Stationary or Non-Stationary Random Excitation for Vibration-Based Structural Damage Detection? An exploratory study

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Abstract
In many vibration-based structural damage detection cases where the excitation is user specified, stationary random signals (typically white noise) are chosen as excitation. The question explored in this study is whether or not the use of non-stationary random excitation may lead to improved damage detectability. The latter is expressed as a distance, in the space defined via the detection method's characteristic quantity, between the damaged and healthy structural states. For exploring this issue, a parametric statistical damage detection method is employed, with the model's Power Spectral Density (PSD) constituting the characteristic quantity. The use of conventional stationary white noise excitation is contrasted to non-stationary excitation produced via a Time-dependent AutoRegressive Moving Average (TARMA) model. A simulated structure, derived from experimental signals obtained from a laboratory composite beam, is employed under two structural states: healthy and damaged (delamination). The results of the study, based on Monte Carlo simulations, interestingly suggest that non-stationary excitation may potentially lead to improved damage detectability.

Keywords: Vibration-based damage detection, non-stationary random vibration, damage detectability, statistical methods

1. Introduction

Vibration-based Structural Health Monitoring (SHM) uses vibration signals to diagnose structural damage at an early stage. The fundamental principle upon which vibration-based methods are founded is that small changes (damage or incipient damage) in a structure cause discrepancies in its vibration response which may be detected and associated with a specific cause [1,2,3]. Measured vibration signals may be then used in order to detect, identify/localize and quantify changes in the underlying structural dynamics which are attributed to damage.

In many SHM cases where the excitation is user specified, stationary random signals (typically white noise) are chosen as excitation. The question explored in this study is whether or not the use of non-stationary random excitation may lead to potential improvements in damage detectability.

It should be noted that ambient (not user specified) excitation is sometimes non-stationary; consider for instance civil infrastructure under seismic excitation, traffic-induced bridge vibration, and so on. In such a framework the question posed is still of interest, but mostly of theoretical rather than practical importance, and concerns whether or not the non-stationary nature of the excitation facilitates damage detection. Of course, this may change and practicality may be restored in case supplemental, user-specified, excitation is provided.

The underlying idea behind the question posed lies with the fact that non-stationary excitation is characterized by richer - compared to its stationary counterpart - structure and more "complex" properties. Indeed, in contrast to the stationary case, its AutoCovariance Function (ACF) and Power Spectral Density (PSD) function are time-dependent [4,5] (the mean may be time-dependent as well, but this is not presently considered), so that the excitation characteristics are not constant but vary with time. The structure is thus excited via different ways over time, and the question explored is whether this richer structure and time-dependent properties of non-stationary excitation may lead to potentially better damage detectability.
In exploring this, a parametric statistical time series \[3\] damage detection method of the output-only type is employed in the present study, with the model's Power Spectral Density (PSD) constituting the characteristic quantity. The use of conventional stationary white noise excitation is contrasted to non-stationary excitation produced via a Time-dependent AutoRegressive Moving Average (TARMA) model \[4,5\]. A simulated structure, derived from experimental signals obtained from a laboratory-scale composite beam simulating the boom in an unmanned aerial vehicle, is used under two structural states: healthy and delamination-type damage. Damage detectability is expressed as a distance, in the space defined via the method's characteristic quantity (presently the model based PSD), between the healthy and damaged structural states. The idea behind this definition is that the higher the distance, the better the discrepancy between the two structural states and thus damage detection ability.

The rest of the article is organized as follows: The methodology is presented in section 2, Monte Carlo damage detectability results are presented in section 3, and the conclusions are summarized in section 4.

2. The methodology

Output-only damage detection is based on a statistical time series parametric method which models the stationary or non-stationary random vibration response signal via a stationary AutoRegressive (AR) or non-stationary Recursive AR (RAR), respectively, model. The method's characteristic quantity is the obtained model's PSD \(S(\omega)\), or Time Varying PSD (TV-PSD) \(S(\omega, t)\), respectively, where \(\omega\) designates frequency and \(t\) normalized discrete time. Thus, when a structure is monitored for potential damage, the latter is detected if and only if the currently obtained model-based PSD (resp. TV-PSD) is significantly deviating from that of the healthy structure (see \[1,3\] for further details).

A non-stationary RAR model representing the non-stationary random vibration response is of the form \[5\]:

\[
y[t] + \sum_{i=1}^{na} a_i[t] \cdot y[t - i] = e[t], \quad e[t] \sim NID(0, \sigma_e^2[t])
\]

with \(y[t]\) representing the vibration response signal modeled, \(e[t]\) a non-stationary, zero-mean, uncorrelated (innovations) signal with variance \(\sigma_e^2[t]\), \(a_i[t]\) the model's time-dependent AR parameters, and \(na\) the model (AR) order. \(NID(\cdot, \cdot)\) stands for Normally Independently Distributed with the indicated mean and variance. In this RAR model form the parameters do not assume any particular time-evolution, but may evolve freely with time. Clearly, the conventional, stationary, AR model is obtained as a special case of the above if the time dependency of the AR parameters and innovations variance is dropped \[6, p 33\].

The RAR model is estimated via the Recursive Least Squares (RLS) algorithm with a forgetting factor \(\lambda\) \[7, p 305\], with three sequential passes over the signal (forward-backward-forward) in order to attenuate the effects of arbitrary initial conditions \[5\].

Once a RAR models is estimated, its "frozen"-type TV-PSD \[5\] is obtained as:

\[
S_F(\omega, t) = \frac{1}{1 + \sum_{i=1}^{na} a_i[t] \cdot e^{-j\omega t_i}} \cdot \sigma_e^2[t]
\]
after substituting the TV AR parameters and innovations variance by their respective estimates. In the above $T_s$ designates the sampling period, $j$ the imaginary unit, and $|\cdot|$ complex magnitude. Like before, the corresponding stationary AR model PSD is obtained by dropping the time dependency.

As already mentioned, damage detection is based on the detection of significant deviation between the currently obtained (estimated) PSD (resp. TV-PSD) $S(\omega, t)$ and that of the healthy structure $S_0(\omega, t)$. The Euclidean distance [8] between the two PSDs (TV-PSDs):

$$d(t) = d(S_0(\omega, t), S(\omega, t)) = \|S_0(\omega, t) - S(\omega, t)\|_{2\omega}$$

is used, with $\|\cdot\|_{2\omega}$ designating $\omega$-based $l_2$ norm. Like before, $S(\omega, t)$ should be changed into $S(\omega)$ in the stationary case. This distance is used as a damage detectability measure in the sequel, as the higher it is the clearer the damage detection.

3. Simulated Monte Carlo damage detectability results

In this section damage detectability is assessed for both stationary and non-stationary excitation using Monte Carlo simulations on a structural simulation model.

3.1 The structural simulation model

The Monte Carlo simulations are based on a structural simulation model derived from experimental signals obtained from a laboratory composite beam; more details in [9]. Two structural states are presently employed: healthy and delamination type damage, the latter induced via impact using a pendulum type impact hammer [9].

The simulation model involves a single force excitation and a single acceleration vibration response (details in [9]), thus an AutoRegressive with eXogenous (ARX) type is employed. A common structure ARX(53,53) model (that is of AR and X orders equal to 53) suffices for both structural states - its parameters and innovations variance being of course different for each state. No noise is added in the simulations. The dynamics of the structural (ARX) simulation model are depicted in Figure 1, where the respective (healthy and damaged) excitation-response Frequency Response Function (FRF) magnitude curves are shown. The delamination effects are indeed evident in the curves, expectedly being more pronounced in the higher frequency range.

Figure 1. ARX-based FRF magnitude for the healthy and damaged states of the beam.
3.2 The stationary and non-stationary force excitation signals

Gaussian force excitation is employed in both the stationary and non-stationary cases. In the stationary case it is conventional broadband (white) noise, the advantage of which is uniform excitation over the whole frequency range.

In the non-stationary case, colored excitation obtained via a non-stationary Time-dependent AutoRegressive Moving Average (TARMA) model is used. This model is of the form [5]:

$$x[t] + \sum_{i=1}^{nb} b_i[t] \cdot x[t-i] = w[t] + \sum_{i=1}^{nc} c_i[t] \cdot w[t-i], \quad w[t] \sim NID(0, \sigma_w^2) \quad (4)$$

with $x[t]$ designating the force excitation signal produced, $w[t]$ a stationary, zero-mean, uncorrelated (innovations) signal with variance $\sigma_w^2$, $b_i[t]$, $c_i[t]$ the model's time-dependent AR and MA parameters, respectively, and $nb, nc$ the AR and MA orders. The selected force excitation TARMA model details are summarized in Table 1, while a resulting force signal realization is shown in Figure 2 and its "frozen" TV-PSD in Figure 3.

Table 1. Non-stationary excitation signal characteristics

<table>
<thead>
<tr>
<th>Sampling frequency</th>
<th>$f_s = 4654.5$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal length</td>
<td>10.7423 s (N = 50 000 samples)</td>
</tr>
<tr>
<td>Model innovations</td>
<td>Gaussian, zero mean, $\sigma_w^2 = 1.25$</td>
</tr>
<tr>
<td>Model</td>
<td>TARMA(6,4)</td>
</tr>
<tr>
<td>AR characteristics</td>
<td>$\omega_{n1}[t]=2 \cdot \pi \cdot f_s \cdot (0.08+0.05 \cdot \sin((12 \cdot \pi/N) \cdot t+\pi/2))$</td>
</tr>
<tr>
<td></td>
<td>$\omega_{n2}[t]=2 \cdot \pi \cdot f_s \cdot (0.5+0.05 \cdot \sin((16 \cdot \pi/N) \cdot t+3\pi/2))$</td>
</tr>
<tr>
<td></td>
<td>$\omega_{n3}[t]=2 \cdot \pi \cdot f_s \cdot (0.8+0.05 \cdot \sin((24 \cdot \pi/N) \cdot t+\pi/4))$</td>
</tr>
<tr>
<td>Damping ratios: $\zeta_1 = 0.05$, $\zeta_2 = 0.05$, $\zeta_3 = 0.05$</td>
<td></td>
</tr>
<tr>
<td>MA characteristics</td>
<td>$\omega_{n1}[t]=2 \cdot \pi \cdot f_s \cdot (0.7+0.2 \cdot \sin(2 \cdot \pi/N \cdot t))$</td>
</tr>
<tr>
<td></td>
<td>$\omega_{n2}[t]=2 \cdot \pi \cdot f_s \cdot (0.2+0.2 \cdot \sin(4 \cdot \pi/N \cdot t))$</td>
</tr>
<tr>
<td>Damping ratios: $\zeta_1=0.2$, $\zeta_2=0.2$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Sample realization of the synthesized non-stationary force excitation.
3.3 Stationary and non-stationary modeling of the random vibration response

Stationary case. In the stationary case the vibration acceleration response signal is modeled via an AR representation. Various model orders are considered using the RSS/SSS and BIC criteria [10], with the analysis leading to an AR(62) model (single realization case presented in Figure 4). Estimation details are provided in Table 2, while AR(62) model based PSD curves for the healthy and damaged cases are (for a single realization) shown in Figure 5. The discrepancies between the healthy and damaged PSDs are evident, in particular in the higher frequency range.

Table 2. AR modeling of the stationary random vibration response signal

<table>
<thead>
<tr>
<th>Method</th>
<th>Implementation Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR modeling</td>
<td>Model order search: find best BIC, RSS/SSS for nb = 20, ..., 140</td>
</tr>
<tr>
<td></td>
<td>Estimation: Least squares (MATLAB arx.m)</td>
</tr>
<tr>
<td>Model</td>
<td>Performance</td>
</tr>
<tr>
<td>AR(62)</td>
<td>RSS/SSS (%)</td>
</tr>
<tr>
<td></td>
<td>29.7945</td>
</tr>
</tbody>
</table>

*Samples Per Parameter
Non-Stationary case. In the non-stationary case the vibration acceleration response signal is modeled via a RAR representation. Various model orders and forgetting factors are considered using the BIC and RSS/SSS criteria [10], with the analysis leading to a RAR(68) model with forgetting factor $\lambda=0.998$ (Figure 6). Estimation details are provided in Table 3. A contour plot of the RAR(68) model based "frozen” TV-PSD is presented, for both the healthy and damaged states of the structure, in Figure 7. These plots are rich in details and exhibit discrepancies between the two structural states, especially in the higher frequency range.

<table>
<thead>
<tr>
<th>Method</th>
<th>Implementation Details</th>
</tr>
</thead>
</table>
| RAR modelling| Model order search: find best BIC, RSS/SSS for $n_a=40,...,80$ and forgetting factor $\lambda=0.92:0.001:0.999$  
Estimation: Recursive Least Squares (MATLAB rarx.m), initial parameter vector 0, initial covariance vector $10^6$I |
| Model        | Forgetting factor $\lambda=0.998$                                                      | Performance |
| RAR (68)     |                                                                                       | RSS/SSS (%) | BIC  | SPP*  |
|              |                                                                                       | 9.604       | 8.385 | 144.92 |

*Samples Per Parameter
3.4 Damage detectability assessment via Monte Carlo simulations

As indicated, damage detectability is quantified via the spectral distance of Eq. (3). Two pairs (stationary and non-stationary) of single realization results are presented in Figures 8 and 9. In each, the employed non-stationary force excitation signal is also shown. In both cases the distance obtained in the non-stationary excitation case is, for most time instants, higher than that obtained in the stationary case. It is also interesting that, in both cases, the non-stationary distance is highest at those time instants for which the excitation exhibits high volatility.

Multiple (50) realization, Monte Carlo, results are then presented in Figure 10. Likewise, higher (for most time instants) spectral distances between the healthy and the damaged states are observed in the non-stationary excitation case. The results thus indicate that, indeed, non-stationary excitation may potentially lead to improved damage detectability.
Figure 7. RAR(68) model based "frozen" TV-PSD of the non-stationary random vibration response signal: (a) Healthy state, (b) damaged state (single realization).

Figure 8. Spectral distance between the healthy and damaged structural states in the stationary (red) and non-stationary excitation (green) cases (single realization). The non-stationary force excitation is also depicted in grey (single realization).

Figure 9. Spectral distance between the healthy and damaged structural states in the stationary (red) and non-stationary excitation (green) cases. The non-stationary force excitation is also depicted in grey (single realization).
4. Conclusions

The results of this exploratory study indicate that while both stationary and non-stationary excitation may be used for vibration-based damage detection, non-stationary excitation may potentially lead to improved damage detectability. This is an important finding, that may open up some interesting possibilities, and also raise some important questions. Yet, it should not be misinterpreted to mean that any non-stationary is always better than a stationary one for damage detection. This study has only served to provide an indication on potentially improved detectability. It is clear that any kind of generalization will require further work that is far outside the scope of an exploratory study.

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References


