Two-dimensional analysis of subharmonic ultrasound generation at closed cracks by damped double nodes

Kazushi YAMANAKA, Yohei SHINTAKU, Miyuki OGUMA and Yoshikazu OHARA
Tohoku University, Sendai, Japan
Phone: +81 22 795 7359, Fax: +81 22 795 4298; e-mail: yamanaka@material.tohoku.ac.jp

Abstract
Evaluation of closed cracks is the most serious issue for safety of important structures such as power plants or air planes. To overcome the problem, subharmonics in nonlinear ultrasound is particularly useful because of the excellent selectivity for closed cracks and high temporal resolution, which has been verified by the subharmonic phased array for crack evaluation (SPACE), developed by the authors. In this study, we propose the first two dimensional (2D) model to reproduce subharmonic generation at closed cracks using damped double nodes (DDN). Numerical simulation using finite difference time domain (FDTD) method was performed using DDN and time frequency analysis was performed based on the wavelet analysis. The results were able to reproduce essential features observed in the experiments.

Keywords: Ultrasonic Testing (UT), FDTD, Nonlinear, Double Node, Subharmonic, Closed Crack

1. Introduction

Ultrasonic testing is an effective method for evaluating cracks in structures. Recently, in particular, an ultrasonic phased array (PA) has become a promising technique. However, there is a problem of underestimation or overlook of fatigue and stress corrosion cracks in weld metals of atomic power plants because of crack closure. To solve the crack closure problem, nonlinear ultrasound [1]-[4] is the most promising. Here, analyses of super- and subharmonics are needed for both scientific understanding and the practical design of testing equipment. So far, it has been reported using a one-dimensional model [2]-[4]. Although two-dimensional (2D) analysis on subharmonic vibration of shallow delamination in a thin plate was reported [6], subharmonic generation at deep closed cracks in thick objects [5],[7] has not been realized, which is required for practical simulation, design and evaluation of testing equipment and conditions in UT.

In this paper, we describe 2D analysis of subharmonic generation at deep closed cracks using damped double nodes (DDN) [8] by the finite-difference time-domain (FDTD) method.

2. Formulation

The principle of nonlinear ultrasound is dictated in Fig. 1. It is based on the detection of nonlinear components, e.g., superharmonic waves (2f, 3f,..) or subharmonic waves (f/2, f/3,..) generated by the interaction of large-amplitude ultrasound with closed cracks[1]-[7]. We have developed a novel imaging method named subharmonic phased array for crack evaluation (SPACE) using short bursts of subharmonic waves and a phased array algorithm.

In 2D analysis of closed cracks [8], staggered grids (Fig. 2) are used for the calculation of particle velocities and stresses by the FDTD method. In Fig. 1, \( \dot{u} \) and \( \dot{w} \) are particle velocities in the \( x \) and \( z \) directions, \( T_s = T_{ss} = T_{zz} \) are the tensile stresses and \( T_s = T_{sc} \) is the shear stress. The fundamental equations for isotropic elastic solids are Hooke’s law and Newton’s second law of motion, and the calculation of FDTD method follows the standard approach such as described in ref. [9].
In the closed state, crack faces are represented by normal nodes. In the open state, the normal nodes are split into double nodes consisting of the displacement of incidence-side crack face \( \hat{w} \) and displacement of transmission-side crack face \( \hat{w}^+ \). To simulate closed crack faces with residual compression stress or strain, we introduce viscous damping into the double nodes. The transition between the open and closed states is defined using the following criterion.

\[
\Delta w = w^+ - w^-
\]

- If \( \Delta w > 0 \), the crack becomes open.
- If \( \Delta w < 0 \), the crack continues to be open.

Fig. 1 Principle of nonlinear ultrasound

Fig. 2 Proposed FDTD model of closed cracks using DDN
(1) At the closed state, a flag used in the computing program IClose is set to 1. The tensile stress of the node at \( z = j \) is calculated as the average of the stress of the upper and lower nodes of \( z = j - 1/2 \) and \( z = j + 1/2 \), such that
\[
T_{3M} = \frac{1}{2} (T_{3i+1/2}^{k-1} + T_{3i+1/2}^{k+1}) ,
\]

If \( T_{3M} \leq T_{th} \), the nodes remain close (a flag in the program IClose = 1) and if \( T_{3M} > T_{th} \), the nodes are opened (IClose=0), where \( T_{th} \) is the residual compression stress giving the threshold for transition.

(2) At the open state (IClose = 0), the particle velocity nodes \( \dot{w}^- \) and \( \dot{w}^+ \) have viscous damping proportional to the particle velocity difference between \( \dot{w}^- \) and \( \dot{w}^+ \) at \( z = j - 1/2 \) as well as \( \dot{w}^+ \) and \( \dot{w}^- \) at \( z = j + 1/2 \), so that
\[
\dot{w}^{+k+1}_{i+1/2} = \dot{w}^{+k}_{i+1/2} + 2V_{Pl}T_{3i+1/2}^{k} - \gamma(\dot{w}^{+k}_{i+1/2} - \dot{w}^{+k}_{i+1/2}) ,
\]
\[
\dot{w}^{-k+1}_{i+1/2} = \dot{w}^{-k}_{i+1/2} - 2V_{Pl}T_{3i+1/2}^{k} - \gamma(\dot{w}^{-k}_{i+1/2} - \dot{w}^{-k}_{i+1/2}) ,
\]
where \( \gamma \) is the damping coefficient, and \( V_{Pl} \) is the normalized longitudinal wave velocity.

The crack opening displacement (COD) is given as
\[
\Delta w^{+k+1}_{i+1/2} = w^{+k+1}_{i+1/2} - w^{-k+1}_{i+1/2} ,
\]
where \( w^{+k+1}_{i+1/2} \) and \( w^{-k+1}_{i+1/2} \) are displacements of the incidence side (-) and transmission side (+) crack faces, calculated as an integral of the particle velocities \( \dot{w}^- \) and \( \dot{w}^+ \), respectively.

If \( \Delta w^{+k+1}_{i+1/2} > 0 \), the crack remains open (IClose = 0), and if \( \Delta w^{+k+1}_{i+1/2} \leq 0 \), the crack is closed (IClose = 1).

Calculation was carried out with a time interval of 2 ns, and node interval of 0.02 mm; Gaussian envelope sine signal of about 8 carriers at 8 MHz was excited by a 32- element array at the surface, with the incident wave amplitudes at the crack face of 40 nm and 80 nm. The residual compression stress \( T_{th} \) was set to 100 MPa.

![32-element array](image)

**Fig. 3 Model for numerical examples**
Without the damping ($\gamma=0$), significant high-frequency noise appeared when the crack faces were separated from each other by overcoming the residual stress. The frequency range of the noise was around the second and third harmonic frequencies, disturbing the nonlinear analysis. However, the noises were suppressed when the damping coefficient $\gamma$ was set to 0.7 in the unit used in the program.

3. Numerical examples

Snapshot of displacement waveform is shown in Fig. 4, with the excitation amplitude of 40 nm (a) and 80 nm (b). In the 40 nm case, the wave is totally transmitted through the crack face. It is explained by the too small 40 nm amplitude of the incident wave to open the crack by overcoming the compression residual stress. Thus, the incident wave is completely transmitted and no reflection is observed as shown in the image of 300 grids (6 mm$^2$ square). This result correctly reproduces the dangerous situation of overlooking the crack.

However, the crack face was opened in the 80 nm case (b), and peculiar scattered wave was observed. It is explained by the large amplitude of 80 nm is enough to open the crack overcoming the compression residual stress. This result suggests the possibility of elimination of overlooking the crack.

![Fig. 4 Snap shot of FDTD simulation](image)

(a) 40 nm  
(b) 80 nm

The waveforms of $w^-$, $w^+$ and the crack opening displacement (COD) $w^+ - w^-$ for 80 nm excitation at the center of the crack face are plotted in Fig. 5 (a). Here, the COD is still zero in period A. However, in period B, $w^+$ is pushed up by $w^-$, and $w^+$ cannot follow $w^-$ owing to the inertia of the node. In periods C and D, $w^+$ is pushed up before returning to the equilibrium position. The detailed behaviors in B, C and D are different because the time of impact is different for each period. Thus, the COD is not zero and there are scattered waves as shown in the image. This example represents the fact that closed cracks are opened by a large-amplitude wave and could be selectively detected from open cracks.

Power spectra for the waveforms in Fig. (a) are shown in Fig 5 (b). In all the spectra, super harmonic component at $2f$ is observed. Moreover, subharmonic component at $f/2$ is clearly observed particularly in the spectre for $w^+$. 
To investigate the nonlinear response of the cracks in time domain, we performed the time-frequency wavelet analysis. Fig. 6 shows wavelet time-frequency images in the normal incidence. Subharmonic generation caused at the frequency of $f/2$ is clearly observed for $w^+$. In the case of oblique incidence shown in Fig. 3 (b), results similar to those in the case of normal incidence shown in Figs. 5 and 6 were obtained as shown in Fig. 7. However, a clearer subharmonic generation is observed in the case of oblique incidence than in the normal incidence case.
4. Conclusions

We described the first two-dimensional model to reproduce subharmonic generation at closed cracks using damped double nodes (DDNs).

Numerical simulation using finite-difference time-domain (FDTD) method was performed and successfully compared with experimental waveforms and time–frequency wavelet analysis. The results were able to reproduce essential features observed in the experiments. The DDN model will be useful in designing and evaluating testing condition and probes. The physical origin of the damping might include the energy dissipation due to adhesion and/or friction between the crack faces. However, it is a subject of further study.

References