

## FEEDBACK RESONANCE FREQUENCY AS AN SHM INDICATOR (THE LARSEN EFFECT)

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### Abstract

*The most efficient way of building up energy in an oscillating system is to do it in phase with a resonant frequency of the system. If the resonant frequency changes for any reason this is difficult to obtain with a static frequency generator. It is of course possible to sweep over a frequency interval to find the maximum repeatedly, but a frequency sweep takes time and it is performed at discrete frequencies. A better approach is to set the device under test in feedback resonance. This will guarantee that the frequency is always a peak in the spectrum and the adaptation to change is immediate and continuous in every aspect. A continuous observation of feedback frequency can conceivably serve as an SHM indicator. Experiments with geophones as actuators are performed.*

*A 9-day test revealed smooth frequency variations in the 236.901 to 237.353 Hz interval. These smooth variations are believed to be caused by thermal and humidity changes in the laboratory. An audible tension release in the test device during this period was clearly indicated by a momentary step in the resonant frequency.*

*A second experiment revealed that as the feedback gain was adjusted in steps, the feedback frequency followed suit. This establishes indirectly the relation between deformation and resonant frequency. It is thus possible to determine non-linearity with the controlled feedback resonance method.*

### 1 INTRODUCTION

Structural damage can be detected by changes in natural frequency according to [1]. This parameter is however, due to ambient conditions, not a constant. That makes it difficult to precisely excite the structure with an oscillator with a static frequency. The most effective way of exciting the structure should be by feedback resonance, which is suggested by [2,3], see Figure 1. The feedback is generated by a pick-up transducer which signal is amplified until feedback (howling) occurs. Feedback is also known as the Larsen Effect. This phenomenon automatically adjusts the frequency to parameter changes in the device under test (DUT) and it is consequently the most efficient way of exciting vibrations in large structures with very small actuators / transducers. Since the resonance is continuous it will produce a standing wave pattern in the DUT.

Since variations in the parameters of the structure are reflected by the resonant frequency, this frequency is in itself an indicator of the state of the structure.



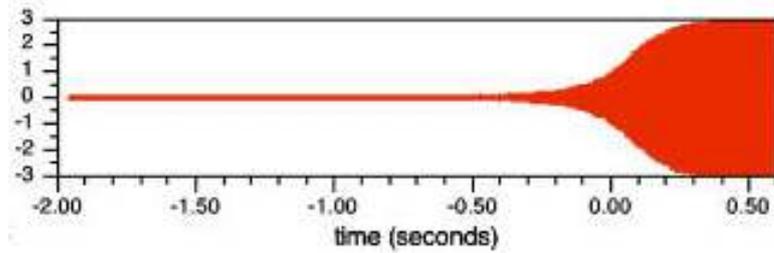


Figure 1: Feedback resonance amplitude build-up. (Reproduced from WEAVER, R., L. and LOBKIS, O.,I., "On the Linewidth of the ultrasonic Larsen effect in a reverberant body", J. Acoust. Soc. Am **120** (1), July 2006, pp 102-108 with the permission of the Acoustical Society of America.)

## 2 EXPERIMENTAL SETUP

Most experiments with feedback resonance have been performed in small test specimens. This experiment is, at least, in mesoscale in that it is performed on a 2 x 8 x 0.08 m concrete plate supported by four interactive glulam (timber) beams, thus approaching the scale of e.g. a bridge deck.

Previously we have investigated different voice-coil and piezoceramic transducers regarding their efficiency as reciprocal elements in an SHM network [4]. The result was that below about 3 kHz the voice coil transducer of the geophone type is superior to the piezoelectric transducers and some other voice coil transducers. We found a transfer function in a geophone to geophone system according to Figure 2, with a totally unexpected peak at around 3 kHz. The geophones have a nominal resonance frequency of 14 Hz. It is rather uniform in output between 500 and 2500 Hz.

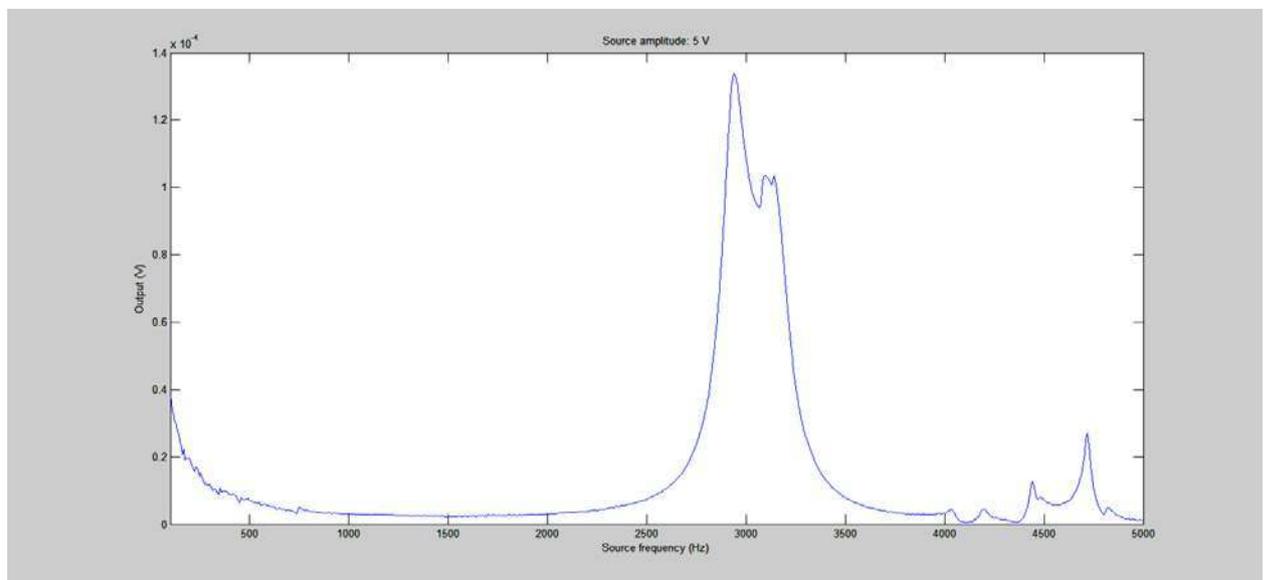


Figure 2: Geophone-geophone response through a heavy concrete block

One geophone was installed at a corner of the 8 x 2 m plate and another one in the opposite diagonal corner. These locations were chosen because they were assumed to be the most excitable points on the specimen. One geophone acted as transmitter, the other as receiver. Following the circuit

from the receiver geophone towards the transmitter geophone we have used the following components to drive the feedback:

1. A general 10x preamplifier with input from the receiving geophone
2. A Krohn-Hite band-pass filter to select a suitable frequency band, adding 20 dB gain
3. A specially built electronic circuit converting the filter AC output to a DC level.
4. An SRS SIM 960 analog PID controller comparing the DC level to the SETPOINT
5. A specially built amplifier using the SIM 960 controller DC output to set the AC gain
6. A bridge connecting the K-H filter output to the amplifier controlled by the SIM 960
7. A 50W battery operated 12V car amplifier
8. An output transformer intended for tube-amplifier impedance conversion operated backwards in order to feed a couple of hundred volts to the transmitting geophone which has an impedance of 3500 Ohms at DC.

The amplification of stage 5 is controlled by the SETPOINT of the controller. If the level is too low the gain is increased and if it is too high the gain is decreased. The controller operates with sampling of the signal at 100 kSa/s. Only gain was used, no integration or derivation terms. The gain was set at 50, producing a stable feedback after 5-10 oscillations and at a level determined by the SETPOINT value. These components constitutes what is similar to a Robinson oscillator. Between stage 6 and 7 an Agilent 53230A counter measured the frequency, using a gate-time of 60s, zero-crossing, producing 12 decimals of frequency data averaged for the gate time.

Before the experiments started a transfer function measurement between the geophones was made, with the result presented in Figure 3.

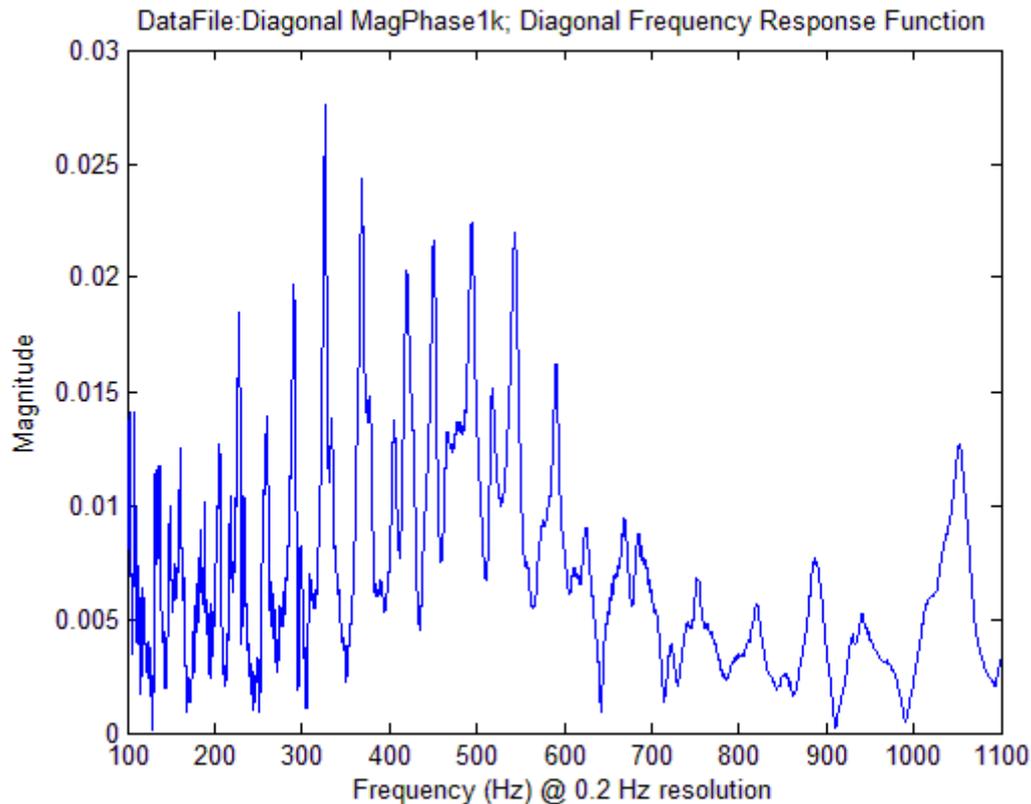


Figure 3: Transfer function diagonally across the 8 x 2 x 0.08 m concrete plate.

We select the peak at about 230 Hz for this experiment because it is least challenged by other strong nearby peaks. We therefore set the Krohn-Hite filter band-pass to [ 50,250 ]. When the filter was accidentally switched to all-pass after the experiment a new stable feedback resonance frequency was established at 309 Hz, the major peak in the transfer function.

### 3 LONG PERIOD RESULTS

The Larsen-effect appeared within a few seconds with no need to introduce any tapping to the DUT or injecting an electronic start signal. We assume that the ambient noise field contain enough amplitude to start the growth of the feedback. The Agilent counter was programmed to take a maximum of 20000 readings and storing them in internal memory, for subsequent transmission to a USB-stick. Two combined temperature and humidity loggers were placed on the test specimen.

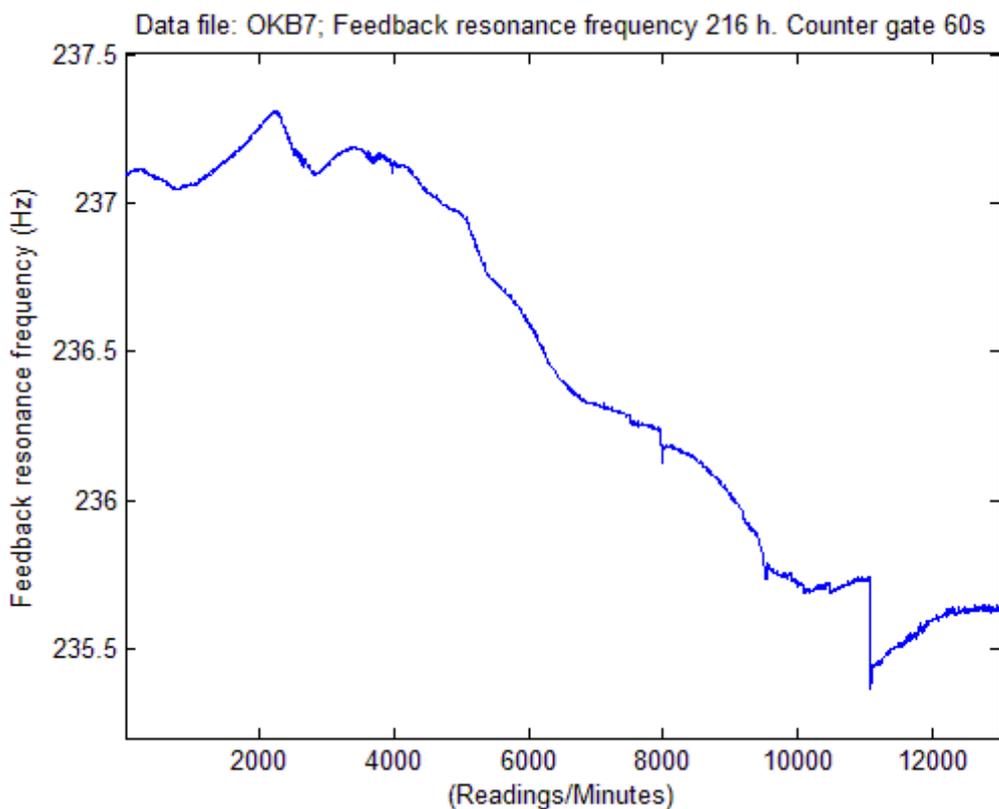


Figure 4: Feedback frequency variation is within 2Hz throughout a 216 hour period.

The resulting variation in feedback resonance frequency is presented in Figure 4. The curve is generally smooth with a couple of transient events. Similar events are reported in [3]. The event at about 11000 minutes was accompanied by an observed loud acoustic emission. It is believed to be a strain release between the concrete and the glulam (timber) beams supporting the plate.

It is interesting to see that the resonance frequency changes with time and that consequently a fixed frequency would have failed to input as much energy to the DUT as is made using the feedback resonance procedure. This may have implications in attempts to measure non-linearity.

Another interesting observation is that the vibration of the large plate sets up a mode pattern in the air in the laboratory where the experiment is performed. It is clear that there are nodes and

anti-nodes also in the air. This means, *of course*, that changes of the acoustic properties of the room itself could influence the feedback resonance frequency of the plate.

The variation in feedback resonance frequency can be used as a measure of degradation of the construction [1], once the ambient conditions can be controlled.

A magnification of a small part of the data is presented in Figure 5, demonstrating that the short-time variations are in the mHz region, as measured with the 60 s gate time. The short time variations were also observed in [2] which gave no explanation. We can only speculate that it may be an instability generated by competing overlapping modes of vibration.

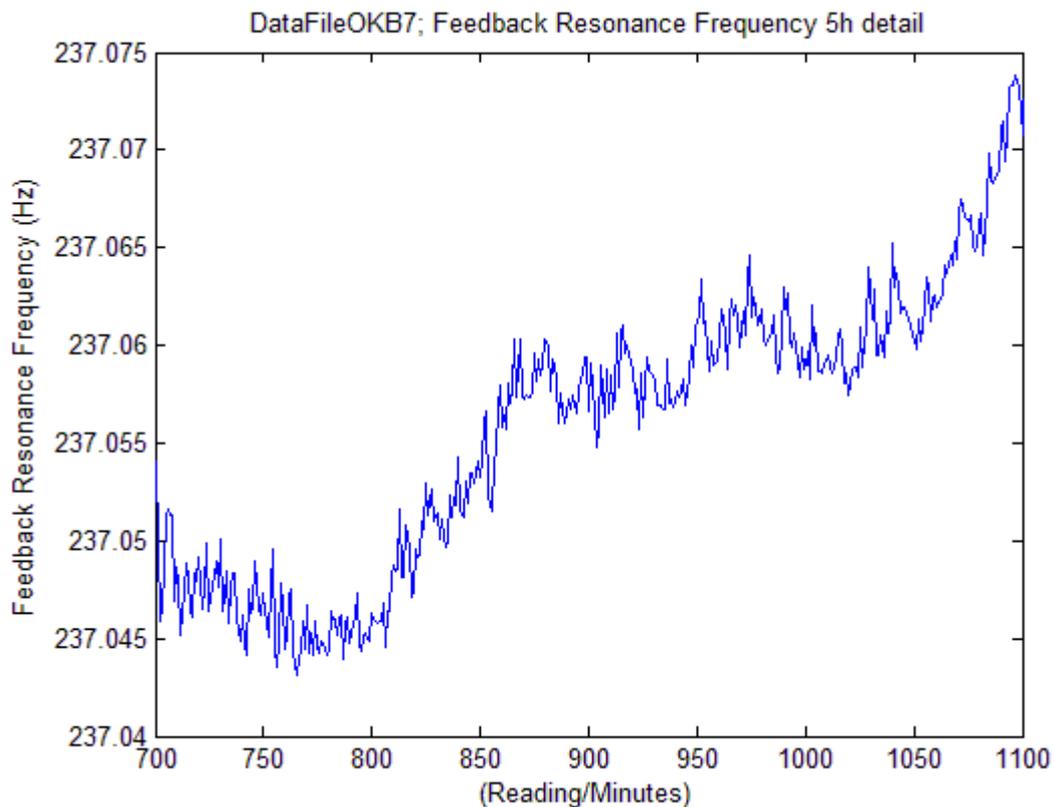


Figure 5: Zoom-in on a 5 hour interval demonstrating the local accuracy

It is interesting to note that after the observed acoustic event at about reading 11000 in Figure 4 there seems to be a slow dynamics recovery of the feedback resonance frequency. The event is magnified in Figure 6. According to [3] the recovery should follow a  $\log(t)$  development, at least for the later parts, which seems to be confirmed. Remembering that the samples are spaced 1 minute apart we still identify a much faster recovery at the “early” times after the event. We speculate that this could be a rebound effect of an overshoot in connection with the strain release between the concrete plate and the glulam beams related to the acoustic event. In [3] it is observed that transients always produce a negative change in the resonant frequency, which is also confirmed by this experiment.

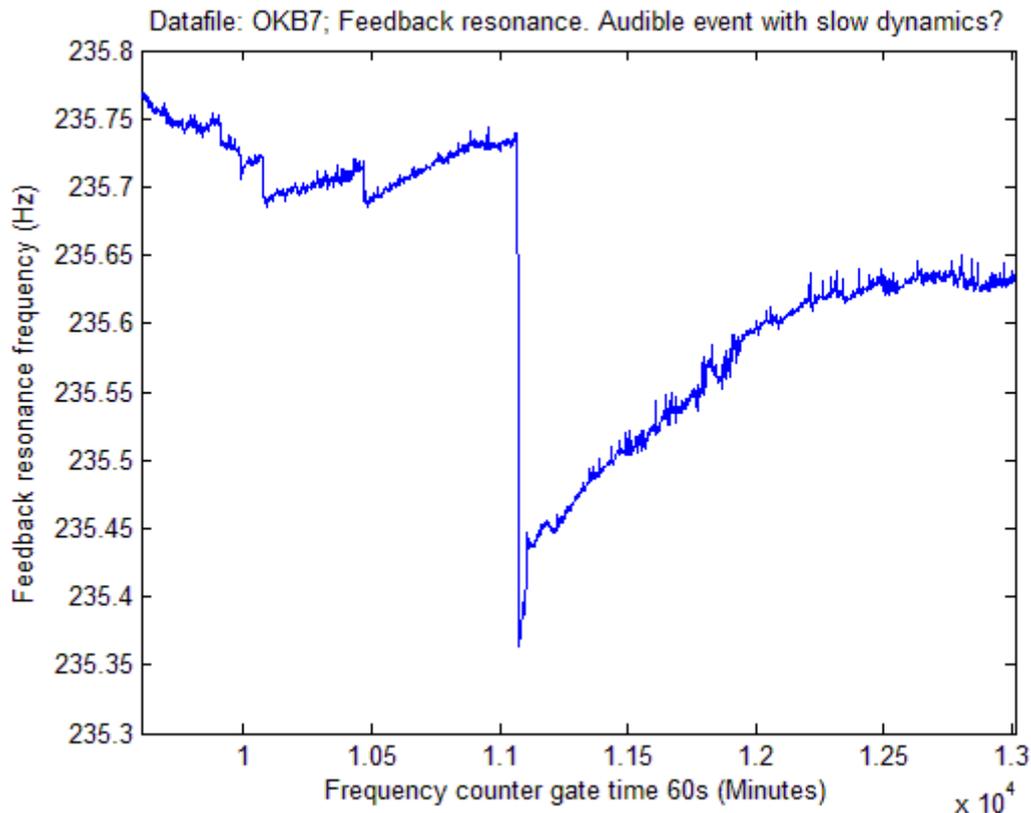


Figure 6: A transient accompanied by acoustic emission and a slow dynamics recovery which approximately is proportional to  $\log(t)$ , except in the early stage after the transient occurs.

In order to use the feedback resonance phenomenon for structural monitoring it is necessary to know what the influence of environmental factors is.

In our experiment we have been able to compare frequencies to temperature and humidity measured as rh in %, Figure 7. Since the parameters have very different values they are plotted as normalized values, each confined to the range 0-1. The peaks in the temperature curve are spaced about 24 hours indicating a relation to the sunlight illuminating the specimen. The frequency follows suit with the humidity to reading 7000, but then the relation is broken.

In Figure 8 the relationship between frequency and temperature is presented. The jagged appearance of the curve is due to the low resolution of the temperature values – 0.1 degrees. The general appearance of the relation suggests an increase in resonance frequency with temperature, which is in opposition to the findings in [3], but there are several peculiarities, a few related to the transient events. The temperature variations are caused by the sun illuminating the laboratory through a large glass window at one end of the specimen. This may produce uneven temperature distributions in the specimen as the sun illuminates the entire specimen at about 9 in the morning and then successively illuminates less and less for leaving the specimen in shadow after noon. Another source of temperature variations is of course the heating system in the building.

Similarly the relation between frequency and humidity is presented in Figure 9. This curve is smoother but it still has a very provoking appearance, regarding the possibilities to establish a relationship between the parameters.

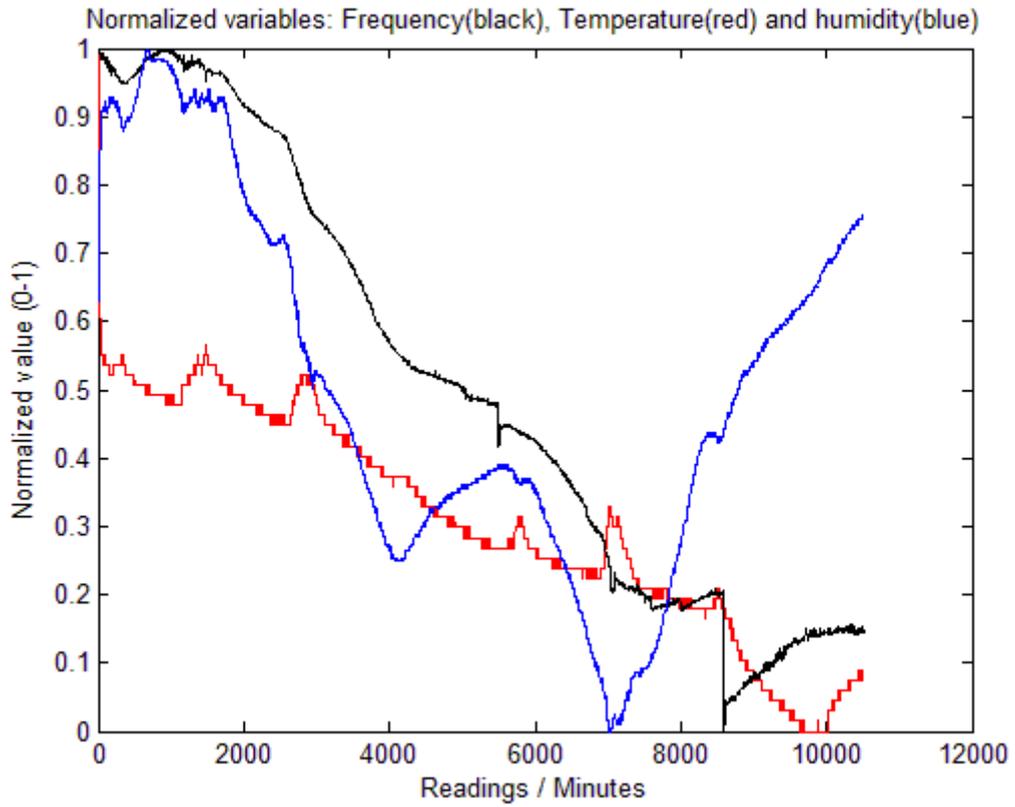


Figure 7: Normalized plots of frequency (black), temperature (red) and humidity (blue)

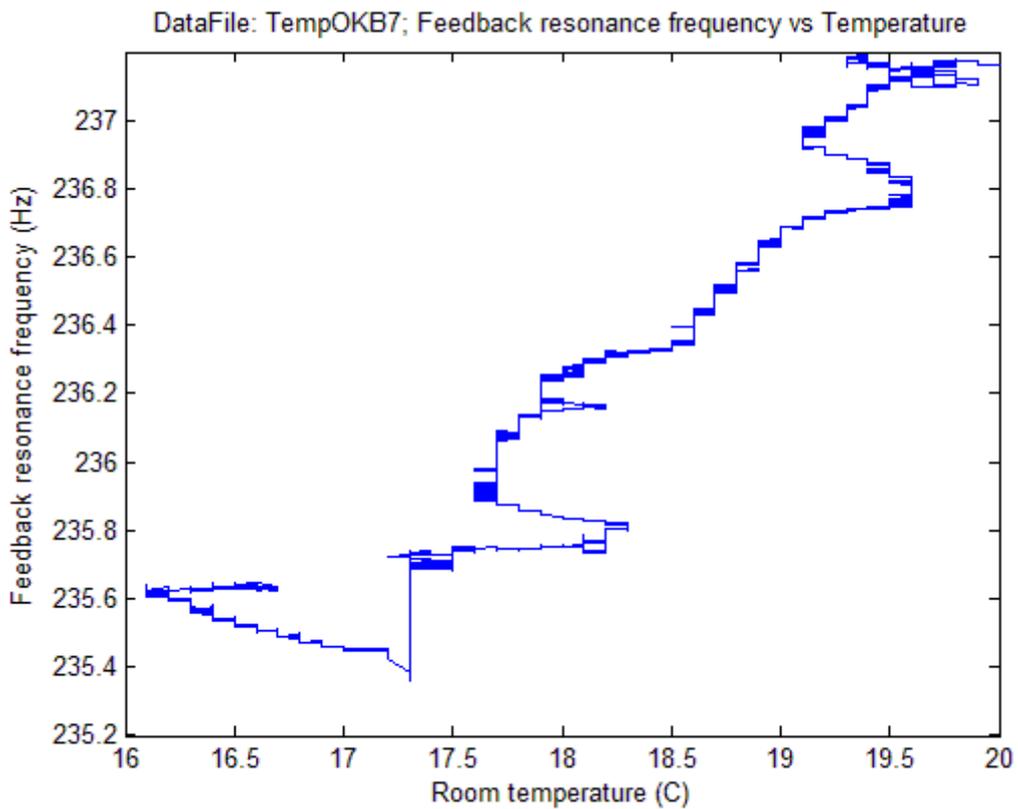


Figure 8: The relationship between measured feedback frequency and temperature.

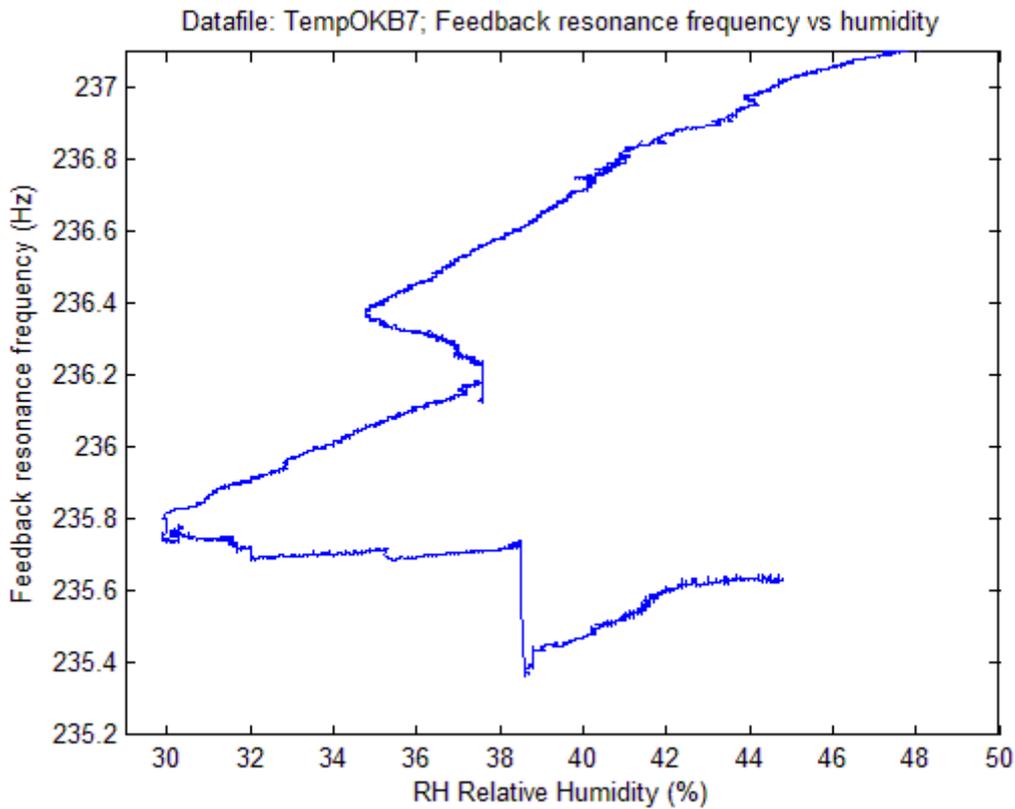


Figure 9: The relationship between measured feedback frequency and humidity.

A COMSOL simulation of the plate generated the mode-shape presented in Figure 10. Excitation in the experiment was in the back left corner and pickup in the front right corner of the figure. The output is out of phase with the input, consequently requiring a 180 degree phase shift in the feedback electronics. It is a standing wave pattern, in reality probably composed of overlapping modes at the same frequency. There may be creeping mode variations tentatively responsible for the micro-variations in the frequency identified in Figure 5. Sensitivity to damage is small in nodes and large in anti-nodes, which requires more than one resonant frequency [1] for the location of damage. The anti-nodes can be regarded as a virtual sensor network.

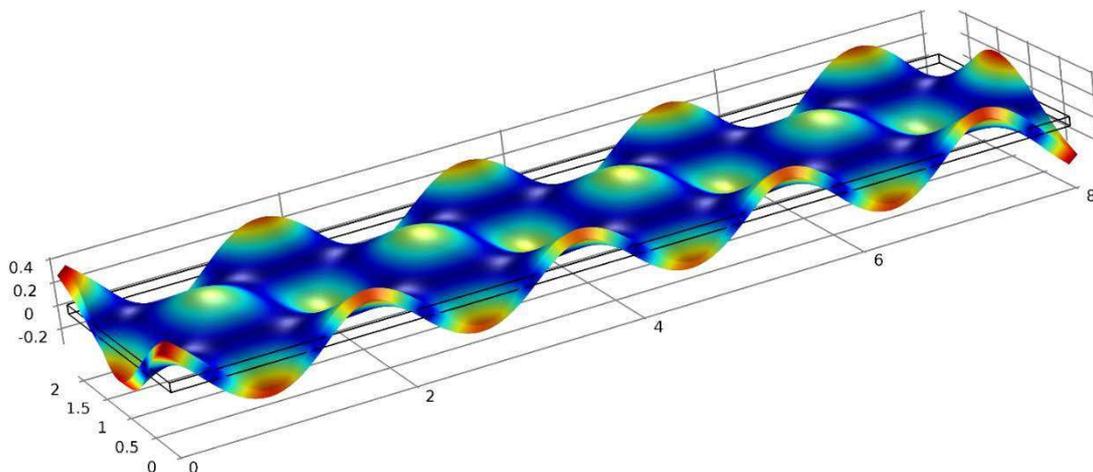


Figure 10: COMSOL modelling of one mode in the concrete plate at a frequency of about 240 Hz.

#### 4 NONLINEARITY RESULTS

A popular way of measuring non-linearity is to make a frequency sweep at different levels of excitation. If the test object is non-linear in wave-propagation terms there will be a shift of resonance peaks towards higher or lower frequencies as the excitation level increases. With the PID-regulator and associated electronics available we do not need to perform sweeps. We can adjust the SETPOINT of the SIM 960 controller and read the frequency shift from the counter measuring the feedback resonance frequency. These frequency shifts are tiny but systematic, as can be seen in Figure 11. Using a 60 s gate time in the counter we see a frequency drop from 237.54 to 237.39 Hz as the SETPOINT value is increased from 0.300 Volt to 1.000 Volt in steps of 0.100 Volt. Since the resonance frequency drops as the excitation / strain increases, the object is said to be softening in non-linear terms.

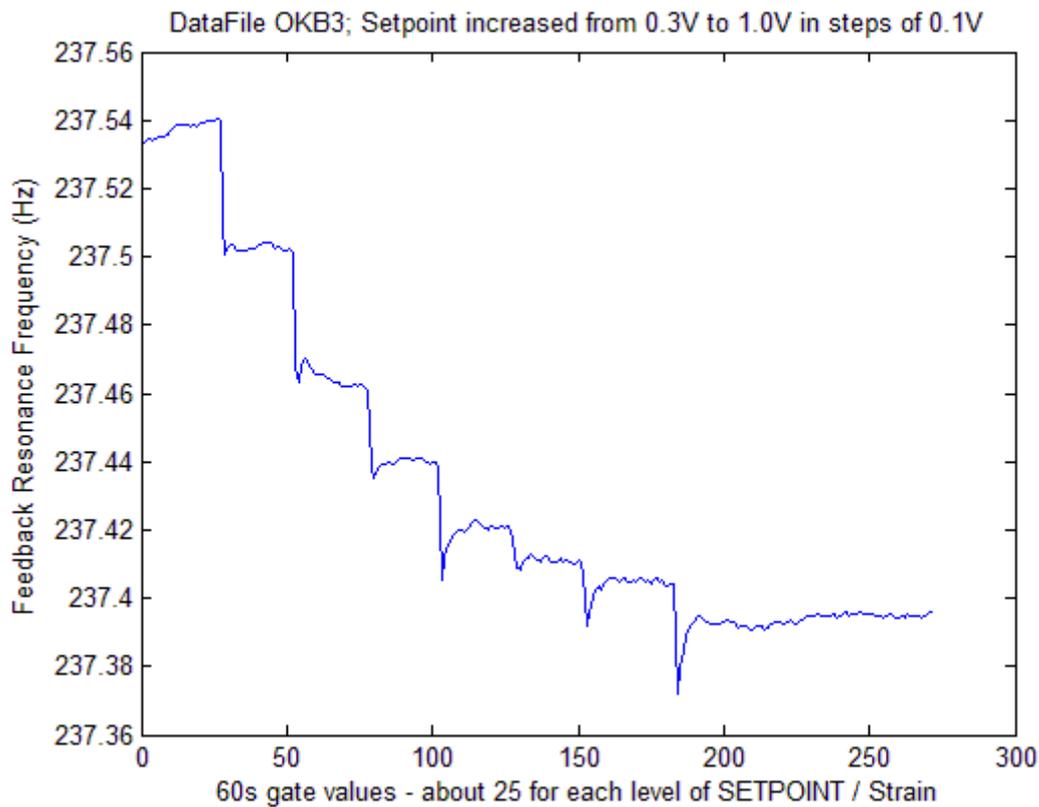


Figure 11: Frequency shift as the PID SETPOINT value / strain is increased

The familiar expression for elastic deformation of the first degree (Hooke's law) is a linear approximation for the more general expression (1).

$$\sigma = K_0 \varepsilon (1 + \beta \varepsilon + \delta \varepsilon^2 + \dots) \quad (1)$$

$K_0$  = the linear modulus

$\beta$  = first order non-linear modulus

$\delta$  = second order non-linear modulus

Averaging the frequency readings for each level of SETPOINT value generates the data in Figure 12, where the data points have been connected with a 2<sup>nd</sup> degree polynomial regression. The expression for the curve is, Equation 2:

$$f(v) = 237.6870 - 0.5881*v + 0.2990*v^2 \quad (2)$$

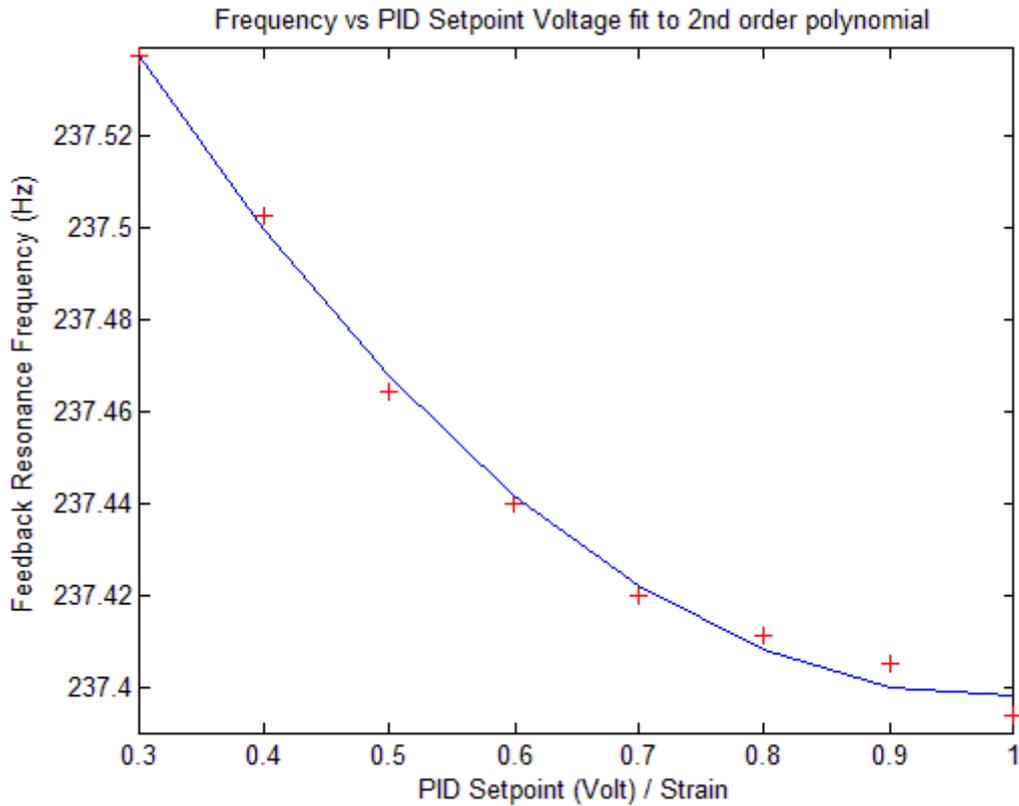


Figure 12: Nonlinear behavior of the concrete plate – a softening object. The SIM 960 SETPOINT value is proportional to strain, but the relation has not been established at this time.

## 5 DISCUSSION

The transient event recorded that was accompanied by an observed acoustic emission did not in any way reduce the strength of the construction. We may thus assume that the frequency variations seen in this 9-day experiment are smaller than what may appear in a strength reducing event. We have not verified that the nonlinearity is not due to the frequency response of the power amplifier.

## 6 CONCLUSIONS

Controlled feedback resonance seems suitable for long-term structural health monitoring applications of full-scale objects. The excitation can be made with very small transducers since it is done at resonance. Controlling the amplitude of the oscillations may give information about non-linearity parameters of the object. We see no relation between transducer distance and frequency and believe it is the most excitable standing wave pattern that controls the frequency.

It remains to understand the influence of ambient conditions and their magnitude relative to real deterioration of concrete objects that are exposed to outdoor conditions for many years.

## ACKNOWLEDGEMENT

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