Biot waves in porous ceramic plates: influence of boundary conditions

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Abstract
This work deals with elastic wave propagation in fluid saturated porous media and interactions with macroscopic defects inside the media. Porous media under consideration are open-cell media consisting of a solid phase (skeleton) and a continuous fluid phase as it is the case for ceramic plates. Generally, the sound wave propagation in such media is described by the Biot’s theory accounting for skeleton vibrations interacting with propagation inside the fluid phase. In a first time, we study the influence of the boundary conditions (open or sealed surfaces) on the fast and slow waves. This problem has already been tackled by different authors but their conclusions are conflicting. In a second time, a macroscopic volume defect is considered. It can be interpreted as a change of boundary conditions but inside the porous media leading to a fluid slab in between two porous regions. We study the effect of this defect on the reflection and transmission coefficients and also the possibility to characterize the defect parameters (size and position) from the measured coefficients.

Keywords : Biot theory, porous media, boundary conditions, ultrasound, non destructive testing (NDT)

1 Introduction

The Biot theory describes sound propagation in a fluid saturated porous media. At normal incidence, the theory predicts two types of waves: fast wave, and slow wave referred as Biot waves. We are interested in the analysis of the velocity fields associated to these two waves. Porous media under consideration are open-cell media consisting of a solid phase (skeleton) and a continuous fluid phase as it is the case for ceramic plates. In the context of wave propagation in trabecular bone, a specific attention has to be accorded to the boundary conditions. In previous works, Rasolofosaon [1] who had performed experimental measurements in the case of open interfaces plate as well as sealed interfaces plate. He showed that, in the case of sealed pores plate, the slow wave disappears. Johnson et al [2], have studied wave propagation in porous media and they were interested on open-sealed pores plate, they showed that the slow wave always exists into the porous plate. These analysis are conflicting and do not clearly conclude on the existence of the slow wave. In this purpose, we use Biot model to explain the existence of slow wave according to boundary conditions, we represent also fast and slow velocity fields inside the porous media in regards to explain the behaviour of these two waves propagating into the solid and make a point on mode conversion phenomena. This paper is organized as follows: in the first section we present the Biot model and give the hypothesis of our model which enable us to establish the boundary conditions on open and sealed pores cases. The second section concerns numerical simulations: transmission and reflection coefficients are presented for the two cases mentioned (open or sealed) as well as velocity fields into the porous media. Second section is devoted to experimental results obtained through a ceramic porous plate open and sealed pores and compare it with numerical simulations which allows to make a conclusion according to the effect of boundary conditions. In final section we introduce a volume defect and we discuss the influence of the defect size and position.
2 Biot model

We consider a fluid-saturated porous media. Biot’s theory predicts three types of waves: two compressional waves which are identified as fast and slow waves, and one shear wave. At normal incidence, only compression waves propagate in the material. In this section, we give the Biot equations and boundary conditions used to calculate fast and slow velocities and the transmission and reflection coefficients.

2.1 Biot equations and velocities

In vector form, Biot equations can be written [3]:

\[ \tilde{\rho}_{11} \frac{\partial^2 \mathbf{u}}{\partial t^2} + \tilde{\rho}_{12} \frac{\partial^2 \mathbf{U}}{\partial t^2} = P \nabla (\nabla \cdot \mathbf{u}) + Q \nabla (\nabla \cdot \mathbf{U}), \tag{1} \]

\[ \tilde{\rho}_{22} \frac{\partial^2 \mathbf{U}}{\partial t^2} + \tilde{\rho}_{12} \frac{\partial^2 \mathbf{u}}{\partial t^2} = R \nabla (\nabla \cdot \mathbf{U}) + Q \nabla (\nabla \cdot \mathbf{u}), \tag{2} \]

where \( \tilde{\rho}_{11}, \tilde{\rho}_{22} \) are respectively, the effective density of fluid and solid phases, and \( \tilde{\rho}_{12} \) is the massic coupling term:

\[ \tilde{\rho}_{11} = (1 - h) \rho_s - \tilde{\rho}_{12}, \quad \tilde{\rho}_{12} = -h \rho_0 (\alpha(\omega) - 1), \quad \tilde{\rho}_{22} = h \rho_0 - \tilde{\rho}_{12}. \]

\( h \) is the porosity, \( \alpha(\omega) \) is the dynamic tortuosity of the medium [3]:

\[ \alpha(\omega) = \alpha_\infty (1 + \frac{h \sigma}{\omega \alpha_\infty \rho_f}) \left( 1 + j \frac{4 \alpha_\infty \eta \rho_f \omega}{\sigma^2 \Lambda^2 h^2} \right). \tag{3} \]

In the equations (eq.1) and (eq.2), \( \mathbf{u} \) and \( \mathbf{U} \) are respectively the displacement of solid and fluid and \( P, Q, R \) are the elastic coefficients related to the elastic moduli of the skeleton \( k_s \) and the fluid \( k_f \) [3]:

\[ P = \frac{(1 - h)(1 - h - \frac{k_b}{k_s})k_s + h \frac{k_b}{k_f} k_b}{1 - h - \frac{k_b}{k_s} + h \frac{k_b}{k_f}} + \frac{4}{3} N, \]

\[ Q = \frac{(1 - h) - \frac{k_b}{k_s}}{1 - h - \frac{k_b}{k_s} + h \frac{k_f}{k_f}}, \]

\[ R = \frac{h^2 k_s}{1 - h - \frac{k_b}{k_s} + h \frac{k_f}{k_f}}. \]

Because of the normal incidence, we are interested only in the compressional wave, then, the displacements are expressed in terms of solid and fluid potentials velocities:

\[ \mathbf{u} = \nabla \Phi^s, \quad \mathbf{U} = \nabla \Phi^f. \]

The solid and fluid displacements are related to the fast (\( \Phi_1 \)) and slow (\( \Phi_2 \)) waves displacements as follow:

\[ \Phi^s = \Phi_1 + \Phi_2, \quad \Phi^f = \mu_2 \Phi_1 + \mu_2 \Phi_2, \]

where \( \mu_1 \) and \( \mu_2 \) are weight functions.

The fast and slow wave velocities into the porous medium are given by [3]:

\[ V^2 = \frac{P \tilde{\rho}_{22} + R \tilde{\rho}_{11} - 2Q \tilde{\rho}_{12} + \sqrt{\Delta}}{2(\tilde{\rho}_{11} \tilde{\rho}_{22} - \tilde{\rho}_{12}^2)}, \tag{4} \]
\[
V_2^2 = \frac{P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12} - \sqrt{\Delta}}{2(\tilde{\rho}_{11}\tilde{\rho}_{22} - \tilde{\rho}_{12}^2)}.
\]

where:
\[
\Delta = (P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12})^2 - 4(PR - Q^2)(\tilde{\rho}_{11}\tilde{\rho}_{22} - \tilde{\rho}_{12}^2).
\]

Fig.1 shows the velocities of free fast and slow waves in a ceramic material as function of frequency:

Figure 1: Compressional wave velocities predicted by Biot depending of the frequency. (a) : Fast wave velocity \(V_1\) (4), (b) : slow wave velocity \(V_2\) (5).

### 2.2 Boundary conditions

As explained previously, we deal with two cases : open pores case (fig.2-a) and sealed pores case (fig.2-b).

In open pores case, the pores are directly connected to the ambient media, in this case, solid and fluid velocities (\(V^s, V^f\)) and stresses (\(\tau^s_{zz}, \tau^f_{zz}\)) are related by the following conditions :

\[
\begin{align*}
\tau^s_{zz}(z_i, \omega) + \tau^f_{zz}(z_i, \omega) + p^f(z_i, \omega) &= j\omega\Sigma z_i v^f(z_i, \omega) \\
v^f(z_i, \omega) &= V^s(z_i, \omega) \\
v^f(z_i, \omega) &= V^f(z_i, \omega)
\end{align*}
\]

In sealed pores case, the surface is recovered by an impervious layer as shown on the figure (fig.2-b). In that case, solid and fluid velocities and stresses are coupled by the following conditions :

\[
\begin{align*}
\tau^s(z_i, \omega) + \tau^f(z_i, \omega) + p^f(z_i, \omega) &= j\omega\Sigma z_i v^f(z_i, \omega) \\
v^f(z_i, \omega) &= V^s(z_i, \omega) \\
v^f(z_i, \omega) &= V^f(z_i, \omega)
\end{align*}
\]
where \( z_i = 0 \), \( L \) refers to the interfaces positions, and \( u^{f_e} \) and \( p^{f_e} \) refers to the velocity and pressure in the external fluid, \( \Sigma_{z_i} \) is surface density of the impervious layer which depends on its thickness \( d \).

### 3 Numerical simulations

For this study we use physical parameters (Table 1) of water saturated ceramic plate of silice QF-20 produced by Filtros®, Ferro Corporation. The formalism developed in this work enables us to calculate transmission and reflection coefficients and velocity fields inside the ceramic plate.

<table>
<thead>
<tr>
<th>parameters</th>
<th>notations (units)</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid compressibility modulus</td>
<td>( k_s (\text{Pa}) )</td>
<td>( 36.6 \cdot 10^9 )</td>
</tr>
<tr>
<td>solid shear coefficient</td>
<td>( N (\text{Pa}) )</td>
<td>( (7.63 + 0.1i) \cdot 10^9 )</td>
</tr>
<tr>
<td>bulk modulus</td>
<td>( k_b (\text{Pa}) )</td>
<td>( (9.47 + 0.3i) \cdot 10^9 )</td>
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<td>solid density</td>
<td>( \rho_s (\text{kg.m}^{-3}) )</td>
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</tr>
<tr>
<td>porosity</td>
<td>( \rho_f (\text{kg.m}^{-3}) )</td>
<td>0.595</td>
</tr>
<tr>
<td>tortuosity</td>
<td>( \alpha_\infty )</td>
<td>1.844</td>
</tr>
<tr>
<td>permeability</td>
<td>( k_0 (\text{m}^2) )</td>
<td>( 1.68 \cdot 10^{-11} )</td>
</tr>
<tr>
<td>pores radius</td>
<td>( a (\text{m}) )</td>
<td>( 3.26 \cdot 10^{-5} )</td>
</tr>
<tr>
<td>water density</td>
<td>( \eta (\text{kg.m}^{-1}.\text{s}^{-1}) )</td>
<td>( 1.14 \cdot 10^{-3} )</td>
</tr>
<tr>
<td>water compressibility modulus</td>
<td>( \mu_f (\text{Pa}) )</td>
<td>( 2.22 \cdot 10^9 )</td>
</tr>
<tr>
<td>wave celerity in water</td>
<td>( c_0 (\text{ms}^{-1}) )</td>
<td>1450</td>
</tr>
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</table>

Table 1: Physical parameters of water saturated ceramic plate QF-20

#### 3.1 Transmission and reflection coefficients

Following figures show transmission and reflection coefficients in time and frequency domain. To identify waves which appears in simulated signals, we compare their velocities with fast and slow velocities given by (4) and (5).

##### 3.1.1 Open pores case

As expected, transmission coefficient (fig.3-a) decreases toward zero in the high frequency limit. On the transmitted signal (fig.3-b) we observe the fast wave arrival time A \((t = 3.26\mu s)\), a fast wave echo B \((t = 20.55\mu s)\) and the slow wave C \((t = 25.26\mu s)\). The high frequency limit of the reflection coefficient (fig.4-a) is the constant value \((0.3)[4]\). The reflected signal (fig.4-b) shows a first wave A which is the reflected incident wave at the interface \((z = 0)\) followed by a fast wave echo B \((t = 13.48\mu s)\) having a lower amplitude than the wave A.
3.1.2 Sealed pores case

In this case, the amplitude of the coefficients is higher than those in the open pores case, the transmission coefficient decreases to zero in the high frequency limit (fig. 5-a) and the reflection coefficient tends toward a constant value (0.6 in this case) (fig. 6-a). Simulated signals show less echoes than those in the first case. On the transmitted signal (fig. 5-b) we note that the slow wave is not present anymore, we can only see the two fast waves A and B. The reflected signal (fig. 6-b) presents reflected incident wave at the interface ($z = 0$) A followed by a fast wave echo B.
These results show that the existence of the slow wave depends on boundary conditions: in open pores plate slow wave exist in the transmitted signal while it disappears when the interfaces of the plate are sealed.

### 3.2 Fast and slow waves velocity fields

This part is devoted to the study of the wave velocity fields into the ceramic plate. We represent these fields in terms of boundary conditions. The numerical simulations show that the amplitude ratio of the slow wave to the fast wave is strongly dependant on the boundary conditions (open or sealed).

#### 3.2.1 Open pores case

Figure 7 shows amplitudes of the fast (7-a) and the slow (7-b) waves for three frequencies (0.3 MHz, 1 MHz and 5 MHz). As expected, the wave potential is more attenuated as the frequency increases. The amplitude ratio of fast to slow waves is approximately 20, we see also that the amplitude decreases as the waves run through the plate, the slow wave attenuation being more significant than this of the fast one.

#### 3.2.2 Sealed pores case

In the case of sealed pores, we have same comments concerning the wave potentials attenuation, but in this case the amplitude ratio of fast to slow waves is approximately 200. As well, we can observe...
the mode conversion at lower (0.3MHz) and medium (1MHz) frequencies in the vicinity of the second interface figure 8-b.

![Figure 8: Fast (a) and slow (b) wave potential into sealed pores plate](image)

In conclusion, we can predict that the slow wave exist into the porous media in the two cases open or sealed pores, but it can not be transmitted in the case of sealed pores plate.

4 Experimental results

Experimental measurements were performed in a water tank, two similar transducers are used as an emitter and receiver, they are manufactured by PANAMETRICS. Our sample is a water saturated porous ceramic plate QF-20 whose parameters are reported in the table 1, we keep half of surface plate with open pores, and we spread cement over the second half to obtain the sealed pores condition, the thickness of plate is 25.8mm.

![Figure 9: Experimental setup (a), reference signal in water without sample (b)](image)

At first time we record the emitted signal in water reported in figure (9-b), this signal arrives at $t_0 = 186.757\mu s$. Then we place the plate in the water tank and we obtain the following signals according to open pores plate (fig.10-a) and sealed pores plate (fig.10-b).

From these signals, wave velocities are calculated using the relation: $v = \frac{e}{t_1-t_0+\frac{e}{c}}$, where $t_1$ is the arrival time of the transmitted wave which we look to identify. The transmitted signal through an open pores plate shows that the fast wave A arrives at $t_1 = 176.722\mu s$, a fast wave echo B arrives at $t_1 = 189.99\mu s$ followed by the slow wave C. Those waves have been identified in the theoretical part on the figure (3-b). Now we placed the sealed pores plate in the tank and the transmitted signal shows only the fast wave A and a fast wave echo B, those waves were identified in the theoretical part figure 5-b.
The experimental results are in good agreement with the numerical results which allows us to validate our model in open and sealed pores plate.

5 Application to a volume defect

As an application in this context, we consider a porous plate presenting a local inhomogeneity which can be considered as a volume defect. In this case, the analysis of the transmitted and reflected waves enables the characterization of the defect (size and localization). A model of the volume defect is represented as follows (fig.11):

![Defect Model](image)

The defect can be interpreted as a change of boundary conditions but inside the porous media leading to a fluid slab in between two porous regions. The interfaces $z = 0$ and $z = L$ are modeled by sealed boundary conditions, while we applied open boundary conditions to those of the defect. In this study the thickness of the plate is $L = 24mm$.

5.1 Influence of the defect size

Let’s consider following configurations where we change size of the defect while we keep its position in the middle of plate : i) plate without defect having $L = 24mm$, ii) plate with $3mm$ defect $[10.5mm − 3mm − 10.5mm]$ and iii) plate with $5mm$ defect $[9.5mm − 5mm − 9.5mm]$. The amplitude of the transmission coefficient is attenuated by the defect which introduces multiple scattering in the plate. The figure 12 shows the transmission coefficient (fig12-a) and reflection coefficient (fig12-b). The presence of the defect destroys the resonant peaks and shifts them to higher frequencies as the size of defect...
increases. Figure 12-b shows the reflection coefficient, we see that the presence of the defect increases its amplitude: this is due to the multiple scattering phenomena. By another way the number of resonants peaks is reduced and they are shifted toward the higher frequencies. This shift allows to localize the first interface of defect.

5.2 Influence of the defect position

In this part we keep the same size defect (3mm), and we change its position. We consider the following configurations : i) in the middle of plate [10.5mm − 3mm − 10.5mm], ii) [12.5mm − 3mm − 8.5mm], iii) [14.5mm − 3mm − 6.5mm].

The most important information is given by the reflection coefficient: we observe that more defect get closer to the second interface of plate (z = L), more reflection coefficient is shifted to lower frequencies. In this case we can determine position of defect.

6 Conclusion

By applying Biot’s theory to a ceramic plate having open and sealed interfaces, we have shown theoretically and experimentally that both fast and slow waves always exist inside the porous plate. However, for the ceramic porous plate considered, the slow wave is only visible on the transmitted signal when considering open interfaces. When considering a volume defect inside a porous plate having sealed pores,
numerical results show the influence of both size and position of the defect. Analysis of the reflection and transmission coefficients allows for characterization of the defect parameters which is helpful in a context of Non Destructive Testing of biphasic materials.

References


