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A formal approach to structural health monitoring design

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Abstract
While the objective of structural design is to achieve stability with an appropriate level of safety, the design of structural health monitoring (SHM design) is performed to identify a configuration that enables acquisition of data with an appropriate level of precision in order to understand the condition state of a structure. Nevertheless, a practical and standardized approach for SHM design is not fully available. In this contribution, we address SHM design by proposing a method for the estimation of the effectiveness of SHM (monitoring effectiveness) based on information available a priori—i.e. before the acquisition of data from sensors. The proposed method is developed with the aim of easing SHM design in real-life settings and maintaining an analogy with structural design. The expected monitoring effectiveness relies on the calculation, performed a priori, of the variance that will affect the estimate of a target variable a posteriori. Since no real observations are available a priori, the estimation of variance is carried out by considering the observations as a random variable. With the aid of two real-life applications, we show how the proposed method can be used in order to evaluate a monitoring system.

1 INTRODUCTION

In structural health monitoring (SHM), we employ our resources in order to acquire information about a structure [1][2]. The effectiveness of SHM (monitoring effectiveness) depends on the capability of the monitoring system to provide information about one or more target variables: variables that are used to assess the structural condition and performance [3]. Damage sensitive features such as the curvature of a beam or the size of cracks could be considered as target variables. Usually, the information is obtained from measurements that are provided by the sensors installed on the structure itself. The designer of the monitoring system must choose the number of sensors, the type and the position.

When designing a structure, civil engineers follow a well-established rational procedure

[4][5], whereby the performance of the design concept is predicted through structural analysis and compared to the target performance. Conversely, when engineers carry out the design of an SHM system (SHM design), the approach is often based on engineering judgement [3]. Herein, we propose a rational procedure for SHM design, which is the counterpart (Table 1) of the semi-probabilistic method used in structural design and proposed by Eurocode 0 [4]. Structural design aims to ensure stability with an appropriate safety factor and includes definition of design loads, calculation of structural demand and choice of a solution that offers the required capacity. The design is satisfactory if the capacity is greater than the demand. In SHM, the object is to estimate the structural state with an appropriate precision or level of confidence. SHM design includes definition of the target monitoring effectiveness, calculation of the expected effectiveness using a probabilistic method and choice of the monitoring solution that ensure the target effectiveness.

The reason why SHM design is often carried out based on intuition is that a practical and standardized procedure for the estimation monitoring effectiveness is not fully available. The literature contains mostly academic methods for SHM design based on optimal sensor placement (OSP) techniques that are focused on monitoring for structural dynamics [1][6][7][8][9][10]. Those techniques use the principle of minimizing the information entropy [10] in order to define computationally-efficient heuristic algorithms for sequential sensor placement. Herein, we want to provide a tool for the solution of any monitoring problem in real-life settings, whose practicality can be compared to that of traditional structural design. Our approach is based on the assumption that Bayesian statistical inference provides the variance of the target variable, which we take as an indicator of monitoring effectiveness. In real-life applications, the square root of the expected variance—the standard deviation—is a value that can be easily interpreted with engineering judgement, compared with the variance of the prior distribution and with a required threshold. We posit that the variance of the target variable is a clear and simple metric for describing the demand in the SHM design.

The method we propose here provides, a priori, the distribution and therefore the expected value of the posterior variance, which can be used to evaluate the effectiveness of a tentative SHM system in the design stage. We consider the observations, normally a given value only affected by sensor noise, as a random variable whose distribution reflects the uncertainty of the structural properties a priori. This approach is called ‘pre-posterior analysis’ [11].

In section 2, we show the difference between posterior and pre-posterior analysis. In section 3, we define in mathematical terms the expected value of the posterior variance a priori, which we call ‘expected pre-posterior variance’. In section 4 and 5, we show how the formulation presented in section 3 can be applied in real life settings to estimate the monitoring effectiveness.

	Structural design	SHM design
Objective	Structural stability with appropriate safety.	Knowledge of structural state with appropriate confidence.
Demand	Effects of design loads (e.g. bending moments, axial forces).	Required precision of knowledge about the structural state (e.g. required variance of damage sensitive features).
Capacity	Structural capacity (e.g. bending resistance).	Precision of knowledge given by the SHM solution (e.g. calculated variance of damage sensitive features).
Model	Relationship between material properties and structural capacity.	Relationship between sensor measurements and precision of knowledge given by the SHM solution.
Limit state	Effect of design loads vs. structural capacity.	Required precision of knowledge of structural state vs. precision of knowledge provided by the SHM solution.

Table 1: Analogy between SHM design and the semi-probabilistic method proposed by Eurocode 0 [4].

2 PROBLEM STATEMENT AND FORMULATION

The entities involved in the Bayesian parameter estimation problem are the following.

Observations or measurements. We use objective quantitative information in our inference problems. We call the raw data acquired by sensors 'observations' or 'measurements'.

State. This represents the condition of a structure. The proposed method is developed on the assumption that the state parameters $\boldsymbol{\theta}$ are defined in a continuous space (like the Young's modulus of concrete or the stiffness of a steel beam).

Target variable. 'Target variable' α refers to the variable we want to estimate using the SHM system. It can be one of the state parameters $\boldsymbol{\theta}$ or a variable linked to the state parameters through a defined function $\alpha = \mathrm{f}(\boldsymbol{\theta})$. Herein, we investigate only cases of single target variables.

Structural model. This is the relationship linking the observations to the state parameters, without error in the measurements. Typically, in structural engineering problems, the structural model has a physical base (e.g. it is a finite element model or an analytical model of the structure).

We assume that, a priori, we know only: (1) the structural model; (2) the function $\alpha = \mathrm{f}(\boldsymbol{\theta})$; (3) the magnitude of the uncertainties due to the sensor noise and, possibly, to the model approximations; (4) the prior information about the state parameters $\boldsymbol{\theta}$. During SHM design, the prior information can be quantitatively defined using a statistical distribution $\mathrm{p}(\boldsymbol{\theta})$, called 'prior distribution' [12], while the structural model and the uncertainties are used to define the likelihood function $\mathrm{p}(\mathbf{y}|\boldsymbol{\theta})$. The likelihood function is a probability density function that gives the probability of obtaining the observations $\mathbf{y}$, given a value of $\boldsymbol{\theta}$. In SHM design, the observations $\mathbf{y}$ are unknown so they must be considered as a random variable. However, since the structural model is known, we can calculate the distribution $\mathrm{p}(\mathbf{y})$ of the observations $\mathbf{y}$ from the prior distribution $\mathrm{p}(\boldsymbol{\theta})$ of state parameters $\boldsymbol{\theta}$.

The Bayes' formula is

$$\mathrm{p}(\boldsymbol{\theta}|\mathbf{y}) = \frac{\mathrm{p}(\boldsymbol{\theta},\mathbf{y})}{\mathrm{p}(\mathbf{y})} = \frac{\mathrm{p}(\mathbf{y}|\boldsymbol{\theta})\cdot\mathrm{p}(\boldsymbol{\theta})}{\mathrm{p}(\mathbf{y})}, \tag{1}$$

and shows that the distribution $\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})$ of the state parameters a posteriori is proportional to the product of the likelihood function $\mathrm{p}(\mathbf{y}|\boldsymbol{\theta})$ and the prior distribution $\mathrm{p}(\boldsymbol{\theta})$. The evidence $\mathrm{p}(\mathbf{y})$ can be seen as a normalization factor [12], calculated by integrating over the parameter space $\boldsymbol{\Omega}_{\boldsymbol{\theta}}$:

$$\mathrm{p}(\mathbf{y}) = \int_{\Omega_\theta} \mathrm{p}(\boldsymbol{\theta},\mathbf{y})\cdot d\boldsymbol{\theta} = \int_{\Omega_\theta} \mathrm{p}(\mathbf{y}\,|\,\boldsymbol{\theta})\cdot\mathrm{p}(\boldsymbol{\theta})\cdot d\boldsymbol{\theta}. \tag{2}$$

The probability $\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})$ is a function in the space of $\boldsymbol{\theta}$ and $\mathbf{y}$ that returns the posterior probability density of $\boldsymbol{\theta}$. We can use the Bayes' theorem to calculate the parameter distribution $\mathrm{p}(\boldsymbol{\theta}|\mathbf{y}')$ after acquiring a specific dataset $\mathbf{y}'$ from the monitoring system. In this case, unlike in (1), $\mathbf{y}'$ is a given vector. Thus, $\mathrm{p}(\boldsymbol{\theta}|\mathbf{y}')$ and $\mathrm{p}(\mathbf{y}'|\boldsymbol{\theta})$ are now mere functions of $\boldsymbol{\theta}$ and $\mathrm{p}(\mathbf{y}')$ is a normalization constant that makes the integral of the posterior distribution equal to 1. Once $\mathrm{p}(\boldsymbol{\theta}|\mathbf{y}')$ or $\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})$ are known, we can calculate respectively the distributions $\mathrm{p}(\alpha|\mathbf{y}')$ and $\mathrm{p}(\alpha|\mathbf{y})$ of the target variable with the assumption that $\alpha = \mathrm{f}(\boldsymbol{\theta})$ and $\boldsymbol{\theta}|\mathbf{y} \sim \mathrm{p}(\boldsymbol{\theta}|\mathbf{y})$.

In this paper, we want to assess a priori the features of the distribution $\mathrm{p}(\alpha|\mathbf{y})$ of the target variable, assuming that we will acquire data affected by a certain noise, but before that data is actually available. This process is usually called pre-posterior analysis [11] and we call the distribution p(**θ**|**y**) of (1) 'pre-posterior distribution' of the state parameters, in order to distinguish it from the posterior distribution p(**θ**|**y'**), in which the observations **y'** are a given value. Similarly, we call p(α|**y**) and p(α|**y'**) respectively 'pre-posterior' and 'posterior' distributions of the target variable.

3 CALCULATION OF EXPECTED MONITORING EFFECTIVENESS

The pre-posterior distribution p(**θ**|**y**) of the state parameters **θ**, a function of the observations **y**, can be used to calculate the expected value of the target variable $\mu_{\alpha|\mathbf{y}}$ and variance $\sigma^2_{\alpha|\mathbf{y}}$, which both depend on **y**. Their values are

$$\mu_{\alpha|\mathbf{y}}(\mathbf{y}) = \mathrm{E}_{\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})}\left[\mathrm{f}(\boldsymbol{\theta})\right] = \int_{\Omega_{\boldsymbol{\theta}}} \mathrm{f}(\boldsymbol{\theta}) \cdot \mathrm{p}(\boldsymbol{\theta}\,|\,\mathbf{y}) \cdot \mathrm{d}\boldsymbol{\theta}, \tag{3}$$

$$\sigma^2_{\alpha|\mathbf{y}}(\mathbf{y}) = \mathrm{V}_{\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})}\left[\mathrm{f}(\boldsymbol{\theta})\right] = \int_{\Omega_{\boldsymbol{\theta}}} \left[\mu_{\alpha|\mathbf{y}}(\mathbf{y}) - \mathrm{f}(\boldsymbol{\theta})\right]^2 \cdot \mathrm{p}(\boldsymbol{\theta}\,|\,\mathbf{y}) \cdot \mathrm{d}\boldsymbol{\theta}, \tag{4}$$

where $\mathrm{E}_{\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})}[\mathrm{f}(\boldsymbol{\theta})]$ and $\mathrm{V}_{\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})}[\mathrm{f}(\boldsymbol{\theta})]$ are respectively the expected value and the variance operator applied to f(**θ**). The posterior variance can be seen as a random variable with expected value equal to

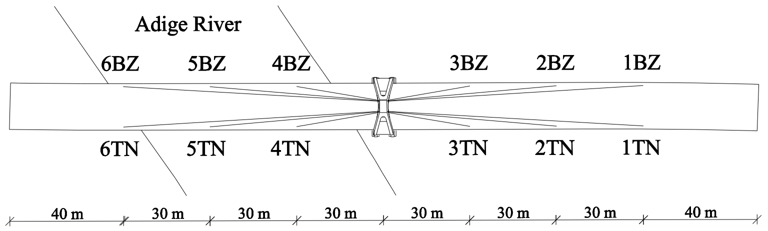
$$\sigma^2_{\alpha(\mathbf{y})} = \mathrm{E}_{\mathrm{p}(\mathbf{y})}\left[\mathrm{V}_{\mathrm{p}(\boldsymbol{\theta}|\mathbf{y})}\left[\mathrm{f}(\boldsymbol{\theta})\right]\right] = \int_{\Omega_{\mathbf{y}}} \sigma^2_{\alpha|\mathbf{y}}(\mathbf{y}) \cdot \mathrm{p}(\mathbf{y}) \cdot \mathrm{d}\mathbf{y}, \tag{5}$$

where $\Omega_{\mathbf{y}}$ is the domain of the observations. In practice, $\sigma^2_{\alpha(\mathbf{y})}$ represents what we expect the variance of the target variable to be after that monitoring is performed. We refer to this quantity as the 'expected pre-posterior variance'. The value of $\sigma^2_{\alpha(\mathbf{y})}$ does not depend on a specific value of **y** but can be calculated a priori using a distribution of the observations p(**y**). Therefore, we can take $\sigma^2_{\alpha(\mathbf{y})}$ as an indicator of expected monitoring effectiveness and compare $\sigma^2_{\alpha(\mathbf{y})}$ with a required threshold to check if the required performance is satisfied.

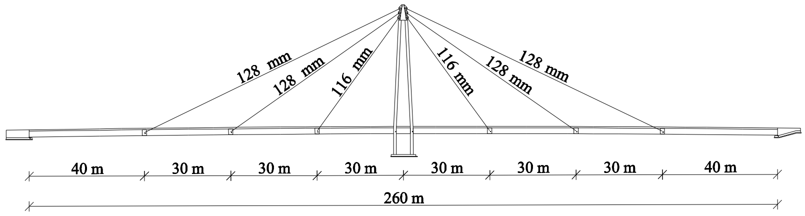
4 APPLICATION TO A SINGLE-OBSERVATION PROBLEM

In this example, we deal with a single observation y and a single state parameter θ, which is also the target variable, so $\alpha = \theta$. The monitoring design concept consists of a single PCB 393B12 piezoelectric accelerometer, installed on a steel cable for the measurement of the first natural frequency (observation) and the consequent estimation of the cable tension force (target variable/state parameter). The cable is among those that support the deck of Adige Bridge [13][14]; a cable-stayed bridge built in 2008 near the city of Trento, Italy. The bridge, depicted in Figure 1, is supported by 12 stay cables, 6 for each side, anchored to the bridge tower. Herein, for simplicity we consider only cable 1BZ, which has length L = 95.06 m, diameter 128 mm, linear mass density ρ = 88.62 kg/m, and was designed for a service load of 7,818 kN—48% of its nominal capacity. Vibrational tests on the cables need to be performed in order to check the tensioning force. During SHM design, we calculate that the monitoring solution is satisfactory if the expected standard deviation of the uncertainty that will affect the force a posteriori is below $\bar{\sigma}_{\alpha(\mathrm{y})}$ = 200 kN. The threshold of 200 kN is obtained considering that the expected value of the force a posteriori will be compared with the cable nominal capacity in order to evaluate the structural reliability. In the SHM design, we want to predict the uncertainty that will affect the estimate of the tension force α in cable 1BZ, calculated based on the observation y of its first natural frequency, and compare the expected pre-posterior variance with the threshold $\bar{\sigma}^2_{\alpha(\mathrm{y})}$ = 40,000 kN^2, which represents the target monitoring effectiveness.

The first natural frequency of the cable is then calculated by applying a Hann window to the recorded accelerations and performing a fast Fourier transform (FFT). Based on previous tests carried out with this SHM methodology, we expect to obtain an observation of the natural frequency affected by an uncertainty with standard deviation $\sigma_{\mathrm{y}|\theta}$ = 0.01 Hz.



(a) Top view [13].



(b) Lateral view [13].


(c) Monitoring concept: a PCB 393B12 piezoelectric accelerometer installed on cable 1BZ.

Figure 1: Geometry of Adige Bridge and monitoring concept.

4.2 Structural health monitoring design

We assume the following model in order to calculate the theoretical value of the first natural frequency $\hat{\mathrm{y}}$, given the tension force θ:

$$\hat{\mathrm{y}}(\theta) = \sqrt{\frac{\theta}{4\rho \cdot \mathrm{L}^2}}\,. \tag{6}$$

The prior distribution p(θ) of the state parameter is assumed to be a log-normal probability density function, with mean value set equal to the design service load of the cable, 7,818 kN, and coefficient of variation 0.20.

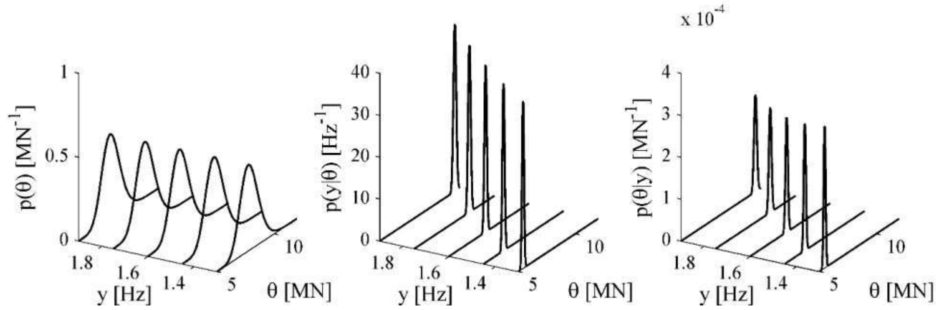
If the natural frequency is calculated using (6), the result is the theoretical value of frequency $\hat{\mathrm{y}}(\theta)$ corresponding to the force θ. Instead, if the natural frequency is provided by the FFT, based on the recorded accelerations, it may be different from the theoretical value $\hat{\mathrm{y}}$, because it is affected by a random noise that we assume to be normally distributed with mean zero and standard deviation $\sigma_{\mathrm{y}|\theta}$. We can define the likelihood function p(y|θ), which gives the probability of observing y given a value of tension force θ, as a normal distribution:

$$\mathrm{p}(\mathrm{y}\,|\,\theta) = \mathcal{N}\left(\sqrt{\frac{\theta}{4\rho \cdot \mathrm{L}^2}}, \sigma^2_{\mathrm{y}|\theta}\right) \tag{7}$$

Figure 2 depicts the prior distribution, the likelihood function and the pre-posterior distributions p(θ|y). First, we should notice that the prior distribution (Figure 2a) does not depend on the observation, while the likelihood function (Figure 2b) depends on both y and θ. However, although we showed p(y|θ) for constant values of y, we should notice that p(y|θ) is a probability distribution with respect to y, not to θ, and the mode of p(y|θ) in the domain of y is proportional to $\sqrt{\theta}$. In contrast, the pre-posterior distribution (Figure 2c) is a function of both y and θ but provides the probability density of the state parameter θ. As shown by Figure 2c, for different values of y, the peak of the pre-posterior distribution changes in magnitude. The reason for this is that the variance of the pre-posterior distribution, defined in the domain of θ, changes with the value of the observation y because the model is non-linear.

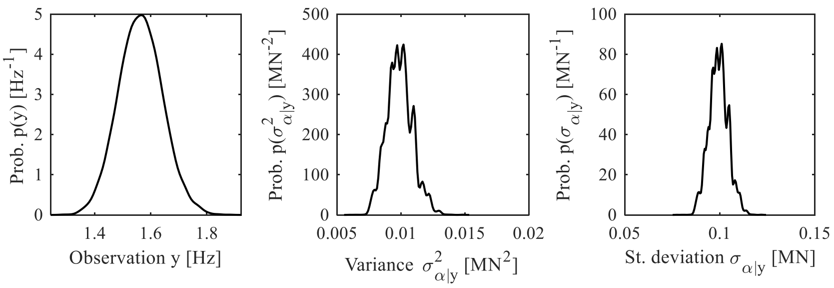
Using a Monte Carlo algorithm, we calculated a sample of pre-posterior variances of the target variable. Then, we applied kernel density estimation (KDE) [15] to the sample of pre-posterior variances in order to obtain the probability density functions depicted in Figure 3.

The expected value of pre-posterior variance resulted in the value $\tilde{\sigma}^2_{\alpha(\mathrm{y})}$ = 10,000 kN^2, corresponding to a standard deviation of $\tilde{\sigma}_{\alpha(\mathrm{y})}$ = 100 kN. Therefore, we can conclude the SHM design by saying that the SHM method is satisfactory because the expected pre-posterior variance is lower than the required monitoring effectiveness: $\tilde{\sigma}^2_{\alpha(\mathrm{y})}$ = 10,000 kN^2 $\leq$ $\bar{\sigma}^2_{\alpha(\mathrm{y})}$ = 40,000 kN^2.



(a) Prior distribution. (b) Likelihood function. (c) Pre-posterior distribution.

Figure 2: Distributions involved in the prediction of the uncertainty that affects the tension force of stay cable 1BZ.



(a) Marginal distr. of the observation. (b) Distribution of the pre-posterior variance. (c) Distribution of the pre-posterior standard deviation.

Figure 3: Marginal distributions resulting from the pre-posterior analysis.

5 APPLICATION TO A MULTI-OBSERVATION PROBLEM

In this example, the aim of monitoring is the estimation of a single target variable, a function of four state parameters, from a set of two observations. The case study is the US202/NJ23 overpass (Figure 4), located near the town of Wayne, NJ, USA. The overpass, previously described in [16], is made of a steel-concrete composite deck. The position of neutral axis was chosen as a damage sensitive feature and target variable, while the monitoring concept provides observations through 12 SMARTEC MuST FBG deformation sensors. Herein, we present the evaluation of the monitoring concept for cross section 5.2 (Figure 5), located at midspan. The sensors of section 5.2 were named '5.2 Down' and '5.2 Up', respectively for the sensor installed on the bottom flange and the one on the top flange, and provide the observations of strain $\mathrm{y_b}$ and $\mathrm{y_t}$. From $\mathbf{y} = [\mathrm{y_t} \quad \mathrm{y_b}]^T$, we want to estimate the position of the neutral axis α in section 5.2. The target monitoring effectiveness is set to a standard deviation of $\bar{\sigma}_{\alpha(\mathrm{y})}$ = 60 mm, based on the fact that the expected value of α will be compared, a posteriori, with the theoretical value calculated using the features of the healthy cross section.

5.2 Structural health monitoring design

Through the SHM system, we want to estimate the position of neutral axis α in section 5.2, based on a single set of two relative strains $\mathbf{y} = [y_t \quad y_b]^T$. The structural model that links the state parameters to the observed strains is based on the linear model of the cross section illustrated in Figure 5. The state parameters are $\boldsymbol{\theta} = [b_c \quad E_s \quad E_c \quad B]^T$, where b_c is the effective width of the concrete slab, E_s the Young's modulus of steel, E_c the Young's modulus of concrete and B the bending moment. The other parameters are assumed to be deterministic and are described in Table 2. The position of neutral axis α is defined as the distance between the neutral axis and the bottom flange (Figure 5). Based on the model, the function f($\boldsymbol{\theta}$) of the state parameters that must be used to calculate the target variable is:

$$\alpha = f(\boldsymbol{\theta}) = \frac{A_{sb} z_{sb} + s_w b_w z_{sw} + A_{st} z_{st} + A_{rt} z_{rt} + A_{rb} z_{rb} + s_c b_c z_c E_c / E_s}{A_{sb} + s_w b_w + A_{st} + A_{rt} + A_{rb} + s_c b_c E_c / E_s} . \quad (8)$$

Based on the state parameters, we can also calculate the moment of inertia J:

$$J(\boldsymbol{\theta}) = A_{sb}\left[z_{sb} - f(\boldsymbol{\theta})\right]^2 + A_{st}\left[z_{st} - f(\boldsymbol{\theta})\right]^2 + A_{rt}\left[z_{rt} - f(\boldsymbol{\theta})\right]^2 + A_{rb}\left[z_{rb} - f(\boldsymbol{\theta})\right]^2 + \\ + \left[b_c s_c \left[z_c - f(\boldsymbol{\theta})\right]^2 + \frac{b_c s_c^3}{12}\right]\frac{E_c}{E_s} + b_w s_w \left[z_{sw} - f(\boldsymbol{\theta})\right]^2 + \frac{b_w s_w^3}{12}. \quad (9)$$

Then, we can calculate the theoretical values of strain $\hat{\mathbf{y}}(\boldsymbol{\theta})$:

$$\hat{\mathbf{y}}(\boldsymbol{\theta}) = \begin{bmatrix} \hat{y}_t(\boldsymbol{\theta}) \\ \hat{y}_b(\boldsymbol{\theta}) \end{bmatrix} = \frac{B}{E_s J(\boldsymbol{\theta})} \cdot \begin{bmatrix} f(\boldsymbol{\theta}) - h_t \\ f(\boldsymbol{\theta}) - h_b \end{bmatrix}, \quad (10)$$

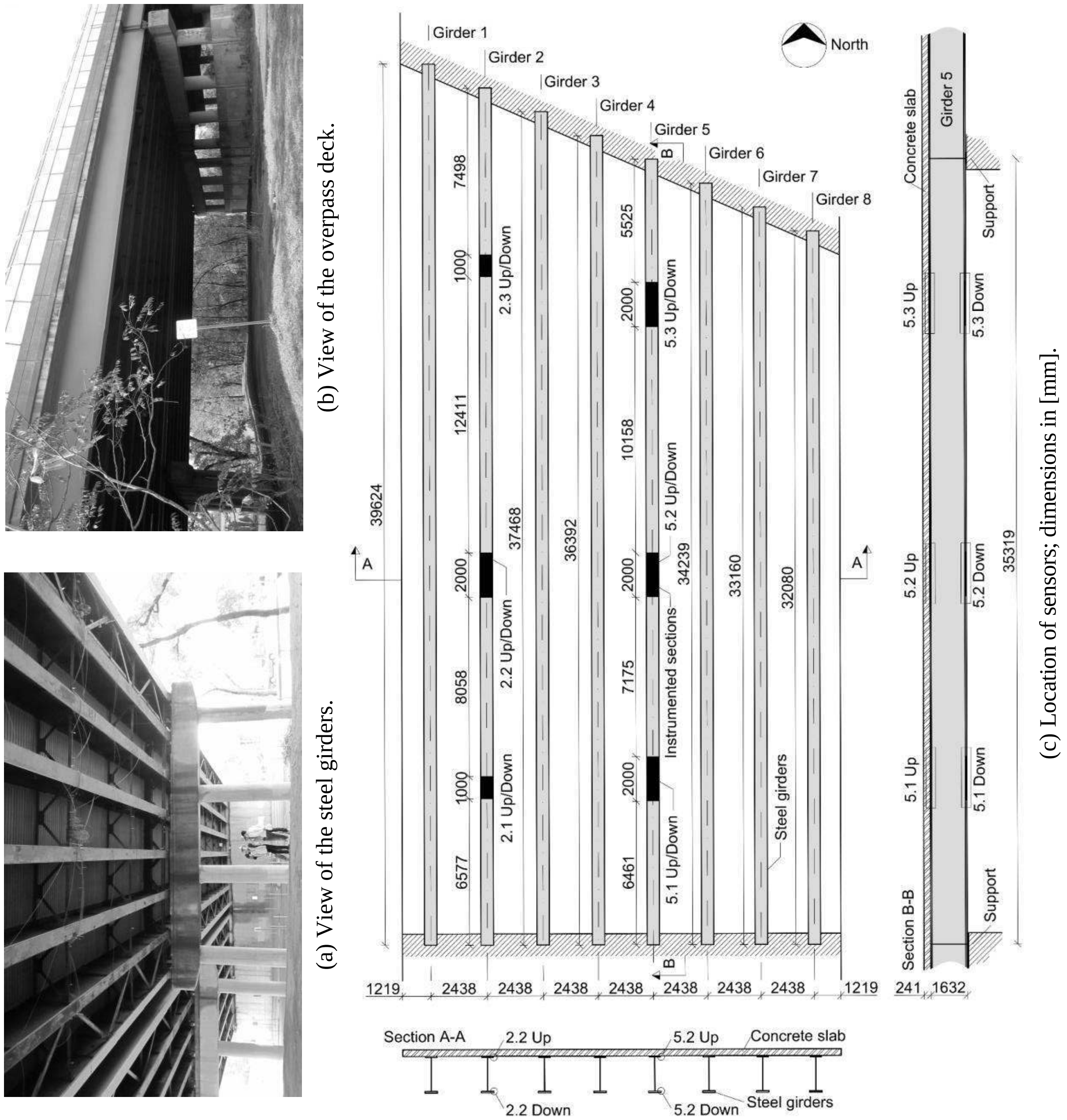
in which $\hat{y}_t$ is the theoretical strain where the top sensor is installed and $\hat{y}_b$ is the theoretical strain where the bottom sensor is installed. The prior distribution of the state parameters is made of four independent log-normal distributions, whose statistical parameters are shown in Table 2.

Based on our experience with FBG sensors, we assume that each relative strain is characterized by a noise of standard deviation $\sigma_{y|\boldsymbol{\theta}} \cong 3\ \mu\varepsilon$. We can therefore say that the probability of observing a realization **y**, given a set of state parameters **θ**, is

$$p(\mathbf{y} | \boldsymbol{\theta}) = \mathcal{N}\left(\begin{bmatrix} \hat{y}_t(\boldsymbol{\theta}) \\ \hat{y}_b(\boldsymbol{\theta}) \end{bmatrix}, \begin{bmatrix} \sigma_{y|\theta}^2 & 0 \\ 0 & \sigma_{y|\theta}^2 \end{bmatrix}\right). \quad (11)$$

where $\mathcal{N}(\boldsymbol{\mu},\boldsymbol{\Sigma})$ denotes a multivariate normal probability density function with mean **μ** and covariance **Σ**. The distribution of (11), which depends on **y** and **θ**, is the likelihood function.

The pre-posterior analysis is performed once again through a Monte Carlo algorithm, which provides the probability density function of the pre-posterior variance depicted in Figure 6b. The expected pre-posterior variance (i.e. the mean value of the pre-posterior variance) of the neutral axis results in $\tilde{\sigma}^2_{\alpha(\mathbf{y})}$ = 2,490 mm^2, corresponding to a standard deviation of $\tilde{\sigma}_{\alpha(\mathbf{y})}$ = 50 mm. Therefore, we can conclude the SHM design by checking that the expected pre-posterior variance is lower than required: $\tilde{\sigma}^2_{\alpha(\mathbf{y})}$ = 2,490 mm^2 ≤ $\bar{\sigma}^2_{\alpha(\mathbf{y})}$ = 3,600 mm^2.

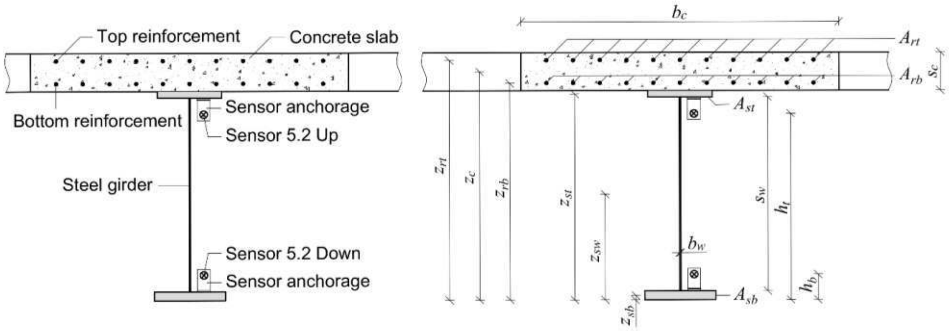


(a) View of the steel girders.

(b) View of the overpass deck.

(c) Location of sensors; dimensions in [mm].

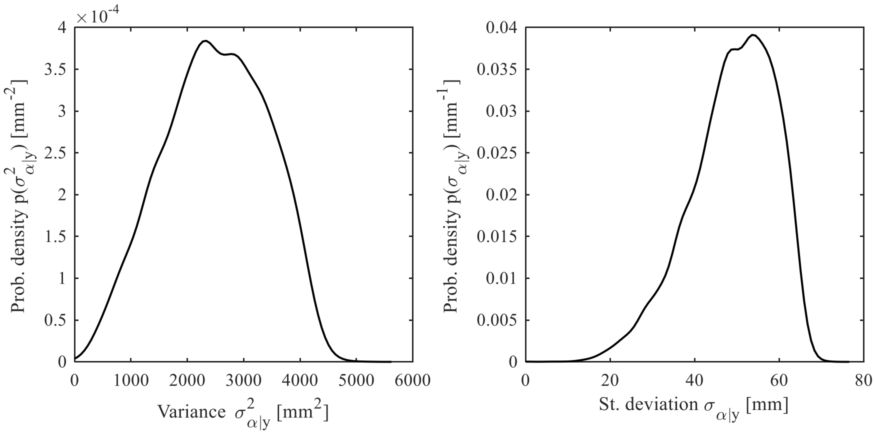
Figure 4: Pictures and geometry of US202/NJ23 overpass.



(a) Section 5.2.

(b) Geometrical features of section 5.2 involved in the structural model.

Figure 5: Geometry of the composite section in position 5.2.



(a) Variance. (b) Standard deviation.

Figure 6: Probability density function of the pre-posterior variance and standard deviation.

Description	Parameter	Type	Mean/nominal value	Coefficient of variation
Area of the bottom steel flange	A_{sb}	deterministic	29,032 mm^2	–
Area of the top steel flange	A_{su}	deterministic	18,085 mm^2	–
Global area of the bottom reinforcement	A_{rb}	deterministic	2,819 mm^2	–
Global area of the top reinforcement	A_{ru}	deterministic	2,109 mm^2	–
Centroid of the bottom steel flange	z_{sb}	deterministic	31.8 mm	–
Centroid of the top steel flange	z_{su}	deterministic	1,609.8 mm	–
Centroid of the bottom reinforcement	z_{rb}	deterministic	1,667.0 mm	–
Centroid of the top reinforcement	z_{ru}	deterministic	1,800.0 mm	–
Centroid of the steel web	z_{sw}	deterministic	825.5 mm	–
Centroid of the concrete slab	z_c	deterministic	1,752.7 mm	–
Steel web depth	s_w	deterministic	1,524.0 mm	–
Steel web thickness	b_w	deterministic	9.5 mm	–
Concrete slab thickness	s_c	deterministic	241.3 mm	–
Position of the bottom sensor	h_b	deterministic	80.5 mm	–
Position of the top sensor	h_t	deterministic	1,570.5 mm	–
Effective width of the concrete slab	b_c	log-normal	2,438.4 mm	0.20
Young's modulus of steel	E_s	log-normal	200,000 MPa	0.02
Young's modulus of concrete	E_c	log-normal	35,000 Mpa	0.20
Bending moment	B	log-normal	500.0 kNm	0.50

Table 2: Prior information about the parameters of the structural model.

9 CONCLUSIONS

In this paper, we propose a rational method for the design of structural health monitoring systems (monitoring design) that is the counterpart of the structural design process proposed by most design codes, such as Eurocode 0. In structural design, the demand is the effect of the loads (e.g. bending moment or axial compression), while the capacity is the resistance of the structural elements. In monitoring design, we identify the demand as the required variance of a selected target variable (e.g. deformed shape or crack size), while the capacity is the variance of the target variable calculated a posteriori, i.e. after the acquisition of monitoring data. Herein, we formalize how the expected variance of a single target variable can be calculated when a monitoring system is being designed. The proposed method is a pre-posterior analysis, in which the monitoring data is considered as a random variable. It provides a distribution of the variance of the target variable, whose mean value is taken as an indicator of monitoring effectiveness. The monitoring solution is satisfactory if the expected variance of the target variable is less than the required value, which is set a priori based on the monitoring purpose.

After the formulation of our monitoring design method, we show two applications. In the first case study, the monitoring system is designed to provide the force in a stay cable based on the first natural frequency of the cable. In the second case study, the target variable is the position of neutral axis in a composite cross section while the observations are two strain values measured along the height of the section. These two practical cases show the applicability of the proposed method in real-life structural health monitoring design problems.

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