Uncertainty and Error Analysis for NDE Performance Evaluation

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Background

Qualification of Non-Destructive Examination (NDE) techniques has been an integral part of the Periodic Inspection Program (PIP) for CANDU nuclear generating stations. The qualification of an inspection technique is an important evidence that demonstrates the NDE technique meets the capability required by the Canadian Standard Association standard CSA N285.4-05.

The limits of measurement uncertainties and/or errors are usually used as criteria to qualify a technique. For examples,

COG-JP-4027-V11, Steam Generator Tube Inspection Specification

The flaw sizing performance is specified by the standard error of regression, correlation coefficient, root-mean-square (RMS) error, and mean error.
Background

COG-JP-4027-V00-R01, Inspection Specification for CANDU Fuel Channels

The measurement accuracy requirement is specified/defined by a 2 Standard Deviation (2σ) value unless otherwise specified.

COG-JP-4059-V3, Feeder Pipes Wall Thickness and Cracking Inspection specification

The wall thickness is required to be measured with +/- 0.006 mm (2 standard deviations).

Measurement uncertainties and errors need to be evaluated for inspection techniques and it helps engineers to interpret measurement results.
Methodology

Guide to the Expression of Uncertainty in Measurement (GUM) published by the Joint Committee for Guides in Metrology (JCGM) has been widely accepted and supported by renowned international institutions:

- BIPM: International Bureau of Weights and Measures
- IEC: International Electrotechnical Commission
- IFCC: International Federation of Clinical Chemistry
- ISO: International Organization for Standardization
- IUPAC: International Union of Pure and Applied Chemistry
- OIML: International Organization of Legal Metrology
Methodology

The uncertainty analysis method, as is described in GUM, requires a measurement model relating a measurement result to the other quantities through a function relationship.

\[ y = f(x_1, x_2, \cdots, x_N) \]
Methodology

The standard deviation of measurement can be estimated from $M$ measurements of $y$

$$\hat{\sigma}(y) = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} (y_i - \bar{y})}$$

If $f(x_1, x_2, \ldots, x_N)$ is a linear function and the variation of $x_1, x_2, \ldots, x_N$ are very small around their means,

$$\hat{\sigma}(y) \approx \sqrt{\sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 (\sigma(x_i))^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \rho_{ij} \sigma(x_i) \sigma(x_j)}$$
Methodology

If \( x_1, x_2, \ldots, x_N \) are not correlated, the standard deviation is given by

\[
\hat{\sigma}(y) \approx \sqrt{\sum_{i=1}^{N} \left( \frac{df}{dx_i} \right)^2 (\sigma(x_i))^2}
\]

If the \( f \) is nonlinear, higher order terms of the Taylor series expansion must be included.

The GUM provides a systematic approach for uncertainty analysis.
Application of GUM

Although GUM provides a framework for assessing uncertainty, the application of uncertainty analysis methods based on GUM requires detailed knowledge on the influential parameters and the their effects on the final measurement result. The success of uncertainty analysis depends greatly on the understanding on the measurement and the effects of influential parameters.

It is often impractical to obtain the uncertainty by experimentally varying all of the influential parameters due to the time and cost constrains. Mathematical modelling and analysis may be used to overcome such a difficulty.
Application of GUM

The uncertainty components can be classified into Type A and Type B, two categories based on the method of evaluation.

**Type A:** The standard uncertainty is *obtained* from a probability density function from observed frequency distribution.

**Type B:** The standard uncertainty is obtained from an *assumed* probability density function based on the degree of belief that an event will occur.

The estimation methods for each uncertainty components should be selected based on each individual cases.
The advantage and disadvantage of Type A evaluation

*Type A* uncertainty analysis requires to **obtain** a probability density function from experiments.

**Advantage:** It does not require any priori knowledge on the probability density function.

**Disadvantage:** it requires experiments to be performed under many random conditions. A large amount of experiments may be required in order to accurately estimate a probability density function. When the effects of influential parameters are correlated, it may become practically very difficult do.
The advantage and disadvantage of Type B evaluation

**Type B** uncertainty analysis requires to **Assume** a probability density function based on belief.

**Advantage:** It does not require experiments which in some cases are difficult to implement. A probability density function can often be assumed correctly based on the knowledge of measurement and the influential parameters.

**Disadvantage:** It requires a thorough understanding on the measurement and all of the influential parameters. A mathematical measurement model is required.
The Propagation of Distributions

With the linear approximation of the measurement system, a measurement is a sum of contributions from each influential parameters.

If all of the influential parameters obeys Gaussian distributions, the measurement obeys a Gaussian distribution.

If some or all of the influential parameters do not obey Gaussian distributions, the distribution of measurement may be approximated by a Gaussian distribution due to the Central Limit Theorem.
The Propagation of Distributions

The Central Limit Theorem may not be valid when the major uncertainty components do not obey Gaussian distributions and dominate.

An simple example of such case is a UT wall thickness measurement when the time resolution is the dominate source of uncertainty.
The Propagation of Distributions

The wall thickness measurement error is given by

\[ \varepsilon_H = C(\varepsilon_2 - \varepsilon_1) \]

where \( \varepsilon_2 = t_2 - \bar{t}_2 \) and \( \varepsilon_1 = t_1 - \bar{t}_1 \).

If \( t_1 \) and \( t_2 \) have equal chances to be located at any position within a resolution cell, \( \varepsilon_1 \) and \( \varepsilon_2 \) obey rectangular distributions.

When \( \varepsilon_1 \) and \( \varepsilon_2 \) are statistically independent, it is well known that the combined distribution \( \varepsilon_H \) obeys a triangular distribution.
Application of Monte Carlo Methods

An analytical expression of a measurement distribution may not always be easily derived for situations where the measurement has complex relationships to the influential parameters.

The following example is presented to illustrate an application of Monte Carlo method to estimate the uncertainty of flaw depth measurement.

In steam generator UT inspection, a flaw depth measurement is commonly reported as the Through-Wall (TW) ratio.
Application of Monte Carlo Methods

The TW ratio is defined as the percentage of wall loss with respect to a full wall thickness. This ratio is usually determined from the remaining wall thickness and full wall thickness measurements as following:

\[ \alpha = 1 - \frac{x}{y} \]

where \( x \) is the remaining wall thickness and \( y \) is the full wall thickness. It can be shown that the error of a TW ratio is

\[ \varepsilon_\alpha = -\frac{1}{y} \varepsilon_x + \frac{x}{y^2} \varepsilon_y \]
Application of Monte Carlo Method

The full wall thickness measurement is not only affected by the time-of-flight measurement uncertainty but also by the wall thickness variations due to manufacture tolerance. We can decompose $\varepsilon_y$ into two components as following:

$$\varepsilon_y = \varepsilon_w + \varepsilon_T$$

where, $\varepsilon_w$ and $\varepsilon_T$ account for the errors due to time resolution for thickness measurement and manufacturing tolerance of wall thickness respectively. Since $\varepsilon_w$ and $\varepsilon_T$ are independent, the statistical distribution of $\varepsilon_y$ can be obtained by convolution. Instead of analytical derivation, we estimated the probability density distribution by using a Monte Carlo method.
Application of Monte Carlo Method

PDF for a 80% TW flaw with no wall thickness variation
Application of Monte Carlo Method

PDF for a 80% TW flaw with 5% wall thickness variation
Conclusions

- GUM provides a framework and general guides for uncertainty. It is well accepted by Metrology laboratories and can be used as a standard approach for NDE systems.

- The uncertainty analysis of an inspection system requires a thorough understanding of the system, detailed knowledge of all the influential parameters and their effects.

- Type A and B approaches may complement each other. An analysis approach needs to be selected on each parameter basis.

- For a complex system, the distribution of measurement is rarely possible to be obtained by experiment only. Type B evaluation should be applied whenever it is feasible.

- Monte Carlo methods are very useful to estimate the PDF.

- The current inspection specifications may not be complete in terms of defining the requirement for a measurement uncertainty. Two standard deviation does not necessarily always correlate to a 95% CL. Detailed PDF information is required.
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