Vibration-Based Damage Detection for a Population of Like Structures 
via a Multiple Model Framework

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Abstract
This study focuses on the problem of vibration-based damage detection for a population of like structures. Although nominally identical, like structures exhibit variability in their characteristics due to variability in the materials and manufacturing. This inevitably leads to variability in the dynamics, which may be so significant as to mask deviations due to damage. Damage detection, conventionally employing a single nominal (baseline) model for the population, thus becomes challenging, with available methods exhibiting inadequate performance. The use of a new, Multiple Model (MM) framework, is presently explored for the potential alleviation of the problem. Within it, four response-only methods are considered and exhaustively tested via a population of laboratory-scale like composite beams and delamination type damage. The results illustrate the improvement in performance attained within the postulated framework, with one parametric method achieving impressive performance characteristics. The general conclusion is that the MM framework offers some new and exciting possibilities for effectively tackling the problem of damage detection for a population of like structures.

Keywords: Damage detection, vibration-based methods, material & manufacturing variability, population of like structures, uncertainties

1. Introduction

Vibration-based methods for damage detection have been widely used in a multitude of cases [1], as vibration is often naturally arising, while the measurement and processing of vibration signals is technically mature. There are two main families of vibration-based damage detection methods, depending upon the type of model employed: Those based on detailed analytical models, such as Finite Element Models [2], and those based on data-based models [3-5]. In addition, depending upon whether the excitation is measurable or not, vibration based damage detection methods may be further classified as either excitation-response or response-only (output-only). The latter case applies to many situations where the excitation is ambient and difficult to measure.

The present study focuses on the problem of vibration-based damage detection for a population of like structures. Although nominally identical, like structures exhibit variability in their characteristics due to variability in the materials and manufacturing. This typically leads to variability in the dynamics, which may be so significant as to mask deviations due to damage. Damage detection, conventionally employing a single nominal (baseline) model for the whole population, thus becomes challenging, with available methods exhibiting inadequate performance.

Unlike other uncertainty factors, like those associated with environmental and operational variability [6], the variability associated with damage detection in a population of like structures appears essentially unexplored in the current literature. It was recently posed in a study by the present authors [7], where conventional vibration-based, response-only, statistical time series methods were employed for damage detection in a population of 26 composite beams using a multitude of inspection (test) cases.
The results of the study have served to illustrate the poor performance of the various methods in this context. One parametric method, namely an AR parameter based method, exhibited good performance and robustness. Yet, a subsequent, more detailed, analysis with many more inspection cases (produced by rotating the experiment selected as baseline and representing the healthy state) has revealed that performance is very much dependent on the selected baseline model, and may degrade significantly as this selection is altered.

The present study thus focuses on a new, Multiple Model (MM) framework, recently introduced by the second author and a co-worker [8], which is explored for the potential alleviation of the problem of damage detection for a population of like structures. The main idea behind this framework is on the use of several, rather than one, models for more effectively representing the population's nominal (healthy) structural dynamics. Four response-only methods are considered within this framework, and are exhaustively tested via the population of laboratory-scale composite beams and delamination type damage employed in [7]. Unlike [7] though, the evaluation of the methods is now exhaustive, in the sense that the baseline model(s) is(are) not uniquely selected and employed for all inspection cases, but is(are) instead rotating among available healthy beam models. This eliminates the effects of baseline selection, plus leads to a drastically higher number of inspection cases to assess the methods with.

The rest of this article is organized as follows: The experimental set-up and the damage scenarios are briefly reviewed in section 2, the Multiple Model (MM) damage detection framework is elaborated upon in section 3, damage detection performance is explored in section 4, and the conclusions are finally summarized in section 5.

2. The experimental set-up and the damage scenarios

The experimental set-up & damage scenarios are described in detail in reference [7]; only a brief summary is provided here for completeness of presentation. The study is based on 26, nominally identical, composite beams, consisting of several layers of woven and unidirectional (UD) fabric. Each beam (of dimensions LxWxH 600x65x65mm, wall thickness 3mm, corner radius 8mm) represents the topology of a commercial UAV boom. In the experimental setup each beam is tightly clamped at one end, simulating its connection to the fuselage, while its free end is attached to an aluminium mass representing the aircraft tail (Figure 1). The beam is excited at its free end with a random, approximately white, Gaussian
force applied vertically at Point X via an electromechanical shaker, while vibration acceleration responses are measured at points Y1 and Y2 via lightweight accelerometers.

Delamination type damage is injected in 3 of the 26 beams at Point D via a pendulum type impact hammer using a distinct impact energy level for each; the corresponding damages are referred to as Damage A, B, C, respectively (see Table 1). Welch-based FRF magnitude estimates for the healthy and damaged beams, revealing significant variability in the dynamics of the healthy beams, as well as changes due to damage, are presented in Figure 2.

### Table 1. The damage scenarios and experimental details

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Description</th>
<th>Number of beams</th>
<th>Number of experiments per beam</th>
<th>Total number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>---</td>
<td>23</td>
<td>7</td>
<td>161</td>
</tr>
<tr>
<td>Damage A</td>
<td>Impact (5J)</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage B</td>
<td>Impact (10J)</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage C</td>
<td>Impact (15J)</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Sampling frequency $f_s = 4654.5$ Hz; signal bandwidth $[3 - 2327.25]$ Hz

![Figure 2. Welch-based FRF magnitude estimates for healthy and damaged beams: (a) Five healthy composite beams (single experiment per beam); (b) 23 healthy and 3 damaged beams (7 experiments per beam) [7].](image)

### 3. The Multiple Model (MM) damage detection framework

The Multiple Model framework [8] is postulated to address damage detection under significant variability – presently variability associated with a population of like structures. The main idea behind it is on the use of several models, instead of the conventional single model, for representing the nominal (healthy) structural dynamics. Like with all vibration-based methods [3-4], the framework includes two phases: (a) The baseline (training) phase, and (b) the inspection (damage detection) phase.
**Baseline phase.** In this phase (non-parametric/parametric) modeling of the healthy structural dynamics is carried out based on vibration signals from a number of like healthy structures. Let \( v \) be the number of healthy structures used and \( Y_o = \{ y_{o,k}[t] \} \) the respective response signals with \( t = 1, \ldots, N \) and \( k = 1, \ldots, v \) (the subscript \( o \) is used to designate the healthy state). Employing the available signals, a set of models \( m_o = \{ m_{o,1}, \ldots, m_{o,v} \} \) are obtained for representing the population’s healthy dynamics.

**Inspection phase.** In this phase an inspection (test) case is considered, with vibration response signal(s) obtained from the current structure under inspection (thus in unknown structural state). The objective is to decide whether or not the current structure is healthy.

For doing so, let \( Y_u[t] \) designate the current response signal(s) and \( m_u \) the respective model (the subscript \( u \) is used to designate unknown state). Then a proper distance metric in the method’s characteristic quantity space (the characteristic quantity being designated as \( Q [3] \)) is used, and the distances between the model \( m_u \) and each one of the models \( m_{o,1}, \ldots, m_{o,v} \) used as baseline are obtained. Two distance metrics are presently employed, (a) the Itakura-Saito (IS) distance and (b) the Kullback-Leibler (KL) divergence.

The IS distance [9] is employed in conjunction with non-parametric models, and is defined as:

\[
d(Q_u, Q_{o,k}) = d_{IS}(Q_u, Q_{o,k}) = \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \left[ \frac{Q_u(\omega)}{Q_{o,k}(\omega)} - \log \frac{Q_u(\omega)}{Q_{o,k}(\omega)} - 1 \right] d\omega
\]  

(1)

with \( Q_u(\omega), Q_{o,k}(\omega) \) designating the characteristic quantity for the non-parametric models \( m_u, m_{o,k} \), respectively \( (k = 1, \ldots, v) \), which now is a function of frequency \( \omega \) (for instance the power spectral density or transmittance). In the above \( T \) stands for the sampling period. The KL divergence [10] is employed in conjunction with parametric models, and for two quantities, \( Q_u, Q_{o,k} \), each following \( l \)-dimensional Gaussian distribution with means \( \mu_u, \mu_o \) and covariance matrices \( \Sigma_{Q_u}, \Sigma_{Q_{o,k}} \), respectively, is defined as (capital/lower case bold face symbols represent matrix/vector quantities):

\[
d(Q_u, Q_{o,k}) = d_{KL}(Q_u, Q_{o,k}) = \frac{1}{2} \left( \text{tr} \left( \Sigma_{Q_u}^{-1} \Sigma_{Q_{o,k}} \right) + (\mu_u - \mu_o)^T \Sigma_{Q_u}^{-1} (\mu_u - \mu_o) - \ln \left( \frac{\det \Sigma_{Q_u}}{\det \Sigma_{Q_{o,k}}} \right) \right)
\]  

(2)

with \( \text{tr}(\cdot) \) designating trace and \( \det(\cdot) \) determinant of the indicated quantity.

Once the distances between the current model and those of the baseline are available, damage detection is based on the following decision making mechanism that takes all distances to the baseline models into account through an overall distance metric \( D \):

\[
D = \sum_{k} d(Q_u, Q_{o,k}) \leq l_{\text{lim}} \quad \rightarrow \quad \text{Healthy structure}
\]

\[
\text{Otherwise} \quad \rightarrow \quad \text{Damaged structure}
\]  

(3)

with \( l_{\text{lim}} \) designating a user defined threshold. Four damage detection methods, two non-parametric and two parametric, are presently formulated within this MM framework. They are briefly described in the sequel (also see [3] for details) and are summarized in Table 2.
Table 2. The formulated damage detection methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Characteristic quantity</th>
<th>Distance metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-parametric PSD based</td>
<td>PSD</td>
<td>Itakura-Saito</td>
</tr>
<tr>
<td>AR parameter based</td>
<td>AR parameter vector</td>
<td>Kullback-Leibler</td>
</tr>
<tr>
<td>Non-parametric transmittance based</td>
<td>Transmittance magnitude</td>
<td>Itakura-Saito</td>
</tr>
<tr>
<td>ARX (transmittance) parameter based</td>
<td>ARX parameter vector</td>
<td>Kullback-Leibler</td>
</tr>
</tbody>
</table>

3.1 Non-parametric PSD based method

The method employs the Power Spectral Density (PSD) of the vibration response signal as its characteristic quantity \((Q = S(\omega))\); estimation is based on the Welch method [11, p 76].

3.2 Parametric AR parameter based method

The method is based on parametric AutoRegressive (AR) modeling of the vibration response signal. The model form is [12]:

\[
y(t) + \sum_{i=1}^{n_a} a_i y(t - i) = e(t), \quad e(t) \sim NID(0, \sigma_e^2)
\]  

(5)

with \(t = 1, ..., N\) designating normalized discrete time, \(y(t)\) the response signal, \(n_a\) the model order, \(a_i\) the \(i\)-th AR parameter, and \(e(t)\) the model residual that is a white Gaussian zero-mean with variance \(\sigma_e^2\) sequence. The method’s characteristic quantity is the AR model parameter vector \((Q = \theta)\) [3]. Estimation is based on the Least Squares (LS) method.

3.3 Non-parametric transmittance based method

The method is based on the transmittance function which is defined as [13]:

\[
T_{lm}(j\omega) = \frac{S_{ml}(j\omega)}{S_{ll}(\omega)}
\]

(6)

with \(j\) designating the imaginary unit, the subscripts \(l, m\) the corresponding measurement points on the structure, \(S_{ml}(j\omega)\) the Cross Spectral Density (CSD) between signals obtained at points \(l\) and \(m\), and \(S_{ll}(\omega)\) the PSD of the vibration response at point \(l\). In this case the transmittance is obtained via non-parametric Welch-based CFD and PSD estimators. The method’s characteristic quantity is the transmittance function magnitude \((Q = |T(j\omega)|)\).

3.4 Parametric ARX (transmittance) parameter based method

In this method the transmittance is represented via an AutoRegressive with eXogenous excitation (ARX) model relating (as excitation and response) the vibration responses at two points on the structure. The ARX model form is [12]:

\[
y_m(t) + \sum_{i=1}^{n_a} a_i y_m(t - i) = \sum_{i=0}^{n_h} b_i y(t - i) + e(t), \quad e(t) \sim NID(0, \sigma_e^2)
\]

(7)
with $t = 1, ..., N$ designating normalized discrete time, $y_m[t], y_l[t]$ the response signals from measurement points $m$ and $l$, respectively, $n_a, n_p$ the model orders, $a_i, b_i$ the $i$-th AR and X parameter, respectively, and $e[t]$ the model residual that is a white Gaussian zero-mean with variance $\sigma^2_e$ sequence. The method’s characteristic quantity is the ARX model parameter vector ($Q = \theta$), while estimation is based on the Least Squares (LS) algorithm.

4. Damage detection performance

The estimation of the non-parametric models (PSD and transmittance function) is based on $N = 112\,000$ sample long ($\approx 24\,s$) vibration response signals (Table 3). The vibration response signals acquired from measurement points $Y1$ and $Y2$ are used in the non-parametric transmittance function and parametric ARX (transmittance) parameter based methods, while the signal from point $Y2$ is used in the non-parametric PSD based and parametric AR parameter based methods (see Figure 1). Estimation details are summarized in Table 3.

<table>
<thead>
<tr>
<th>Table 3. Non-parametric and parametric model estimation details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-parametric methods</strong></td>
</tr>
<tr>
<td>Characteristic quantity</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>PSD</td>
</tr>
<tr>
<td>Transmittance</td>
</tr>
<tr>
<td><strong>Parametric methods</strong></td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>AR</td>
</tr>
<tr>
<td>ARX (transm.)</td>
</tr>
</tbody>
</table>

MATLAB functions: pwelch.m (PSD), tfestimate.m (transmittance)

On the other hand, the identification of the parametric AR and ARX models is based on $N = 10\,000$ sample long ($\approx 2.15\,s$) data records. Model estimation is based on Ordinary Least Squares with QR implementation [14, pp 318-320], while the model order selection is based on the Bayesian Information Criterion (BIC) and the Residual Sum of Squares / Signal Sum of Squares (RSS/SSS) [14, pp 498-514]. The model orders obtained and universally used for all models are indicated in Table 3, along with certain model performance characteristics (these are for individual baseline models).

Damage detection, for an increasing number of baseline models ($v = \{1, 6, 8, 10, 12\}$; each corresponding to a distinct healthy beam) is considered for each method. The inspection cases considered correspond to experiments with beams not used in the baseline phase. By rotating the beams used in the baseline phase, a high number of inspection cases is generated for testing (in a reliable and representative way) the effectiveness of each method. For each
number of baseline beams considered, the number of inspection beams, the inspection experiments, and the number of inspection cases used are summarized in Table 4.

Table 4. Inspection beams, inspection experiments, & inspection cases for various values of $\nu$.

<table>
<thead>
<tr>
<th>No. of baseline beams ($\nu$)</th>
<th>No. of inspection beams</th>
<th>Inspection experiments</th>
<th>Inspection cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>175</td>
<td>4025</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>140</td>
<td>7000*</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>126</td>
<td>6300*</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>112</td>
<td>5600*</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>98</td>
<td>4900*</td>
</tr>
</tbody>
</table>

* 50 rotations are employed.

Indicative damage detection for $\nu = \{1,12\}$ and many inspection cases are, for the non-parametric PSD based method, presented in Figure 3. For each inspection case, the distance metric $D$ from the current model to the baseline model set is provided. Evidently, for $\nu = 1$ (which corresponds to the conventional approach) several distances related to damaged beams are lower than those related to their healthy counterparts, thus rendering damage detection ineffective. The situation improves as $\nu$ is increased to 12 (the MM framework is thus used), but the problems persist. Similar remarks apply to the non-parametric transmittance based method (Figure 4).

Indicative damage detection for $\nu = \{1,12\}$ and many inspection cases are, for the AR parameter based method, presented in Figure 5. Similar remarks apply to the case of $\nu = 1$, yet, for $\nu = 12$ the improvement is dramatic. Indeed, the distances of damaged inspection beams are now clearly higher than their healthy counterparts, rendering detection very effective! Unfortunately, this is not the case with the last, ARX (transmittance) parameter based method, where performance remains inadequate (Figure 6). It is conjectured that this is probably due to the nature and properties of the transmittance function itself.

Potential damage detection performance for all four methods is (for various values of $\nu$) summarized in Figure 7 in terms of Receiver Operating Characteristic (ROC) curves [15, pp 34-35]. These depict the true positive rate (percentage of correct damage detections) versus the false positive rate (percentage of false alarms) for varying decision threshold. Based on them, the observations made earlier are confirmed: the potential improvement when moving from the conventional ($\nu = 1$) to the Multiple Model (MM) ($\nu > 1$) approach is evident for all methods; yet only the AR parameter based method achieves excellent performance!

5. Conclusions

The problem of vibration-based response-only damage detection for a population of like structures has been addressed. The inadequacy of conventional methods based on a single baseline model ($\nu = 1$) has been shown, while the benefits of a Multiple Model (MM) ($\nu > 1$) framework have been experimentally demonstrated. Among the four response-only methods examined within this framework, the AR parameter based method has provided excellent performance. This is in line with previous broad studies indicating the improved performance of parametric methods. Yet, the inferior performance of the ARX parameter based method needs further investigation. The general conclusion of the study is that the MM framework offers some new and exciting possibilities for effectively tackling the problem of damage detection for a population of like structures.
Acknowledgements

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Figure 5. Damage detection by the parametric AR model parameter based method: Kullback-Leibler distance $D$ for each inspection case. (a) Single-model baseline (4 025 inspection cases); (b) 12-model baseline (4 900 inspection cases).

Figure 6. Damage detection by the parametric ARX (transmittance) parameter based method: Kullback-Leibler distance $D$ for each inspection case. (a) Single-model baseline (4 025 inspection cases); (b) 12-model baseline (4 900 inspection cases).

References


