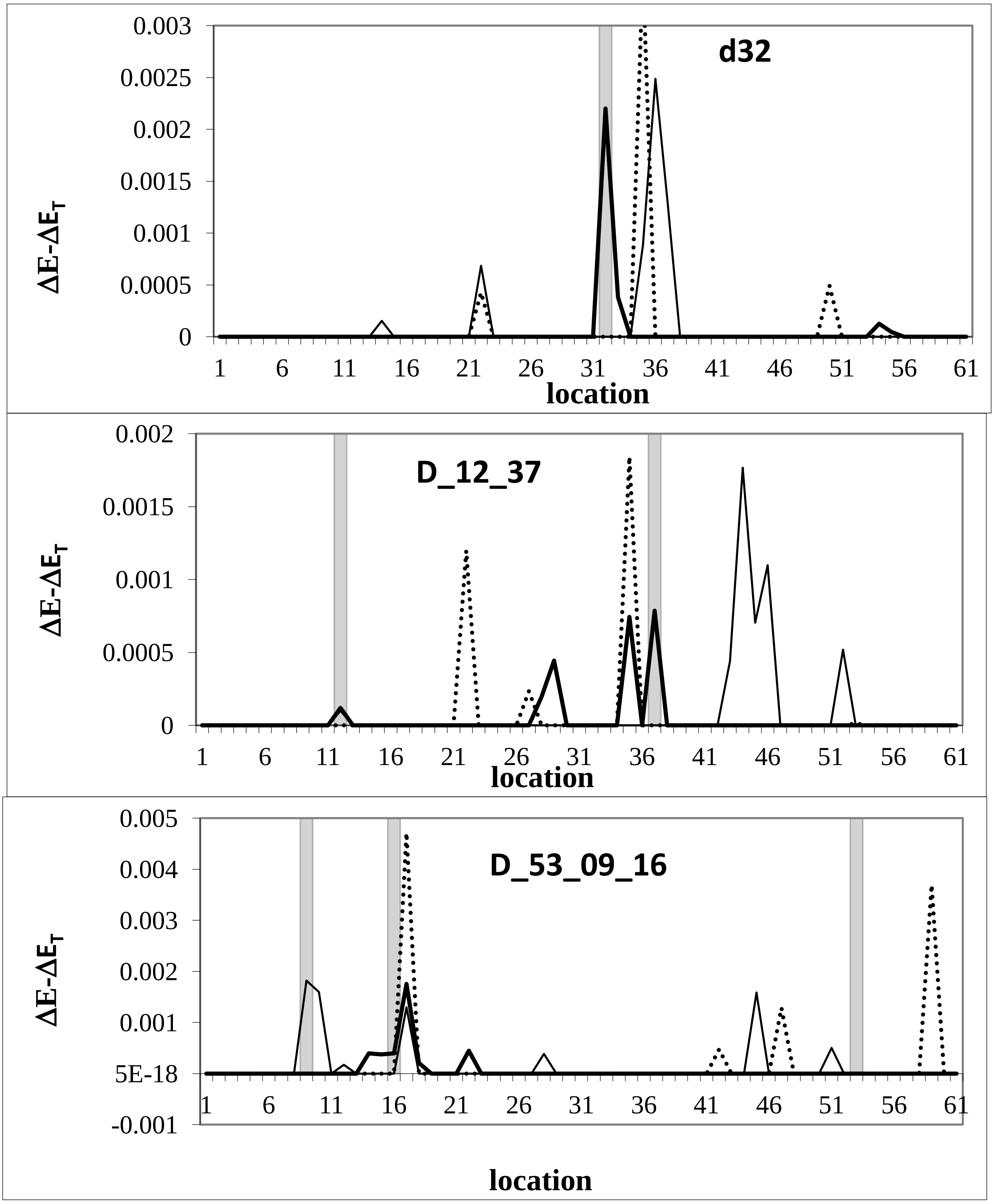

Figure 4: Results of the application of the MIM : noise 1%; noise 2% ; noise 3% ; damage location

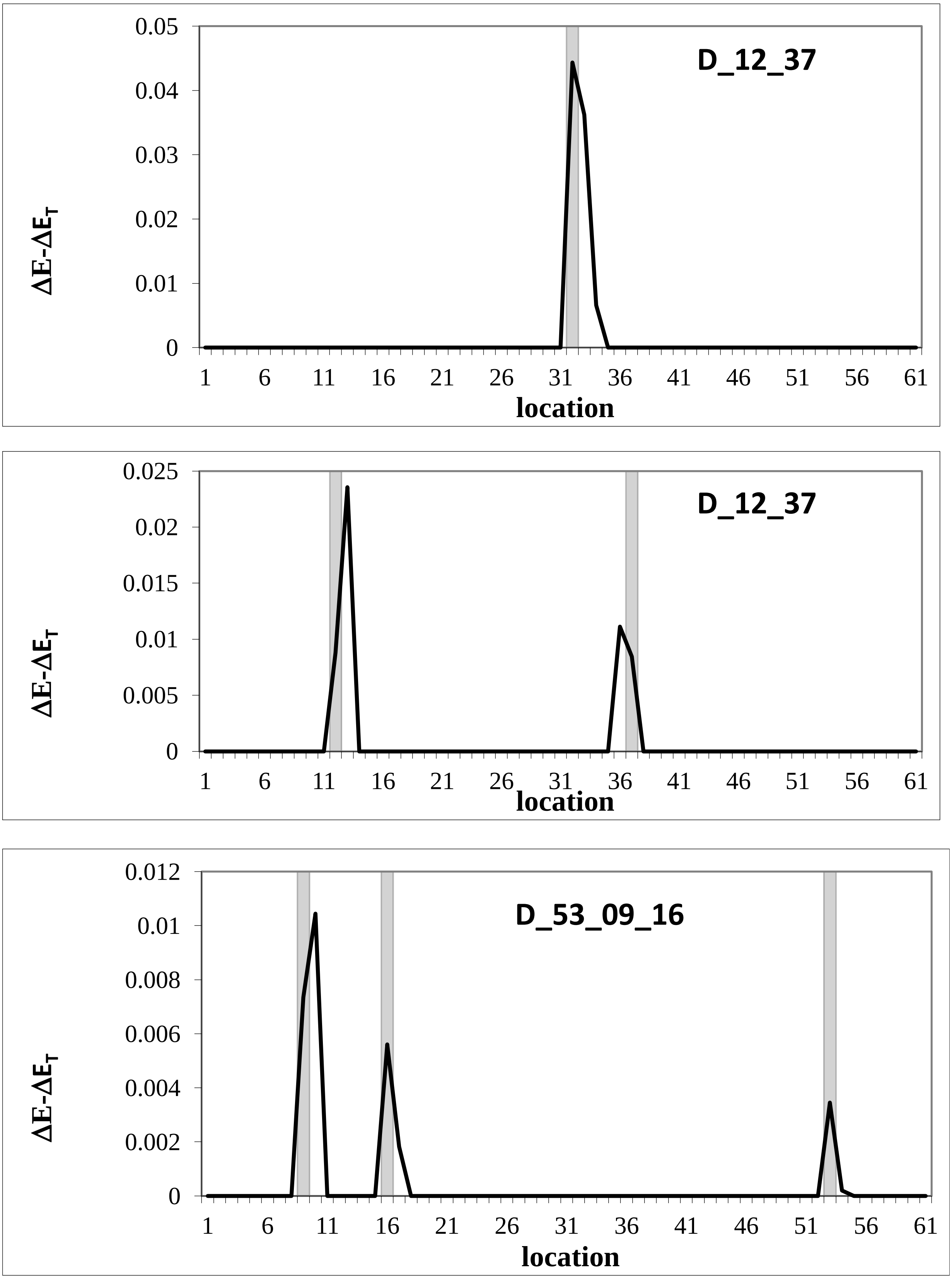

Figure 5: Results of the application of the MIM using a total number of 45 modes. Noise 3%

5 CONCLUSIONS

In this paper the Modal Interpolation Method for localization of damage is introduced and compared with the Interpolation Damage Detection Method. The two methods differ for the definition of the damage feature that in the IDDM is calculated basing on the operational shapes in a given frequency range while in the MIM is calculated basing only on the modal shapes in the same frequency range. The two methods have been applied to a numerical example previously used in literature, considering several levels of noise in responses. Results show that the reliability of both methods decreases with noise and increases with the damage severity. The comparison between the results retrieved by the application of the two methods indicates that the IDDM is more effective for higher levels of noise due to the averaging of the random errors on the total set of operational shapes. In the MIM an increase in the total number of modes considered for the evaluation of the damage parameter leads to an improvement of the reliability of results. A corresponding increase of the frequency range for the computation of the damage parameter in the IDDM does not lead to the same improvement of results probably due to the higher effect of noise on operational shapes at high frequencies, less excited by the considered input.

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